

Research Article

Comparison of Traditional Methods and Fractal Dimension Method in River Pattern Discrimination

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Abstract: The river patterns comprise of straight, meandering, braided and wandering river. They are formed and developed under the action of the long-term certain sediment-water assembly and the geological condition. There are many traditional methods such as single parameter including bend coefficient, width-depth ration and multiple-parameters discrimination methods, to distinguish the river pattern. In recent year, the fractal dimension method based on the fractal theory is developed and popular in the river pattern discrimination. In this study, the discrimination results are compared between the traditional methods and the fractal dimension method in the lower Yellow river, with the field observed data during 1960-2008. There are some differences in the discrimination results either between the traditional methods and the fractal dimension method or among the traditional methods because of the different indicators. But the discrimination results indicate the similar developing trend of the river regime. It means that the fractal dimension method is suitable for the river pattern discrimination and its degree of accuracy is in the same level with the traditional discrimination method. However, the further study should be carried out to find the critical values for the river pattern discrimination in the fractal method.

Keywords: Fractal dimension method, lower yellow river, river pattern, traditional methods

INTRODUCTION

The river patterns are used to describe the channel shape in plan and they are classified into four categories: straight river, meandering river, braided river and wandering river. There are a lot of approaches to distinguish the pattern of a certain river. The traditional methods for the river pattern discrimination include single parameter including bend coefficient, width-depth ration, mainstream swing and multiple-parameters discrimination methods. For each method, there are some critical values to separate the river patterns. In the Yellow River, the traditional methods are widely used for the river pattern discrimination whether in the past or at present. In recent years, a new method based the fractal theory is popular in the river pattern discrimination in many rivers at home and abroad. In this study, the new fractal dimension method is applied in the lower Yellow River for the river pattern discrimination and compared with the results obtained from the traditional methods.

Fractal dimension is based on the morphological characteristics of the non-integer dimension form filling space. After the fractal conception was presented by Mandelbrot (1982), many authors applied a fractal

analysis to river network and showed that the spatial distribution patterns of river network are fractal. The fractal theory is a branch of the nonlinear science and becomes an important tool for science research. For the ubiquitous irregularities in nature, fractal geometry can give a minute description of nature. At the very beginning, fractal theory is inseparable from the research. It has a very wide range of applications, from macromolecules to galaxies of the universe, the natural sciences to the social sciences and the non-linear dynamical systems to fully developed turbulence, earthquakes, econophysics and neuronal networks, etc. In a word, where autocorrelation exists, there is fractal.

The quantitative analysis of river networks has a long history. It can be dated back to the works by Horton (1932, 1945), who firstly studied the origin of river networks and their dendritic structures. La Barbera and Rosso (1989) associated the fractal dimension of the river networks with the bifurcation ratio, area ratio and length ratio defined in the Horton's laws and concerning the number, the average length and the slopes of the rivers in a river networks in a given hierarchical order. Moreover, many studies show that the fractal dimensions can be used to model the hydrologic response of a river basin through the Fractal

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Instantaneous Unit Hydrograph (FIUH) (Fiorentino *et al.*, 2002; Cudennec *et al.*, 2004). In the recent years, the applications of fractal geometry in the study of river network geometry have a great growth. Many researchers apply the fractal analysis to river network and show that the spatial distribution of a river network exhibits the fractal features.

This study will apply the fractal dimension method in the Lower Yellow River and discriminate the river regime.

EVOLUTION OF THE RIVER REGIME IN THE LOWER YELLOW RIVER

The Yellow River with abundant sediment is the second longest river in China. It passes four different tectonic provinces in North China from west to east, which results in widely different rock types and topographic surfaces. The Yellow River flows through the loess plateau region, where a large amount of sand is transported into the river. The channel bed and river bank of Yellow River are erodible and the mainstream is wandering.

The river regime is defined as the flattened form and the development trend of the river flow. It contains the location and tending towards of the axis of the hydrodynamic, distribution and trend of river bend, shoreline, sandbar and the heart of beach. It is a crucial factor sustaining the aquatic and riverine ecosystems. The river regime can be showed an irregular curve and its evolution mainly refers to the change of the flattened form of the river flow.

All the downstream channel of the Yellow River can be divided into the four sections: wandering river, the transitional reach, meandering river and estuary. The reach from Tiexie to Gaocun is the wandering river, Gaocun to Taochengpu is the transitional reach, Taochengpu to Lijin is the meandering river and the reach downstream Lijin station belongs to the estuary. The total length of the wandering reach is 299 km, which comprises of the five reaches: Tiexie to Yiluo River mouth (T-Y reach), Yiluo River mouth to Huayuankou (Y-H reach), Huayuankou to Heigangkou (H-H reach), Heigangkou to Jiahetan (H-J reach) and Jiahetan to Gaocun (J-G reach).

River regime evolution is greatly affected by the incoming water and sediment, boundary conditions and so on. In the Lower Yellow River, the river regime varies in different periods with the variations of the water and sediment conditions and the boundary conditions. Before the construction of Sanmenxia Reservoir in 1960, the Lower Yellow River was basically in the natural state of incoming water and sediment and the boundary conditions. For the wandering reach of the steep slope and large flow rate range, changes in river regime were very intense. Mainstream frequently swings between the levees and

sometimes the northern main channel could switch to the south only after one flood event. According to the statistics, the mainstream swing range is generally 5-7 km, with a maximum of 10 km.

During 1960-1990, with the construction of the Sanmenxia Reservoir, the incoming water and sediment conditions in the Lower Yellow River had greatly changed. Meanwhile, many river training works were planned for regulating the river channel and part of them were built, then the boundary conditions were strengthened and the swing range of the mainstream was limited. During this period, the mainstream swing decreased and the main flow was controlled between river training works. Some researches show that the average mainstream swing of the upper reach from Dongbatou was 3.5 to 4.2 km during 1960-1964 and 2.5 km during 1981-1990.

Since the beginning of the 1990s, especially in the late 1990s, the planned river training works were built and the river training system was greatly improved. The river channel was controlled and the mainstream swing was further reduced. In the Lower Yellow River, the river regime of some reaches such as Baihe to Shendi reach, Mazhuang to Laitongzhai reach and Chanfang to Gaocun reach became more stable than before. However, due to that some river training works didn't work well, the river regime in some reaches was still messy. During 1995-2004, the average mainstream swing in the Lower Yellow River was 1.11 km.

After the operation of Xiaolangdi Reservoir in 2000, the incoming water and sediment conditions in the Lower Yellow River have obviously changed. Most of the water from the upper reach is blocked in the reservoir in flood seasons for the flood defense and power generation and discharged into the lower reach in non-flood seasons. Therefore the daily flow hydrography has a great change. In addition, most of the coarse sediment from the upper reach was intercepted in the reservoir; only the fine sediment was let out in the practice of water and sediment regulations, which was carried out since 2002. For most of a year, the downstream channel is experiencing the clear water or the low sediment-laden water. During 2002-2012, the practice of water and sediment regulations has been executed for 13 times. The river channel in the Lower Yellow River has been scoured for 1-2m. The total amount of the erosion is 1.34 billion m³. With the operation of Xiaolangdi Reservoir, the river regime is getting regulated under the reduction of water and sediment. Coupled with the river training works, which limit the main flow through the course between the opposite engineering, the wandering channel is confined. Figure 1 to 3 are the river regime maps of the T-Y reach, H-H reach and J-G reach in 2000, 2004 and 2008. They lie in the head, middle and rear sections of the wandering reach. The river regime maps are drawn

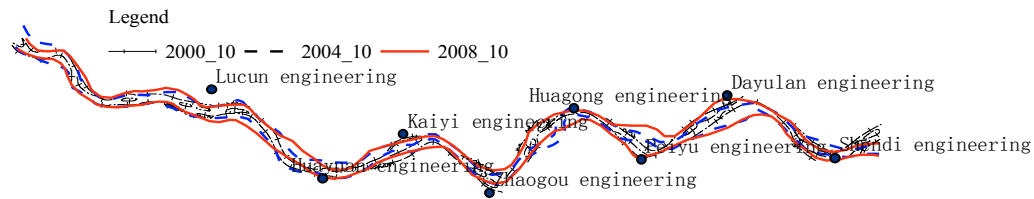


Fig. 1: The river regime in T-Y reach in 2000, 2004 and 2008

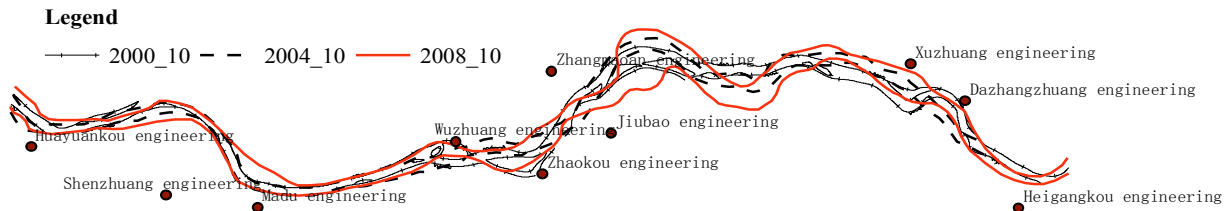


Fig. 2: The river regime in H-H reach in 2000, 2004 and 2008

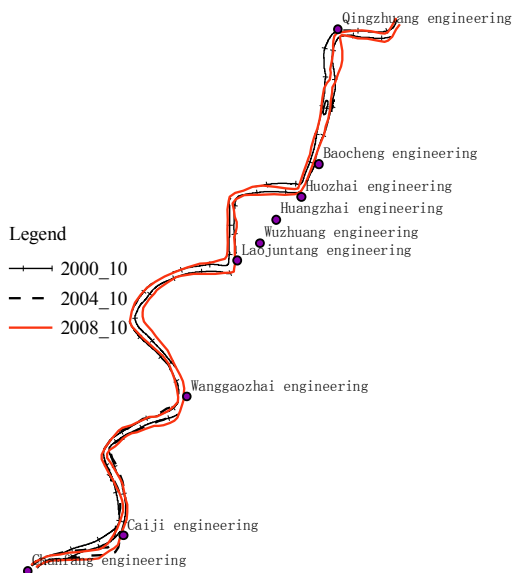


Fig. 3: The river regime in J-G reach in 2000, 2004 and 2008

by the Yellow River Conservancy Commission, Ministry of Water Resources, China and truly reflect the variations of the river regime. According to these maps, the water edges are close proximity to the river training works, which means the river regimes of the wandering reaches are becoming stable.

THE RIVER PATTERN DISCRIMINATED BY THE TRDATIONAL METHODS

Tradational methods: The channel gradient, median diameter, bend coefficient and width-depth ratio are general parameters for the discrimination of the river pattern (Wang *et al.*, 2010; Leopold and Wolman, 1957). However, some researchers insisted that river pattern is not determined by the single factor and put up

with the equations comprising of many parameters to discriminate the river pattern (Qian *et al.*, 1987; Xie, 2004; Zhang *et al.*, 1996). The single factor and the equations are listed as follows.

- **The single parameter for river pattern discrimination:**
 - Lu *et al.* (2002) indicated that bend coefficient ξ could be used to discriminate the river pattern and the corresponding value for the meandering river is $\xi > 1.3$.
 - In the Yellow River, the width-depth ration is defined as \sqrt{B}/h . Lu gave the critical value of \sqrt{B}/h for the different river pattern:
 - i. Meandering river as $3.5 \leq \sqrt{B}/h < 7$
 - ii. Straight river as $7 \leq \sqrt{B}/h < 18$
 - iii. Wandering river as $18 \leq \sqrt{B}/h < 24$

- **The multiple-parameters discrimination equation:** Zhang *et al.* (1996) carried out a series of physical model experiments in which the plastic sand, coal ash and the nature sediment were taken as the model sand, in order to achieve the index for the channel stability. In addition, Zhang has analyzed the field observation data and then concluded that the river pattern is relevant to the longitudinal and lateral stability. Thus, the synthesized stability indicator Z_w was obtained, as shown in Eq. (1).

$$Z_w = \frac{\left(\frac{\gamma_s - \gamma}{\gamma} D_{50} H \right)^{1/3}}{J B^{-2/3}} \quad (1)$$

in which, r is the density of water, $1 \times 10^3 \text{ kg/m}^3$; r_s is the density of the siltation, $2.65 \times 10^3 \text{ kg/m}^3$; D_{50} is the

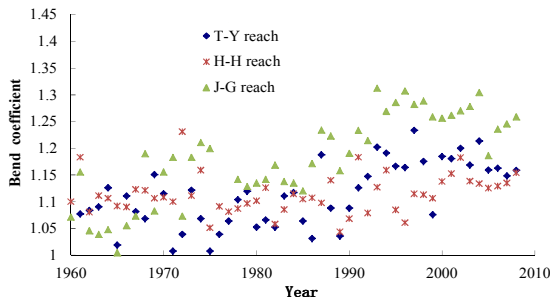


Fig. 4: The bend coefficient of the typical reaches in 1960-2008

median diameter of the bed load, mm; H , B are water depth and channel width respectively corresponding to the bankfull discharge, m; J is bed slope, $\times 10^{-4}$.

The critical values are: 1) $Zw < 5$ corresponding to the wandering river, 2) $5 \leq Zw \leq 15$ corresponding to the braided river and 3) $Zw > 15$ corresponding to the meandering river.

River pattern discriminated by the traditional methods:

- **Discrimination results by the single parameter:** Based on the measured data in the Lower Yellow River, the bend coefficient and width-depth ration of the three studied channel reach are calculated, as shown in Fig. 4

to 5. In the figures, year is the abscissa and the parameters of the river regime are the ordinate. These figures indicate that the bend coefficient has been increasing year by year and the width-depth ration has been decreasing since 1960.

- **Bend coefficient:** In Fig. 4, none of the bend coefficient of the studied reach is beyond 1.3, thus the three reaches are not meandering channel. But the bend coefficient has been increasing year by year. The average bend coefficient of T-Y reach, H-H reach and J-G reach are 1.18, 1.14 and 1.26 respectively in 2000-2008, which is the maximum during 1960-2008. It indicates that the river pattern transforms to the meandering river after the regulation of the reservoir.
- **Width-depth ratio:** Figure 5 shows the width-depth ratio \sqrt{B}/h in 2000-2008 is the minimum during 1960-2008. In the last few years, the width-depth ratio has been experiencing the reduction. Under the water and sediment regulation of Xiaolangdi Reservoir, the riverbed has been scoured continuously for about ten years and the bed elevation has decreased by 1-2m in the Lower Yellow River. Concomitant with it, the channel width has also broadened a little because of the bank slump. Nevertheless, the overall indicator width-depth ratio \sqrt{B}/h has decreased gradually. In the three channel reaches, \sqrt{B}/h reaches the minimum in 2000-2008. Table 1 gives the

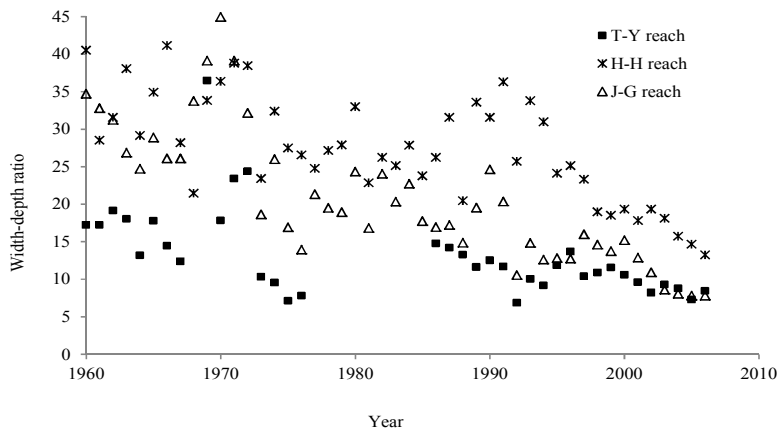


Fig. 5: The width-depth ratio of the studied reach in 1960-2008

Table 1: The width-depth ratio and the river regime features of the studied reaches in 1960-2008

| Period | T-Y reach | | H-H reach | | J-G reach | |
|-----------|--------------------|-----------------------|-------------------|-----------------------|-------------------|-----------------------|
| | Width- depth ratio | River regime features | Width-depth ratio | River regime features | Width-depth ratio | River regime features |
| 1960-1964 | 19 | Wandering | 34 | Wandering | 28 | Wandering |
| 1965-1973 | 31 | Wandering | 33 | Wandering | 30 | Wandering |
| 1974-1985 | 21 | Wandering | 27 | Wandering | 22 | Wandering |
| 1986-1992 | 14 | Straight | 30 | Wandering | 19 | Wandering |
| 1993-1999 | 12 | Straight | 26 | Wandering | 15 | Straight |
| 2000-2008 | 8 | Straight | 15 | Straight | 10 | Straight |

discrimination results by the width-depth ratio. In 2000-2008, the studied reaches have exhibited the features of the straight river.

The variation trend of all the single parameters: bend coefficient and width-depth ratio manifests that the wandering features of the Lower Yellow River are becoming weakened and the river regime is becoming stable. Although the discrimination results of the different parameters are different, all of them have the same conclusion that the river regime is transforming from the wandering and unstable river channel to the stable channel, which indicates that the operation of the reservoir is helpful to stabilize the main channel.

Discrimination results by the equation: According to Eq. (1), the annual values of the indicator Z_w are obtained from 1960 to 2008 and the corresponding river patterns are suggested in Table 2. Since 1993, J-G reach has exhibited the braided features and the indicator value became stable since 2002. Although H-H reach always shows the features of wandering channel, but the indicator values experience increasing, this indicates that the wandering degree has been inhibited. The indicator value of T-Y reach reaches the maximum and the river channel exhibits the braided features.

The discrimination results of the single parameter and the equation are consistent except for the divergence on the indicator values. So the conclusion can be drawn that the wandering river reach is becoming stable.

FRACTAL DIMENSION METHOD FOR RIVER PATTERN DISCRIMINATION

The fractal dimension of a natural river channel may be estimated by two methods: the divider method

(Richardson, 1961) and the box-counting method (Grassberger, 1983). The divider method was first used by Richardson (1961) to measure the length of complex curves and employed by Mandelbrot (1967) to estimate the fractal dimension of a coastline. Snow (1989) later much more precisely developed the divider method, used this method to characterize the fractal sinuosity of 12 single-channel stream segments in the western Ohio River drainage basin. Similarly, Beauvais and Montgomery (1996) used the divider method to reveal that river plan forms in the Cascade and Olympic Mountains of Washington State exhibit fractal scaling properties over scaling ranges bounded by the channel width and the largest meander wavelength and explore their relation to the geomorphological context defined by valley type.

This study focus on the T-Y reach, H-H reach and J-G reach and calculates the fractal dimensions of the mainstream of the studied reaches with the divider method. Its simple formulation as follows: if the relationship between object number N (the number of characteristic scales larger than 1 in a series) and $N-r$ - D is calibrated, the mathematical expression is:

$$N_i = cr^{-D} \tag{2}$$

$$D = \lim [\lg(N) / \lg(r)] \tag{3}$$

The value of D can be calculated from the slope of the line projection of N_i and r_i on double logarithmic coordinate system. The bigger is the value of the fractal dimensions, the more wandering is the channel. In versus, if the value of the fractal dimensions is small, it means the small oscillation range of the mainstream and the river is regulated.

Table 2: The river pattern discriminated by equation 1

| Years | T-Y reach | | H-H reach | | J-G reach | |
|-------|---------------------------|-----------------------|---------------------------|-----------------------|---------------------------|-----------------------|
| | Values of indicator Z_w | River regime features | Values of indicator Z_w | River regime features | Values of indicator Z_w | River regime features |
| 1960 | 2 | Wandering | 2 | Wandering | 2 | Wandering |
| 1973 | 5 | Braided | 3 | Wandering | 3 | Wandering |
| 1974 | 5 | Braided | 2 | Wandering | 3 | Wandering |
| 1976 | 5 | Braided | 3 | Wandering | 4 | Wandering |
| 1977 | 5 | Braided | 3 | Wandering | 4 | Wandering |
| 1978 | 5 | Braided | 3 | Wandering | 3 | Wandering |
| 1980 | 5 | Braided | 2 | Wandering | 3 | Wandering |
| 1981 | 4 | Wandering | 3 | Wandering | 3 | Wandering |
| 1988 | 4 | Wandering | 3 | Wandering | 4 | Wandering |
| 1989 | 5 | Braided | 2 | Wandering | 3 | Wandering |
| 1993 | 4 | Wandering | 2 | Wandering | 5 | Braided |
| 1998 | 4 | Wandering | 4 | Wandering | 5 | Braided |
| 1999 | 4 | Wandering | 4 | Wandering | 6 | Braided |
| 2002 | 4 | Wandering | 3 | Wandering | 5 | Braided |
| 2003 | 3 | Wandering | 3 | Wandering | 5 | Braided |
| 2004 | 4 | Wandering | 4 | Wandering | 5 | Braided |
| 2005 | 3 | Wandering | 3 | Wandering | 6 | Braided |
| 2006 | 3 | Wandering | 4 | Wandering | 4 | Wandering |
| 2007 | 3 | Wandering | 4 | Wandering | 5 | Braided |
| 2008 | 7 | Braided | 4 | Wandering | 5 | Braided |

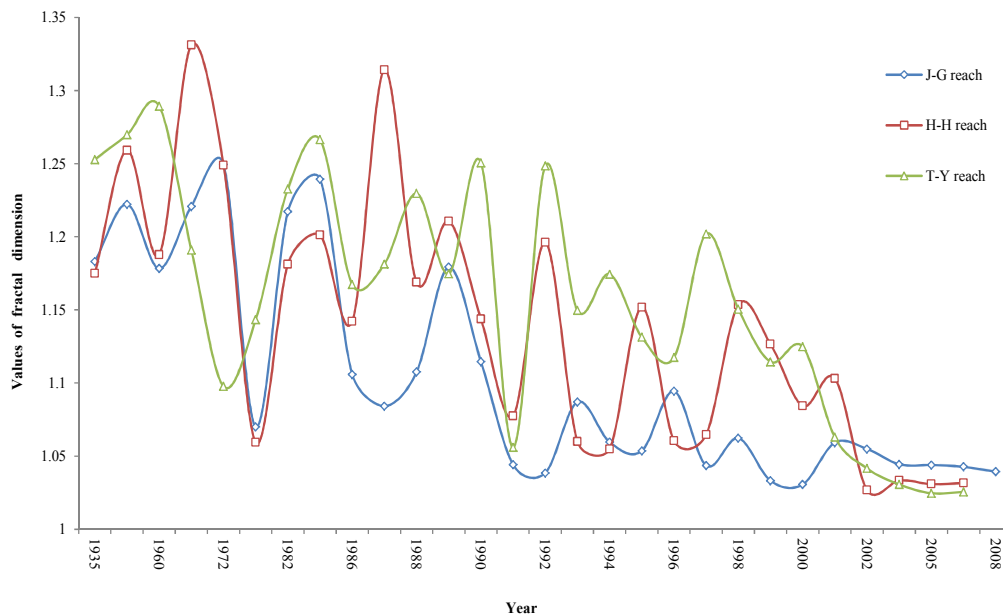


Fig. 6: The fractal dimension values of the studied reaches

The values of the fractal dimension of the three reaches are shown in Fig. 6. The values have exhibited the declining tendency since 1935 and reach the minimum (<1.05) during 2002-2008. In Fig. 4, the values of the fractal dimension of J-G reach are always lower than the other reaches. Since the late 1990s, the values of the fractal dimension of T-Y reach and J-G reach were both obviously decreased. After the operation of Xiaolangdi Reservoir, the values of the fractal dimension of the studied reaches decrease rapidly. The results of the fractal dimension method are consistent with the river regime maps and the previous study, which indicates that the fractal dimension method is suitable for the river regime discrimination.

DISCUSSION

The study attempts to introduce the fractal dimension methods for the river pattern estimation other than the traditional methods. After the fractal conception was presented by Mandelbrot (1982), many authors applied a fractal analysis to river network and showed that the spatial distribution patterns of river network are fractal. However, there are not many studies concerning individual river channels, as most studies were focused on drainage patterns (Hjelmfelt, 1988; La Barbera and Rosso, 1989; Tarboton *et al.*, 1988; Veltri *et al.*, 1996; Tarboton, 1996; Nikora *et al.*, 1993; Schuller *et al.*, 2001; De Bartolo *et al.*, 2006), such as relating the fractal dimension estimated to the bifurcation ratio, length ratio and area ratio defined in Horton's laws (Shen *et al.*, 2011). A natural river channel can be described broadly as sinuous in plan. A magnified view of its any small part is also sinuous in

shape. Fractals, which also have this property, are the useful tool for quantitatively describing channel sinuosity in shape.

The fractal dimensions values of the mainstream of the studied reaches are obtained with the divider method. The bigger is the value of the fractal dimensions, the more wandering is the channel. In versus, if the value of the fractal dimensions is small, it means the small oscillation range of the mainstream and the river is regulated. The values of the fractal dimension of the three reaches are shown in Fig. 6 The values have exhibited the declining tendency since 1935 and reach the minimum (<1.05) during 2002-2008.

The discrimination results of the traditional methods and the fractal dimension method are consistent in the developing trend of the river pattern. However, there are no critical values for the river pattern discrimination in the fractal dimension method. So the further study is necessary to find the critical values for the fractal dimension method.

CONCLUSION

This study introduces the fractal dimension method based on the fractal theory to describe the river regime. The fractal dimension values of the three reaches in the lower Yellow River are obtained and compared with the traditional methods. According to the discrimination results, both the traditional methods and the fractal method indicate that the wandering features of the lower Yellow River have been becoming weakened and the river regime has been becoming stable.

The discrimination results of the fractal dimension method are consistent with that obtained by the traditional methods. It means the fractal dimension method is fit with the river pattern discrimination.

There are no critical values in the fractal dimension method for the river pattern discrimination. And the further study is needed to find the critical values.

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