Research Article
Competition between Commercial Open Source Software Firms under the GNU General Public License

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Abstract: This study compares R&D incentives for commercial open source software (COSS) firms under the GNU General Public License (i.e., GPL). It is found that: (i) although the GPL requires firms open the codes of features, firms have incentives to invest in software features under private optimum; (ii) the firm with high software usability has much higher incentive to invest in software features, sets higher price, obtains more market share and profit than the one with low software usability does; (iii) firms invest too little in software features under GPL from a public policy perspective.

Keywords: Commercial open source software, competition, R&D, general public license, software features, software usability

INTRODUCTION

Since 1990s, the rapid development of open source (e.g., Linux) is a significant phenomenon in software industries. Open Source Software (OSS) is software, whose sources codes are allowed software developers to share, identify and correct errors and redistribute (O'Reilly, 1999). Now more and more companies build commercial products based on open source software, a typical example is Red Hat Inc. Commercial Open Source Software (COSS) is privately developed based on publicly available source code (Kumar *et al*., 2011). A software quality consists of two components: usability (includes ease of installation, documentation, user interface and level of technical support) and features (includes feature set, reliability, security etc) (Choudhary and Zhou, 2007). By improving the features or usability of the existing open source software, firms generate a product that contains both publicly and privately developed components. The total amount of work invested into open source software projects is growing at an exponential rate and can be expected to continue growing at this rate for a while before a slowing down (Deshpande and Riehle, 2008).

Firms must follow corresponding open source license when they develop software based on publicly available source code. The most common license dictating how commercial open source software may be distributed is the GNU General Public License (GPL) (Laurent, 2004). Under the GPL, firms can freely obtain the codes of open source software, but the codes must open when they enhance the software features. This gives rise to the following issues. First, does a commercial open source software firm have an incentive to invest in software features when its competitors can freely obtain its developments under the GPL? Second, from a public policy perspective, are the commercial open source software firms' R&D incentives towards software features just the right or too high (low)? We answer above questions by modifying the vertical differentiation model (Mussa and Rosen, 1978).

The following works are related to our study. Raghunathan *et al*. (2005), Choudhary and Zhou (2007), Lanzi (2009) and Xing (2010) research the quality (innovation) competition between open and closed source software, however they don't consider commercial open source and open source licenses. Sen (2007) investigates the price competition between commercial version of open source software and proprietary software. Dixon (2009) and Riehle (2011) present the core properties of commercial open source business models and discuss how they work. Although their papers relates to commercial open source, they don't involve in R&D competition.

THE BASIC SETUP

There are three types of open source software products in a market. One is from the not-for profit community (called OSS in this study) and the other two are from the commercial open source software (called COSS in this study) firms. Firms can freely derive the codes of software features from the open source community, but they must comply with the relevant open source licenses.

Software users are indexed by their level of technical ability, measured by parameter θ, uniformly distributed with density 1 over interval [0, 1]. Assume

that users with higher level of technical skills have lower θ, while those with lower degree of technical capability have higher θ. Moreover, a user who with lower technical expertise has higher willingness to pay for software usability than a user with higher technical expertise does (Choudhary and Zhou, 2007).

A firm must follow the corresponding open source licenses when develops software based on publicly available source code. The GNU General Public License (GPL) is the most common open source license. This study assumes firms develop the commercial open source software based on the open source software of community under the GPL.

The indirect utility functions for the generic consumer at $\theta \in [0, 1]$ when he/she uses OSS and COSS are respectively given by:

$$u_o = \theta v_o + \theta v_o(f_o + f_1 + f_2) + (f_o + f_1 + f_2) \qquad (1)$$

$$u_1 = \theta v_1 + \theta v_1(f_o + f_1 + f_2) + (f_o + f_1 + f_2) - p_1 \qquad (2)$$

$$u_2 = \theta v_2 + \theta v_2(f_o + f_1 + f_2) + (f_o + f_1 + f_2) - p_2 \qquad (3)$$

where, v_o, v_i is the usability of OSS, COSS i (i = 1, 2), satisfies $0 < v_o < v_1 < v_2$; f_o, f_i is the initial features of OSS and the feature developments of COSS i (i = 1, 2); p_i is the price of COSS i (i = 1, 2). Note that:

- The price of OSS equals zero (i.e., $p_o = 0$) because the open source software can be freely available from the open source community;
- This study assumes open source community and both firms can wholly obtain others' feature developments (because the GPL requires firms open the developments of feature), so all of their software features equal $f_o + f_1 + f_2$.

The marginal consumer who is indifferent between using OSS and COSS 1, indexed by $\hat{\theta}_{o1}$, is given by $u_o = u_1$:

$$\begin{aligned}\hat{\theta}_{o1}v_o + \hat{\theta}_{o1}v_o(f_o + f_1 + f_2) + (f_o + f_1 + f_2) \\= \hat{\theta}_{o1}v_1 + \hat{\theta}_{o1}v_1(f_o + f_1 + f_2) + (f_o + f_1 + f_2) - p_1\end{aligned} \qquad (4)$$

Solving (4), we obtain:

$$\hat{\theta}_{o1} = \frac{p_1}{(v_1 - v_o)(1 + f_o + f_1 + f_2)} \qquad (5)$$

The marginal consumer who is indifferent between using COSS 1 and COSS 2, indexed by $\hat{\theta}_{12}$, is given by $u_1 = u_2$:

$$\begin{aligned}\hat{\theta}_{12}v_1 + \hat{\theta}_{12}v_1(f_o + f_1 + f_2) + (f_o + f_1 + f_2) - p_1 \\= \hat{\theta}_{12}v_2 + \hat{\theta}_{12}v_2(f_o + f_1 + f_2) + (f_o + f_1 + f_2) - p_2\end{aligned} \qquad (6)$$

Solving (6), we obtain:

$$\hat{\theta}_{12} = \frac{p_2 - p_1}{(v_2 - v_1)(1 + f_o + f_1 + f_2)} \qquad (7)$$

The demand functions for open source community and firms are respectively given by:

$$d_o = \hat{\theta}_{o1} - 0 = \frac{p_1}{(v_1 - v_o)(1 + f_o + f_1 + f_2)} \qquad (8)$$

$$d_1 = \hat{\theta}_{12} - \hat{\theta}_{o1} = \frac{1}{(1 + f_o + f_1 + f_2)}(\frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1 - v_o}) \qquad (9)$$

$$d_2 = 1 - \hat{\theta}_{12} = 1 - \frac{p_2 - p_1}{(v_2 - v_1)(1 + f_o + f_1 + f_2)} \qquad (10)$$

The profit functions for firm 1 and firm 2 are respectively given by:

$$\begin{aligned}\pi_1 &= p_1 d_1 - \gamma f_1^2 \\&= \frac{p_1}{(1 + f_o + f_1 + f_2)}(\frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1 - v_o}) - \gamma f_1^2\end{aligned} \qquad (11)$$

$$\begin{aligned}\pi_2 &= p_2 d_2 - \gamma f_2^2 \\&= p_2[1 - \frac{p_2 - p_1}{(v_2 - v_1)(1 + f_o + f_1 + f_2)}] - \gamma f_2^2\end{aligned} \qquad (12)$$

where, γf_i^2 denotes the R&D cost when firm i develops the features of open source software and γ is positive parameter which measures the innovation efficiency.

If the commercial open source software firm is run by a benevolent social planner instead, the levels of software features are chosen to maximize the social welfare, defined as the sum of profits and consumer surplus. The welfare function corresponds to:

$$\begin{aligned}SW &= \pi_1 + \pi_2 + CS \\&= \pi_1 + \pi_2 + \int_0^{\hat{\theta}_{o1}} u_o d\theta + \int_{\hat{\theta}_{o1}}^{\hat{\theta}_{12}} u_1 d\theta + \int_{\hat{\theta}_{12}}^1 u_2 d\theta\end{aligned} \qquad (13)$$

The timing of R&D and price competition is as follows. In the first stage, firms determine the developments of software features. In the second stage, they set price.

THE PRIVATE OPTIMUM

The solutions of the game model are derived by backwards induction. The price stage is analyzed firstly and then the R&D stage is decided.

Solution of Stage 2: The first-order conditions of (11) and (12) with respect to p_1 and p_2 are respectively given by:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{(1+f_o+f_1+f_2)}\left(\frac{p_2-2p_1}{v_2-v_1}-\frac{2p_1}{v_1-v_o}\right)=0 \quad (14)$$

$$\frac{\partial \pi_2}{\partial p_2} = 1 - \frac{2p_2-p_1}{(1+f_o+f_1+f_2)(v_2-v_1)}=0 \quad (15)$$

Solving (14) and (15), we derive the optimal prices for COSS 1 and COSS 2:

$$p_1 = \frac{(v_1-v_o)(v_2-v_1)(1+f_o+f_1+f_2)}{4(v_2-v_o)-(v_1-v_o)} \quad (16)$$

$$p_2 = \frac{2(v_2-v_o)(v_2-v_1)(1+f_o+f_1+f_2)}{4(v_2-v_o)-(v_1-v_o)} \quad (17)$$

Substituting (16) and (17) in (9)-(12), the demand and profit functions for firm 1 and firm 2 are given by:

$$d_1 = \frac{v_2-v_o}{4(v_2-v_o)-(v_1-v_o)} \quad (18)$$

$$d_2 = \frac{2(v_2-v_o)}{4(v_2-v_o)-(v_1-v_o)} \quad (19)$$

$$\pi_1 = \frac{(v_1-v_o)(v_2-v_1)(v_2-v_o)(1+f_o+f_1+f_2)}{[4(v_2-v_o)-(v_1-v_o)]^2}-\gamma f_1^2 \quad (20)$$

$$\pi_2 = \frac{4(v_2-v_1)(v_2-v_o)^2(1+f_o+f_1+f_2)}{[4(v_2-v_o)-(v_1-v_o)]^2}-\gamma f_2^2 \quad (21)$$

Solution of stage 1: Taking the derivatives of (20) and (21) with respect to f_1 and f_2 respectively and setting them equal to zero, we obtain the following equations:

$$\frac{\partial \pi_1}{\partial f_1} = \frac{(v_1-v_o)(v_2-v_1)(v_2-v_o)}{[4(v_2-v_o)-(v_1-v_o)]^2}-2\gamma f_1=0 \quad (22)$$

$$\frac{\partial \pi_2}{\partial f_2} = \frac{4(v_2-v_1)(v_2-v_o)^2}{[4(v_2-v_o)-(v_1-v_o)]^2}-2\gamma f_2=0 \quad (23)$$

Solving (22) and (23), we derive the optimal improvements of feature for firm 1 and firm 2:

$$f_1^* = \frac{(v_1-v_o)(v_2-v_1)(v_2-v_o)}{2\gamma[4(v_2-v_o)-(v_1-v_o)]^2} \quad (24)$$

$$f_2^* = \frac{2(v_2-v_1)(v_2-v_o)^2}{\gamma[4(v_2-v_o)-(v_1-v_o)]^2} \quad (25)$$

Substituting (24) and (25) in (16), (17), (20) and (21), the optimal prices and profits for firm 1 and firm 2 are given by:

$$p_1^* = \frac{(v_1-v_o)(v_2-v_1)}{4(v_2-v_o)-(v_1-v_o)}\times$$
$$\left\{1+f_o+\frac{(v_2-v_1)(v_2-v_o)[4(v_2-v_o)+(v_1-v_o)]}{2\gamma[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \quad (26)$$

$$p_2^* = \frac{2(v_2-v_o)(v_2-v_1)}{4(v_2-v_o)-(v_1-v_o)}\times$$
$$\left\{1+f_o+\frac{(v_2-v_1)(v_2-v_o)[4(v_2-v_o)+(v_1-v_o)]}{2\gamma[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \quad (27)$$

$$\pi_1^* = \frac{(v_1-v_o)(v_2-v_1)(v_2-v_o)}{[4(v_2-v_o)-(v_1-v_o)]^2}\times$$
$$\left\{1+f_o+\frac{(v_2-v_1)(v_2-v_o)[8(v_2-v_o)+(v_1-v_o)]}{4\gamma[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \quad (28)$$

$$\pi_2^* = \frac{4(v_2-v_1)(v_2-v_o)^2}{[4(v_2-v_o)-(v_1-v_o)]^2}\times$$
$$\left\{1+f_o+\frac{(v_2-v_1)(v_2-v_o)[2(v_2-v_o)+(v_1-v_o)]}{2\gamma[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \quad (29)$$

According to (18) and (19), the demand functions don't depend on f_1 and f_2, so the optimal demands for firm 1 and firm 2 are given by (18) and (19).

Comparing the optimal results for two firms, we obtain the following conclusions.

Proposition 1:

- Firm 2 invests more in software features than firm 1 does (i.e. $f_2^* > f_1^*$)
- The price of coss 2 is higher than that of coss 1 (i.e., $p_2^* > p_1^*$)
- Both demand and profit of firm 1 are more than that of firm 1 (i.e. $d_2^* > d_1^*$ and $\pi_2^* > \pi_1^*$).

Proof:

- $f_2^* - f_1^* = \frac{(v_2-v_1)(v_2-v_o)[4(v_2-v_o)-(v_1-v_o)]}{2\gamma[4(v_2-v_o)-(v_1-v_o)]^2}>0$ because of $v_2-v_1>0$, $v_2-v_o>0$ and $4(v_2-v_o)-(v_1-v_o)>0$, so $f_2^* > f_1^*$

- $p_2^*-p_1^* = \frac{(v_2-v_1)[2(v_2-v_o)-(v_1-v_o)](1+f_o+f_1^*+f_2^*)}{4(v_2-v_o)-(v_1-v_o)}>0$ because of $v_2-v_1>0$ and $2(v_2-v_o)-(v_1-v_o)>0$, so $p_2^* > p_1^*$

- $d_2^* - d_1^* = \frac{(v_2-v_o)}{4(v_2-v_o)-(v_1-v_o)}>0$, so $d_2^* > d_1^*$

$\pi_2^*-\pi_1^* = \frac{(v_2-v_1)(v_2-v_o)[1+f_o+(f_1^*+f_2^*)/2]}{4(v_2-v_o)-(v_1-v_o)}>0$, so $\pi_2^* > \pi_1^*$

We use a numerical example to show the results of Proposition 1. When $v_2 = 2v_1 = 4v_o = 1$, $\gamma = 1$ and $q_o = 1$, there are $f_2^* = 12f_1^*$, $p_2^* = 6p_1^*$, $d_2^* = 2d_1^*$ and $\pi_2^* > 11\pi_1^*$.

Proposition 1 show that, although the GPL requires firms open the codes of features, COSS firms have incentives to invest in software features. This well explains why some COSS firms (e.g., Red Hat Inc) make significant contributions to the Linux kernel under the GPL, which implies that they must make publicly available any feature contributions they makes to Linux (Kumar *et al.*, 2011). Moreover, the high-usability firm has higher incentive to invest in features, sets higher price, obtains more market share and profit than the low-usability one does. For example, Red Hat Inc provides users with more documentation, installation and maintenance and support programs for Linux than other COSS firms do (i.e. the usability of Red Hat Linux is higher than that of other COSS firms). As a result, Red Hat Inc is more willing to develop the features of Linux and then sets much higher prices, obtains much more demand and profit than other COSS firms with low usability.

THE SOCIAL OPTIMUM

We now proceed to decide the levels of software features make the social welfare maximization. Assume that the prices are decided by firms and the social planner only chooses the software features. Combining (5), (7), (13) and (16)-(21), the social welfare function is given by:

$$
\begin{aligned}
SW &= \pi_1 + \pi_2 + \int_0^{\hat{\theta}_{o1}} u_o d\theta + \int_{\hat{\theta}_{o1}}^{\hat{\theta}_{12}} u_1 d\theta + \int_{\hat{\theta}_{12}}^1 u_2 d\theta \\
&= \frac{(1+f_o+f_1+f_2)}{2}\left\{v_2 - \frac{(v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)]}{[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \\
&\quad + (f_o+f_1+f_2) - \gamma(f_1^2+f_2^2)
\end{aligned}
\tag{30}
$$

Differentiating (30) with respect to f_1 and f_2 respectively and setting them equal to zero:

$$
\begin{aligned}
\frac{\partial SW}{\partial f_1} &= \frac{1}{2}\left\{v_2 - \frac{(v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)]}{[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \\
&\quad + 1 - 2\gamma f_1 = 0
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
\frac{\partial SW}{\partial f_2} &= \frac{1}{2}\left\{v_2 - \frac{(v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)]}{[4(v_2-v_o)-(v_1-v_o)]^2}\right\} \\
&\quad + 1 - 2\gamma f_2 = 0
\end{aligned}
\tag{32}
$$

Solving (31) and (32), we derive the feature improvements under social optimum:

$$
f_1^s = f_2^s = \frac{1}{4\gamma}\left\{2+v_2 - \frac{(v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)]}{[4(v_2-v_o)-(v_1-v_o)]^2}\right\}
\tag{33}
$$

A brief comparative assessment of the two regimes (profit-seeking firms and social planning) is now in order. We obtain the following results.

Proposition 2: the software feature improvements for both firms under private optimum are less than that under social optimum.
Proof:

$$
\begin{aligned}
f_2^s - f_2^* &= \frac{1}{4\gamma[4(v_2-v_o)-(v_1-v_o)]^2}\Big\{(2+v_2)[4(v_2-v_o)-(v_1-v_o)]^2 \\
&\quad - (v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)+8(v_2-v_o)]\Big\} > 0
\end{aligned}
$$

Because of

$$
\begin{aligned}
&(2+v_2)[4(v_2-v_o)-(v_1-v_o)]^2 \\
&= (2+v_2)[16(v_2-v_1)^2 + 24(v_2-v_1)(v_1-v_o) + 9(v_1-v_o)^2] \\
&> 16(v_2-v_1)^2 v_2 + 24(v_2-v_1)(v_1-v_o)v_2 \\
&> 12(v_2-v_1)^2(v_2-v_o) + 9(v_2-v_1)(v_1-v_o)(v_2-v_o) \\
&= (v_2-v_1)(v_2-v_o)[12(v_2-v_1)+9(v_1-v_o)] \\
&= (v_2-v_1)(v_2-v_o)[(v_1-v_o)+4(v_2-v_1)+8(v_2-v_o)]
\end{aligned}
$$

Therefore, $f_2^s > f_2^*$. Moreover, $f_1^s > f_1^*$ because of $f_1^s = f_2^s$ and $f_2^* > f_1^*$.

We use a numerical example to show the results of Proposition 2. When $v_2 = 2v_1 = 4v_o = 1$, there are $f_1^s > 114f_1^*$ and $f_2^s > 9f_2^*$.

Proposition 2 demonstrates that, contrast to the social optimum, COSS firms invest too little in software features under private optimum. The results of Proposition 1 and Proposition 2 indicate that, COSS firms have incentives to invest in software features under the GPL, but the innovation levels are much lower than the planner expects. The reason is that the GPL requires COSS firms open their any feature developments, which damages to firm' innovation incentives to invest in software features.

CONCLUSION

By extending the vertical differentiation model, this study investigates the R&D incentives for commercial open source software firms to invest in software features in a competitive market. We assume firms must follow the GNU General Public License when they develop the software features and find that:

- The commercial open source software firms have incentives to invest in software features despite the GNU General Public License requires firms make publicly available any feature contributions they make
- The high-usability firm's R&D incentive towards software features is higher than that of the low-usability firm

- From a public policy perspective, the commercial open source software firms' R&D incentive towards software features is too low under the GNU General Public License.

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