Research Journal of Applied Sciences, Engineering and Technology 15(6): 212-218, 2018
DOI:10.19026/rjaset.15.5860
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: December 1, 2017 Accepted: January 25, 2018 Published: June 15, 2018

Research Article

Curve Variations in Non-Stationary Three-Point Subdivision Schemes

[1]Kashif Rehan and [2]Waqas Ali Tanveer
[1]Department of Mathematics, University of Engineering and Technology, KSK Campus,
[2]Department of Mathematics and Statistics, University of Lahore, Lahore, Pakistan

Abstract: Subdivision schemes are acknowledged as an important tool in computer aided geometric design. The new binary non-stationary three-point approximating subdivision schemes have been proposed that generate wide variations of C^1 and C^2 continuous curves using shape control parameter ξ^0. The proposed schemes are the counterpart of stationary schemes introduced by Hormann and Sabin (2008) and Siddiqi and Ahmad (2007). Curve variations using the shape control parameter ξ^0 have been demonstrated by the several examples.

Keywords: Approximating, binary, non-stationary, smooth curves, subdivision

INTRODUCTION

For the generation of smooth curves and surfaces, subdivision schemes have proven itself the building bricks in Computer Aided Geometric Design.

Hassan and Dodgson (2003) proposed a binary three-point approximating subdivision scheme that generates C^3 limiting curve. Siddiqi and Ahmad (2007) introduced another binary three-point approximating subdivision scheme that gives C^2 smooth limiting curve. Hormann and Sabin (2008) developed another binary three-point approximating subdivision scheme that generates the smooth limiting curve of C^1 continuity. The limiting curve generated by the scheme of Hormann and Sabin (2008) always lies outside the control polygon. The geometric behavior of Hassan and Dodgson (2003) scheme, Siddiqi and Ahmad (2007) scheme, Hormann and Sabin (2008) schemes have been illustrated in the Fig. 1.

Dyn *et al*. (1987) developed a binary stationary four-point interpolating subdivision scheme that generates the smooth interpolating limiting curve of C^1 continuous. Beccari *et al*. (2007a) introduced a binary non-stationary four-point interpolating subdivision scheme that generates conics sections quite efficiently. Beccari *et al*. (2007b) proposed a ternary non-stationary four-point interpolating subdivision scheme that generates the smooth interpolating curve of C^2 continuous. Siddiqi *et al*. (2015) introduced a binary non-stationary three and four-point approximating subdivision schemes using hyperbolic function that generate the limiting

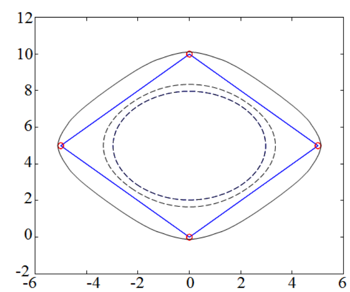

Fig. 1: Comparison of binary stationary three-point approximating subdivision schemes. Continuous curve Harmann and Sabin (2008), dash curve Siddiqi and Ahmad (2007), dotted curve Hassan and Dodgson (2003)

curves of C^1 and C^2 continuous respectively. Siddiqi and Rehan (2010) introduced a new ternary three-point approximating subdivision scheme using quadratic B-spline basis functions that generates a limiting curve of C^2 continuous. Tan *et al*. (2014a) developed a new binary stationary four-point approximating subdivision scheme that generates the shape-preserving limiting curve of C^3 continuous. Tan *et al*. (2014b) also introduced a binary stationary five-point approximating subdivision scheme that preserve the convexity property with a parameter.

Pan *et al*. (2012) introduced a new combined approximating and interpolating subdivision scheme which generates the limiting curve of C^2 continuous. Tan *et al*. (2016) proposed a new type of binary non-stationary three-point approximating subdivision

Corresponding Author: Kashif Rehan, Department of Mathematics, University of Engineering and Technology, KSK Campus, Lahore, Pakistan

scheme that generates the smooth limiting curves of C^3 continuous for the wider range of parameter.

Stationary and non-stationary subdivision schemes are defined as the set of coefficients ξ_i^k determines the subdivision rule at level k and is called the k-th level mask. If the mask ξ_i^k are independent of k, the subdivision scheme is called stationary subdivision scheme otherwise it is called non-stationary. In this study, we present the binary non-stationary three-point approximating subdivision schemes, providing the user with an initial control parameter ξ_0^k which can generate C^1 and C^2 continuous limit curves of different shapes.

BINARY STATIONARY THREE-POINT SUBDIVISION SCHEMES

The binary three-point approximating subdivision techniques to refine the discrete control polygon are:

$$\begin{aligned} r_{2i}^{k+1} &= \xi_0^k r_{i-1}^k + \xi_1^k r_i^k + \xi_2^k r_{i+1}^k, \\ r_{2i+1}^{k+1} &= \xi_2^k r_{i-1}^k + \xi_1^k r_i^k + \xi_0^k r_{i+1}^k, \end{aligned} \tag{1}$$

where, $r^0 = \{r_i^0\}_{i\in Z}$ is the set of initial discrete control point at 0 level and the coefficients of the scheme satisfy the equation $\xi_0^k + \xi_1^k + \xi_2^k = 1$.

Hormann and Sabin (2008) introduced a binary three-point approximating scheme (1) where the coefficients of the scheme are $\xi_0^k = \frac{-3}{32}$, $\xi_1^k = \frac{30}{32}$ and $\xi_2^k = \frac{5}{32}$. The scheme generates the smooth C^1 limiting curve.

Also, Siddiqi and Ahmad (2007) developed a binary three-point approximating scheme (1) where the coefficients of the scheme are $\xi_0^k = \frac{1}{32}$, $\xi_1^k = \frac{22}{32}$ and $\xi_2^k = \frac{9}{32}$. The scheme generates the smooth C^2 limiting curve.

For the above refinement rules, a geometric designer can generate only one limiting curve with a small magnitude and no one can see significant variations (Fig. 1).

Binary non-stationary three-point subdivision schemes: The binary non-stationary three-point approximating subdivision scheme, counterpart of stationary scheme of Hormann and Sabin (2008), is defined by the refinement techniques (1) and the mask of the scheme are given by:

$$\xi_0^k = h\left(\xi^{k+1}\right), \ \xi_1^k = \frac{3}{4} - 2h\left(\xi^{k+1}\right), \ \xi_2^k = \frac{1}{4} + h\left(\xi^{k+1}\right), \quad h\left(\xi^{k+1}\right) = \frac{-3}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} \tag{2}$$

With:

$$\xi^{k+1} = \sqrt{\xi^k + 6}, \qquad \xi^0 \in [-6,-5) \cup (-5,\infty). \tag{3}$$

In this way, given an initial shape control parameter $\xi^0 \in [-6,-5) \cup (-5,\infty)$, the coefficients ξ_i^k at each different level k can be calculated using the above formulae.

Another non-stationary three-point approximating scheme, which is a counterpart of stationary scheme of Siddiqi and Ahmad (2007), is defined by the refinement techniques (1) and the coefficients of the scheme are:

$$\begin{aligned} &\xi_0^k = h\left(\xi^{k+1}\right), \ \xi_1^k = \frac{3}{4} - 2h\left(\xi^{k+1}\right), \\ &\xi_2^k = \frac{1}{4} + h\left(\xi^{k+1}\right), \ h\left(\xi^{k+1}\right) = \frac{1}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} \end{aligned} \tag{4}$$

With:

$$\xi^{k+1} = \sqrt{\xi^k + 6}, \qquad \xi^0 \in [-6,-5) \cup (-5,\infty). \tag{5}$$

From this, using an initial parameter $\xi^0 \in [-6,-5) \cup (-5,\infty)$, the coefficients ξ_i^k at each different level k can be calculated using the above formulae.

Starting from any $\xi^0 \geq -6$, this implies that $\xi^k + 6 \geq 0, \forall k \in Z_+$, so ξ^{k+1} is always well defined. A wide range of shape control parameter allows user to get considerable variations of shapes in curves design (Fig. 2 and 3).

Remark 1: From the Fig. 2 and 3, as the initial shape control parameter ξ^0 increases in its domain, the shape of curve changes a lot at first and then tends to approximate the control polygon as $\xi^0 \to +\infty$.

Remark 2: From (3) and (5), we know:

If $\xi^0 = 3$, then $\xi^k = 3, \forall k \in Z_+$ and the sequence $\{\xi^k\}_{k\in N}$ is stationary. Then the proposed scheme retrogrades to the stationary scheme.

If $\xi^0 > 3$, then $\xi^k > 3$, the sequence $\{\xi^k\}_{k\in N}$ is strictly decreasing and ξ^k converges to 3 as $k \to +\infty$.

If $\xi^0 < 3$, then $\xi^k < 3$, the sequence $\{\xi^k\}_{k\in N}$ is strictly increasing and ξ^k converges to 3 as $k \to +\infty$.

Hence, with any given ξ^0 in its definition domain, we always have $\lim_{k\to+\infty} \xi^k = 3$.

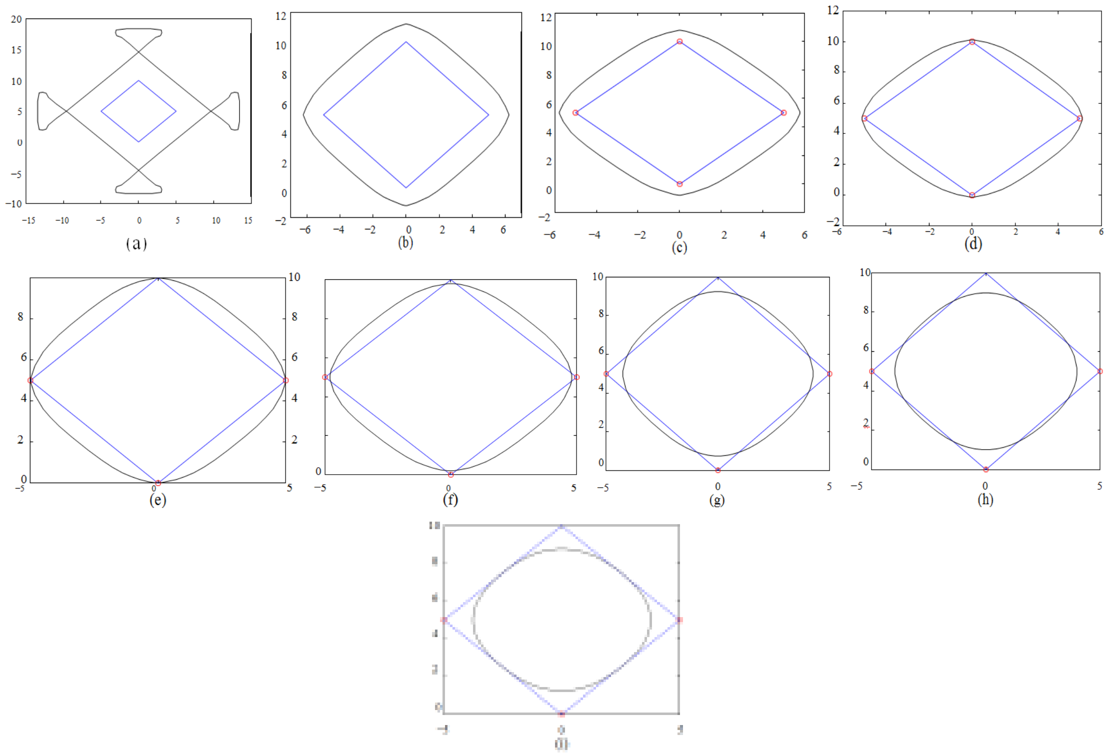

Fig. 2: Generating wide range of C^1-continuous limiting curves using the scheme (3. 1) for different values of parameter ξ^0. (a) $\xi^0 = -5.5$, (b) $\xi^0 = -1$, (c) $\xi^0 = 0$, (d) $\xi^0 = 3$, (e) $\xi^0 = 4$, (f) $\xi^0 = 6$, (g) $\xi^0 = 25$, (h) $\xi^0 = 100$, (i) $\xi^0 = 30,000$

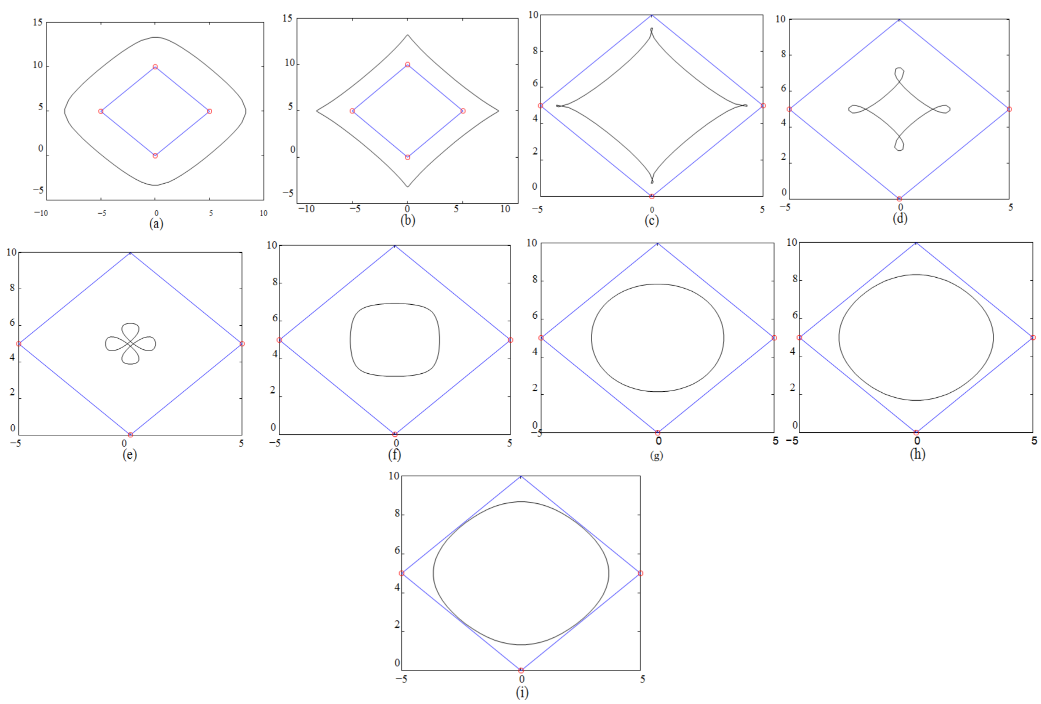

Fig. 3: Generating wide range of C^2-continuous limiting curves using the scheme (3.3) for different values of parameter ξ^0. (a) $\xi^0 = -5.5$, (b) $\xi^0 = -4.8$, (c) $\xi^0 = -4.7$, (d) $\xi^0 = -4.6$, (e) $\xi^0 = -4.5$, (f) $\xi^0 = -3.6$, (g) $\xi^0 = -2$, (h) $\xi^0 = 2$, (i) $\xi^0 = 200$

SMOOTHNESS ANALYSIS

Here convergence of the schemes (2) and (4) are analyzed using the results of Dyn and Levin (1995). Which describes the convergence of a non-stationary scheme to its asymptotically equivalent stationary counterpart.

Theorem 1: The non-stationary scheme (2) is asymptotically equivalent to the stationary scheme of Hormann and Sabin (2008) with the coefficients $\xi_0{}^k = \frac{-3}{32}$, $\xi_1{}^k = \frac{30}{32}$ and $\xi_2{}^k = \frac{5}{32}$. Hence it gives the smooth C^1 curves.

Proof: To verify the smoothness C^1 of the proposed non-stationary scheme (2), its second divided difference mask are required. The coefficients of the scheme are:

$$m^k = \begin{bmatrix} h(\xi^{k+1}), \frac{1}{4}+h(\xi^{k+1}), \frac{3}{4}-2h(\xi^{k+1}), \\ \frac{3}{4}-2h(\xi^{k+1}), \frac{1}{4}+h(\xi^{k+1}), h(\xi^{k+1}) \end{bmatrix}.$$

So, its first divided difference masks are:

$$b_{(1)}{}^k = 2\left[h(\xi^{k+1}), \frac{1}{4}, \frac{1}{2}-2h(\xi^{k+1}), \frac{1}{4}, h(\xi^{k+1})\right].$$

Now applying Remark 2 gives:

$$b_{(1)}{}^\infty = \underset{k\to+\infty}{Limit}\, b_{(1)}{}^k = 2\left[\frac{-3}{32}, \frac{8}{32}, \frac{22}{32}, \frac{8}{32}, \frac{-3}{32}\right].$$

Which are the coefficients of the first divided difference of the stationary scheme having mask in (1) with $\xi_0{}^k = \frac{-3}{32}$, $\xi_1{}^k = \frac{30}{32}$ and $\xi_2{}^k = \frac{5}{32}$. Hormann and Sabin (2008) proved that the stationary scheme associated with $b_{(1)}{}^\infty$ hasC^1 smoothness. Now if:

$$\sum_{k=0}^{+\infty} \left\| b_{(1)}{}^k - b_{(1)}{}^\infty \right\|_\infty, \tag{6}$$

Then the two schemes are asymptotically equivalent. And hence the scheme is C^1 (Dyn and Levin (1995)). Since:

$$b_{(1)}{}^k - b_{(1)}{}^\infty = 2\left[h(\xi^{k+1})+\frac{3}{32}, 0, \frac{-6}{32}-2h(\xi^{k+1}), 0, h(\xi^{k+1})+\frac{3}{32}\right]$$

$$\left\| b_{(1)}{}^k - b_{(1)}{}^\infty \right\|_\infty = 2\max\left\{2\left|h(\xi^{k+1})+\frac{3}{32}\right|, 2\left|h(\xi^{k+1})+\frac{3}{32}\right|\right\}$$

$$= 4\left|h(\xi^{k+1})+\frac{3}{32}\right|.$$

To prove (6), the convergence of the series is required to be prove:

$$\sum_{k=0}^{+\infty} \left|h(\xi^{k+1})+\frac{3}{32}\right|, \tag{7}$$

Which depends on the function $h(\xi^{k+1})$ and $h(\xi^{k+1})$ is expressed in terms of the shape control parameter ξ^{k+1} through relation (2), the behavior of (7) as ξ^{k+1} varies in $[0,+\infty)$. Since:

$$h(\xi^{k+1}) + \frac{3}{32} = 0 \Leftrightarrow \xi^{k+1} = 3,$$

$$h(\xi^{k+1}) + \frac{3}{32} < 0 \Leftrightarrow \xi^{k+1} \in (1,3),$$

$$h(\xi^{k+1}) + \frac{3}{32} > 0 \Leftrightarrow \xi^{k+1} \in [0,1) \cup (3,+\infty).$$

To verify the convergence of (7), the following three cases are:

Case 1: $\xi^0 = 3$ ($i.e.\ \xi^{k+1} = 3$).
Obviously, it gives

$$\left\| b_{(1)}{}^k - b_{(1)}{}^\infty \right\|_\infty = 0.$$

Convergence of (7) is verified.

Case 2: $\xi^0 \in (-5,3)$ ($i.e.\ \xi^{k+1} \in (1,3)$).

Here
$$\left\| b_{(1)}{}^k - b_{(1)}{}^\infty \right\|_\infty = 4\left|h(\xi^{k+1})+\frac{3}{32}\right|.$$
Thus, it suffices to prove

$$\sum_{k=0}^{+\infty} \left|h(\xi^{k+1})+\frac{3}{32}\right| = \sum_{k=0}^{+\infty} \left|\frac{-3}{4\left[(\xi^{k+1})^2-1\right]}+\frac{3}{32}\right| < +\infty.$$

For this, exploit the ratio test. Since $h(\xi^{k+1}) + \frac{3}{32} < 0$ and the sequence $\{\xi^k\}_{k\in N}$ in this case is strictly increasing such that:

$$\frac{\frac{-3}{4\left[(\xi^{k+2})^2-1\right]}+\frac{3}{32}}{\frac{-3}{4\left[(\xi^{k+1})^2-1\right]}+\frac{3}{32}} < 1.$$

Hence the convergence of (7) is verified.

Case 3: $\xi^0 \in [-6,-5) \cup (3,+\infty)$ ($i.e.\ \xi^{k+1} \in [0,1) \cup (3,+\infty)$).

Here

$$\left\| b_{(1)}{}^{k} - b_{(1)}{}^{\infty} \right\|_{\infty} = 4\left| \frac{3}{32} + h\left(\xi^{k+1}\right) \right|.$$

Thus, it suffices to prove:

$$\sum_{k=0}^{+\infty} \left| \frac{3}{32} + h(\xi^{k+1}) \right| = \sum_{k=0}^{+\infty} \left| \frac{3}{32} + \frac{-3}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} \right| < +\infty.$$

Two subcases are:

Case 3.1: $\xi^0 \in (3,+\infty), (i.e.\ \xi^{k+1} \in (3,+\infty))$. Since the sequence $\{\xi^k\}_{k\in N}$ in this case is
strictly decreasing, so:

$$\frac{\frac{3}{32} + \frac{-3}{4\left[\left(\xi^{k+2}\right)^2 - 1\right]}}{\frac{3}{32} + \frac{-3}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]}} < 1.$$

Hence the convergence of (7) is verified.

Case 3.2: $\xi^0 \in [-6,-5) (i.e.\ \xi^1 \in (0,1))$ and $\xi^k \in (1,3), k = 2,3,4,\ldots$ which turns to case 2. Hence the convergence of (7) is verified.
Combining the three cases, the non-stationary subdivision scheme (2) is asymptotically equivalent to the stationary scheme of Hormann and Sabin (2008) with the coefficients $\xi_0{}^k = \frac{-3}{32}$, $\xi_1{}^k = \frac{30}{32}$ and $\xi_2{}^k = \frac{5}{32}$.
And the scheme generates the smooth C^1 curves.

Corollary 1: A proposed binary non-stationary three-point subdivision scheme (2) is not C^t continuous for $t > 1$.

Theorem 2: The non-stationary scheme (4) is asymptotically equivalent to the stationary scheme of Siddiqi and Ahmad (2007) with the coefficients $\xi_0{}^k = \frac{1}{32}$, $\xi_1{}^k = \frac{22}{32}$ and $\xi_2{}^k = \frac{9}{32}$. Hence it gives the smooth C^2 curves.

Proof: To verify the smoothness C^2 of the proposed non-stationary scheme (4), its second divided difference mask are required. The coefficients of the scheme are:

$$v^k = \begin{bmatrix} h\left(\xi^{k+1}\right), \frac{1}{4} + h\left(\xi^{k+1}\right), \frac{3}{4} - 2h\left(\xi^{k+1}\right), \frac{3}{4} \\ -2h\left(\xi^{k+1}\right), \frac{1}{4} + h\left(\xi^{k+1}\right), h\left(\xi^{k+1}\right) \end{bmatrix}.$$

So, its first and second divided difference masks are:

$$b_{(1)}{}^{k} = 2\left[h\left(\xi^{k+1}\right), \frac{1}{4}, \frac{1}{2} - 2h\left(\xi^{k+1}\right), \frac{1}{4}, h\left(\xi^{k+1}\right) \right].$$

$$b_{(2)}{}^{k} = 4\left[h\left(\xi^{k+1}\right), \frac{1}{4} - h\left(\xi^{k+1}\right), \frac{1}{4} - h\left(\xi^{k+1}\right), h\left(\xi^{k+1}\right) \right].$$

Now applying Remark 2 gives:

$$b_{(2)}{}^{\infty} = \underset{k\to+\infty}{Limit}\, b_{(2)}{}^{k} = 4\left[\frac{1}{32}, \frac{7}{32}, \frac{7}{32}, \frac{1}{32} \right].$$

Which are the coefficients of the second divided difference of the stationary scheme having mask in (1) with
$\xi_0{}^k = \frac{1}{32}$, $\xi_1{}^k = \frac{22}{32}$ and $\xi_2{}^k = \frac{9}{32}$. Siddiqi and Ahmad (2007) proved that the stationary scheme associated with $b_{(2)}{}^{\infty}$ has C^2 smoothness. Now if:

$$\sum_{k=0}^{+\infty} \left\| b_{(2)}{}^{k} - b_{(2)}{}^{\infty} \right\|_{\infty} \tag{8}$$

Then the two schemes are asymptotically equivalent. And hence the scheme is C^2 (Dyn and Levin (1995)). Since:

$$b_{(2)}{}^{k} - b_{(2)}{}^{\infty} = 4\begin{bmatrix} h\left(\xi^{k+1}\right) - \frac{1}{32}, \frac{1}{32} - h\left(\xi^{k+1}\right), \\ h\left(\xi^{k+1}\right) - \frac{1}{32}, \frac{1}{32} - h\left(\xi^{k+1}\right) \end{bmatrix},$$

$$\left\| b_{(2)}{}^{k} - b_{(2)}{}^{\infty} \right\|_{\infty} = 4\max\left\{ \begin{matrix} 2\left| h\left(\xi^{k+1}\right) - \frac{1}{32} \right|, \\ 2\left| \frac{1}{32} - h\left(\xi^{k+1}\right) \right| \end{matrix} \right\} = 8\left| h\left(\xi^{k+1}\right) - \frac{1}{32} \right|.$$

To prove (8), the convergence of the series is required to be prove:

$$\sum_{k=0}^{+\infty} \left| h\left(\xi^{k+1}\right) - \frac{1}{32} \right|, \tag{9}$$

Which depends on the function $h(\xi^{k+1})$ and $h(\xi^{k+1})$ is expressed in terms of the shape control parameter ξ^{k+1} through relation (4), the behavior of (9) as ξ^{k+1} varies in $[0,+\infty)$. Since:

$$h(\xi^{k+1}) - \frac{1}{32} = 0 \Leftrightarrow \xi^{k+1} = 3,$$

$$h(\xi^{k+1}) - \frac{1}{32} > 0 \Leftrightarrow \xi^{k+1} \in (1,3),$$

$$h(\xi^{k+1}) - \frac{1}{32} < 0 \Leftrightarrow \xi^{k+1} \in [0,1) \cup (3,+\infty).$$

To verify the convergence of (9), the following three cases are:

Case 1: $\xi^0 = 3$ $(i.e.\, \xi^{k+1} = 3)$.
Obviously, it gives:

$$\left\| b_{(2)}{}^k - b_{(2)}{}^\infty \right\|_\infty = 0.$$

Convergence of (9) is verified.

Case 2: $\xi^0 \in (-5,3)$ $(i.e.\, \xi^{k+1} \in (1,3))$.

Here

$$\left\| b_{(2)}{}^k - b_{(2)}{}^\infty \right\|_\infty = 8\left| h\left(\xi^{k+1}\right) - \frac{1}{32} \right|.$$

Thus, it suffices to prove:

$$\sum_{k=0}^{+\infty} \left| h(\xi^{k+1}) - \frac{1}{32} \right| = \sum_{k=0}^{+\infty} \left| \frac{1}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} - \frac{1}{32} \right| < +\infty.$$

For this, exploit the ratio test. Since $h(\xi^{k+1}) - \frac{1}{32} > 0$ and the sequence $\{\xi^k\}_{k \in N}$ in this case is strictly increasing such that:

$$\frac{\dfrac{1}{4\left[\left(\xi^{k+2}\right)^2 - 1\right]} - \dfrac{1}{32}}{\dfrac{1}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} - \dfrac{1}{32}} < 1.$$

Hence the convergence of (9) is verified.

Case 3: $\xi^0 \in [-6,-5) \cup (3,+\infty)$ $(i.e.\, \xi^{k+1} \in [0,1) \cup (3,+\infty))$.
Here,

$$\left\| b_{(2)}{}^k - b_{(2)}{}^\infty \right\|_\infty = 8\left| \frac{1}{32} - h\left(\xi^{k+1}\right) \right|.$$

Thus, it suffices to prove:

$$\sum_{k=0}^{+\infty} \left| \frac{1}{32} - h(\xi^{k+1}) \right| = \sum_{k=0}^{+\infty} \left| \frac{1}{32} - \frac{1}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]} \right| < +\infty.$$

Two subcases are:

Case 3.1: $\xi^0 \in (3,+\infty)$, $(i.e.\, \xi^{k+1} \in (3,+\infty))$. Since the sequence $\{\xi^k\}_{k \in N}$ in this case is strictly decreasing, so:

$$\frac{\dfrac{1}{32} - \dfrac{1}{4\left[\left(\xi^{k+2}\right)^2 - 1\right]}}{\dfrac{1}{32} - \dfrac{1}{4\left[\left(\xi^{k+1}\right)^2 - 1\right]}} < 1.$$

Hence the convergence of (9) is verified.

Case 3.2: $\xi^0 \in [-6,-5)\, (i.e.\, \xi^1 \in (0,1))$ and $\xi^k \in (1,3),\, k = 2,3,4,\ldots$ which turns to case 2. Hence the convergence of (9) is verified.

Combining the three cases, the non-stationary subdivision scheme (4) is asymptotically equivalent to the stationary scheme of Siddiqi and Ahmad (2007) with the coefficients $\xi_0{}^k = \frac{1}{32}$, $\xi_1{}^k = \frac{22}{32}$ and $\xi_2{}^k = \frac{9}{32}$. And the scheme generates the smooth C^2 curves.

Corollary 2: A proposed binary non-stationary three-point subdivision scheme (4) is not C^t continuous for $t > 2$.

VISUAL INSPECTION

Figure 1, comparison of the binary stationary three-point approximating subdivision schemes Hassan and Dodgson (2003), Siddiqi and Ahmad (2007) and Harmann and Sabin (2008) have been illustrated using initial control points of the square. The schemes of Harmann and Sabin (2008), Siddiqi and Ahmad (2007) generate C^1 and C^2-continuous a fix limiting curve respectively that restrict the user to design the variety of limiting curves. These restrictions have been removed using the binary non-stationary approximating subdivision schemes (2)

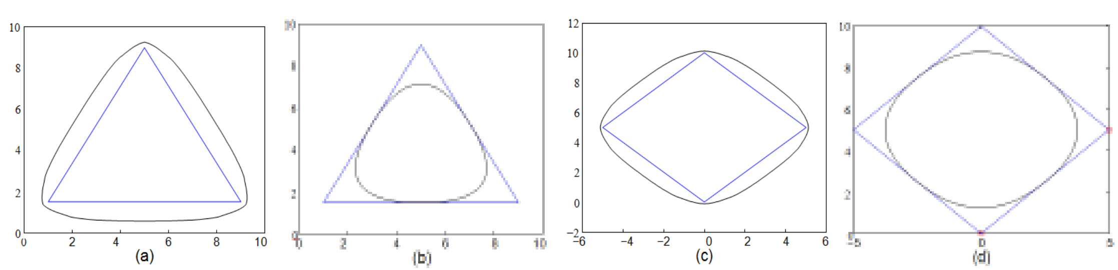

Fig. 4: Comparison of stationary scheme Hormann and Sabin (2008) in (a), (c) and non-stationary scheme (3.1) for $\xi^0 = 30{,}000$ in (b), (d)

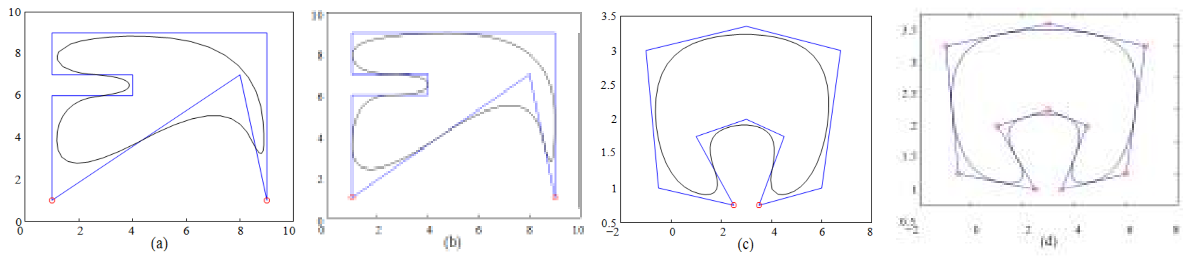

Fig. 5: Comparison of stationary scheme Siddiqi and Ahmad (2007) in (a), (c) and non-stationary scheme (3.3) for $\xi^0 = 1000$ in (b), (d)

and (4) as shown in Fig. 2 and 3. The comparison of the binary stationary three-point schemes Siddiqi and Ahmad (2007) and Harmann and Sabin (2008) have been depicted with binary non-stationary three-point schemes (2) and (4) in Fig. 4 and 5. It is not difficult to observe that the limiting curves generated by the proposed non-stationary schemes outperform those by the stationary schemes.

CONCLUSION

The binary non-stationary three-point approximating subdivision schemes are introduced that generates a family of C^1 and C^2 limiting curves for the wider range of tension parameter. The proposed schemes give wide flexibility to construct C^1 and C^2 considerable variations of shapes in geometric designing. Important feature of the proposed schemes is that the schemes rectified the deficiency of its stationary counter parts (Hormann and Sabin, 2008; Siddiqi and Ahmad, 2007). The derivative continuity of the proposed schemes is proved using the conclusion in Dyn and Levin (1995).

REFERENCES

Beccari, C., G. Casciola and L. Romani, 2007a. A non-stationary uniform tension controlled interpolating 4-point scheme reproducing conics. Comput. Aided Geom. D., 24(1): 1-9.

Beccari, C., G. Casciola and L. Romani, 2007b. An interpolating 4-point C2 ternary non-stationary subdivision scheme with tension control. Comput. Aided Geom. D., 24(4): 210-219.

Dyn, N. and D. Levin, 1995. Analysis of asymptotically equivalent binary subdivision schemes. J. Math. Anal. Appl., 193(2): 594-621.

Dyn, N., D. Levin and J.A. Gregory, 1987. A 4-point interpolatory subdivision scheme for curve design. Comput. Aided Geom. D., 4(4): 257-268.

Hassan, M.F. and N.A. Dodgson, 2003. Ternary and Three Point Univariate Subdivision Schemes. In: Cohen, A., J.L. Merrien and L.L. Schumaker (Eds.), Curve and Surface Fitting: Sant-Malo 2002. Nashboro Press, Brentwood, pp: 199-208.

Hormann, K. and M.A. Sabin, 2008. A family of subdivision schemes with cubic precision. Comput. Aided Geom. D., 25(1): 41-52.

Pan, J., S. Lin and X. Luo, 2012. A combined approximating and interpolating subdivision scheme with C2 continuity. Appl. Math. Lett., 25(12): 2140-2146.

Siddiqi, S.S. and N. Ahmad, 2007. A new three-point approximating C2 subdivision scheme. Appl. Math. Lett., 20(6): 707-711.

Siddiqi, S.S. and K. Rehan, 2010. A ternary three point scheme for curve designing. Int. J. Comput. Math., 87(8): 1709-1715.

Siddiqi, S.S., W. us Salam and K. Rehan, 2015. Binary 3-point and 4-point non-stationary subdivision schemes using hyperbolic function. Appl. Math. Comput., 258: 120-129.

Tan, J., J. Sun and G. Tong, 2016. A non-stationary binary three-point approximating subdivision scheme. Appl. Math. Comput., 276: 37-43.

Tan, J.Q., X.L. Zhuang and L. Zhang, 2014a. A new four-point shape-preserving C3 subdivision scheme. Comput. Aided Geom. D., 31(1): 57-62.

Tan, J.Q., Y.G. Yao, H.J. Cao and L. Zhang, 2014b. Convexity preservation of five-point binary subdivision scheme with a parameter. Appl. Math. Comput., 245: 279-288.