Research Journal of Applied Sciences, Engineering and Technology 5(24): 5588-5591, 2013
DOI:10.19026/rjaset.5.4240
ISSN: 2040-7459; e-ISSN: 2040-7467
© 2013 Maxwell Scientific Publication Corp.
Submitted: October 12, 2012 Accepted: December 12, 2012 Published: May 30, 2013

Research Article

Solution of the Euler Equations Based on Flux Vector Splitting

[1]Ge Xinfeng, [1]Sun Gang and [2]Su Keqin
[1]College of Electrical and Information Engineering, Xuchang University, China
[2]College of Information and Management Science Henan Agricultural University,
Zhengzhou 450002, China

Abstract: The basic properties of the Euler equations are learned by studying hyperbolic PDEs and the Steger-Warming flux vector splitting approach and the NND scheme are given. The NND scheme based on improved Steger-Warming flux vector splitting is used for the solution of 1-D and shown that the numerical oscillation is restrained .The finite difference method has higher order of accuracy and better efficiency.

Keywords: Euler equations, flux vector splitting, NND scheme

INTRODUCTION

Building an effective method to capture shock waves that has been an important part of the study in Computational Fluid Dynamics (CFD) field (Van der Geer *et al*., 2000; Strunk and White, 1979; Baghlani, 2011). John Von Neumann first proposed the use of artificial viscosity method to capture shock and the method of artificial viscosity is still one of the cores in the CFD. The emergence of artificial viscosity means losing the part of information and lowering the computational accuracy. For the non-physical oscillations problems generated near the shock wave, Hearten proposed the total variation difference scheme-the concept of TVD scheme, which the total variation is decreasing and constructed the specifically TVD scheme with second order accuracy. Meanwhile, Steger and Warming proposed a new class of upwind scheme - vector splitting scheme (FVS scheme). Steger-Warming splitting method are often used by other schemes to better capture the shock wave for its characteristics of high efficiency and easy to program. The difference method that can automatically capture the shock wave is proposed based on the combination of improved Steger-Warming splitting method and NND scheme. The algorithm maintained the characteristics of high efficiency and easy to program that Steger-Warming splitting method have had and overcome the numerical oscillation near the shock wave that in Steger-Warming splitting method and had accuracy and manoeuvrability. Method proposed in this study has some reference value to the calculation of vector splitting.

THE FVS SPLIT OF EULER EQUATIONS

Conservation form of one-dimensional Euler equations is:

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}=0 \tag{1}$$

Which

$$U=\begin{pmatrix}\rho\\ \rho u\\ E\end{pmatrix}\quad F(U)=\begin{pmatrix}\rho u\\ \rho u^2+p\\ u(E+p)\end{pmatrix} \tag{2}$$

Here,

$$E=\frac{1}{2}\rho u^2+\frac{p}{\gamma-1}\quad a=\sqrt{\frac{\gamma p}{\rho}}$$

According to Steger-Warming splitting method (Yin and Ge, 2012; Toro, 2009):

$$A=\frac{\partial F}{\partial U}=\begin{pmatrix}0 & 1 & 0\\ \frac{1}{2}(\gamma-1)u^2 & (3-\gamma)u & \gamma-1\\ \frac{1}{2}(\gamma-2)u^3-\frac{a^2u}{\gamma-1} & \frac{1}{2}(3-2\gamma)u^2+\frac{a^2}{\gamma-1} & \gamma u\end{pmatrix} \tag{3}$$

The real eigenvalues are:

$$\lambda_1=u-a\quad \lambda_2=u\quad \lambda_3=u+a \tag{4}$$

The corresponding left and right eigenvector matrix:

$$K^{-1}=\frac{(\gamma-1)}{2a^2}\begin{pmatrix}\frac{1}{2}u^2+\frac{ua}{(\gamma-1)} & -u-\frac{a}{(\gamma-1)} & 1\\ \frac{2a^2}{(\gamma-1)}-u^2 & 2u & -2\\ \frac{1}{2}u^2-\frac{ua}{(\gamma-1)} & \frac{a}{(\gamma-1)} & 1\end{pmatrix} \tag{5}$$

Corresponding Author: Ge Xinfeng, College of Electrical and Information Engineering, Xuchang University, China
This work is licensed under a Creative Commons Attribution 4.0 International License (URL: http://creativecommons.org/licenses/by/4.0/).

$$K = \begin{pmatrix} 0 & 1 & 0 \\ u-a & u & u+a \\ H-ua & \frac{1}{2}u^2 & H+ua \end{pmatrix} \tag{6}$$

Here

$$H = (E+p)/\rho = \frac{1}{2}u^2 + \frac{a^2}{\gamma - 1} = \frac{1}{2}u^2 + \frac{\gamma p}{(\gamma - 1)\rho} \tag{7}$$

The diagonal matrix is satisfied:

$$\Lambda = diag[\lambda_1 , \lambda_2 , \lambda_3] \tag{8}$$

$$A = K\Lambda K^{-1} \tag{9}$$

Now splitting the flux vector and first, the eigenvalues of Jacobian matrix, that are λ_1, λ_2 and λ_3, split the sum form of two:

$$\lambda_i = \lambda_i^+ + \lambda_i^- \;;\; \lambda_i^+ \geq 0 \,,\, \lambda_i^- \leq 0 \quad \text{i}=1,2,3 \tag{10}$$

According to flux vector splitting method:

$$\lambda_i^{\pm} = (\lambda_i \pm |\lambda_i|)/2 \quad , \quad \text{i}=1,2,3 \tag{11}$$

Thus, the eigenvalue matrix after splitting is:

$$\Lambda^{\pm} = diag[\lambda_1^{\pm} , \lambda_2^{\pm} , \lambda_3^{\pm}] \tag{12}$$

And the corresponding Jacobian matrix after splitting is:

$$A^{\pm} = K\Lambda^{\pm}K^{-1} \tag{13}$$

The corresponding flux vector is available after splitting in the end:

$$\mathrm{F}^{\pm} = A^{\pm}U \tag{14}$$

Flux terms can be written as:

$$F = F^+ + F^- \tag{15}$$

Which

$$F^+ = \frac{\rho}{2\gamma}\begin{bmatrix} \lambda_1^+ + 2(\gamma-1)\lambda_2^+ + \lambda_3^+ \\ (u-a)\lambda_1^+ + 2(\gamma-1)u\lambda_2^+ + (u+a)\lambda_3^+ \\ (H-ua)\lambda_1^+ + (\gamma-1)u^2\lambda_2^+ + (H+ua)\lambda_3^+ \end{bmatrix} \tag{16}$$

$$F^- = \frac{\rho}{2\gamma}\begin{bmatrix} \lambda_1^- + 2(\gamma-1)\lambda_2^- + \lambda_3^- \\ (u-a)\lambda_1^- + 2(\gamma-1)u\lambda_2^- + (u+a)\lambda_3^- \\ (H-ua)\lambda_1^- + (\gamma-1)u^2\lambda_2^- + (H+ua)\lambda_3^- \end{bmatrix} \tag{17}$$

Here,

$$\lambda_1 = u - a \,,\, \lambda_2 = u \;\; \lambda_3 = u + a \tag{18}$$

$$\lambda_1^+ = \frac{1}{2}(\lambda_1 + |\lambda_1|) \,,\, \lambda_1^- = \frac{1}{2}(\lambda_1 - |\lambda_1|) \tag{19}$$

$$\lambda_3^+ = \frac{1}{2}(\lambda_3 + |\lambda_3|) \,,\, \lambda_3^- = \frac{1}{2}(\lambda_3 - |\lambda_3|) \tag{20}$$

Because the first derivative of flux vector is no continuous near the changing point of Eigenvalue (Steger and Warming, 1981; Drikakis and Tsangaris, 1993) and leading non-physical oscillations of vector splitting scheme generated in the area. In order to make the scheme transit smoothly near the changing point of Eigenvalue and change the flux vector into a continuous function aboutλ_1 andλ_3 (Zhang, 1988).

$$\lambda_2^+ = \frac{1}{2}(\lambda_1^+ + \lambda_3^+) \quad , \quad \lambda_2^- = \frac{1}{2}(\lambda_1^- + \lambda_3^-) \tag{21}$$

Steger-Warming splitting vector method does not involve matrix operations, positive and negative fluxes have the same expressions and the computation is little. As the derivatives of positive and negative flux are not continuous on the changing point of Eigen value, in the numerical calculation numerical solution oscillation will occur near the changing point of Eigen value for discontinuous of the flux derivative and affecting the precision and stability. Therefore, Steger-Warming splitting vector methods were used to calculating, numerical oscillations near the changing point of Eigen value needed to suppress.

NND SCHEME AND THE NEW SCHEME

The NND scheme (Zhang and Zhuang, 1992) is a computational scheme that has a non-volatile, no free parameters characteristic and can effectively suppress numerical calculation oscillation. From the modified equation in the scheme, different schemes were used according to the location of relative break points and physical meaning is clear. For Eq. (1), explicit scheme of NND scheme can be expressed as follows:

$$
\begin{aligned}
U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x} \cdot \{ & F_j^+ + \frac{1}{2}\operatorname{minmod}(\Delta F_{j-1/2}^+, \Delta F_{j+1/2}^+) + \\
& F_{j+1}^- - \frac{1}{2}\operatorname{minmod}(\Delta F_{j+1/2}^-, \Delta F_{j+3/2}^-) - \\
& F_{j-1}^+ - \frac{1}{2}\operatorname{minmod}(\Delta F_{j-3/2}^+, \Delta F_{j-1/2}^+) - \\
& F_j^- + \frac{1}{2}\operatorname{minmod}(\Delta F_{j-1/2}^-, \Delta F_{j+1/2}^-) \ \}
\end{aligned} \tag{22}
$$

Here,

$$
\Delta F_{j-1/2}^+ = F_j^+ - F_{j-1}^+, \quad \Delta F_{j-1/2}^- = F_j^- - F_{j-1}^- \tag{23}
$$

Others so, which

$$
minmod(x, y) = \begin{cases} 0 & x \cdot y \le 0 \\ sign(x) \cdot min(|x|, |y|) & x \cdot y > 0 \end{cases} \tag{24}
$$

Table 1: Initial data for the flow field

Test	ρ_L	u_L	p_L	ρ_R	u_R	p_R
1	1.0	0.0	1.0	0.125	0.0	0.1
2	5.99924	19.5975	460.894	5.99242	-6.19633	46.0950

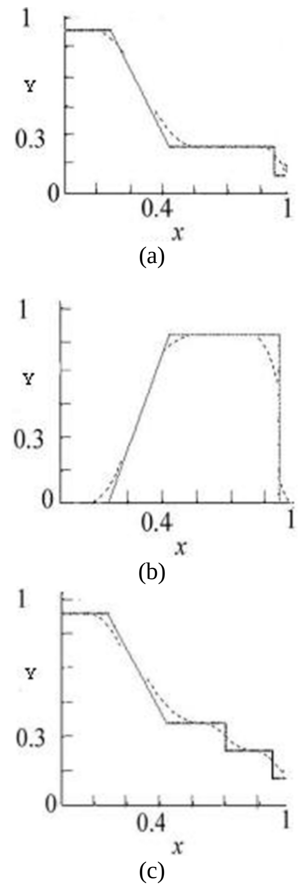

Fig. 1: The method applied to Test 1.Numerical (symbol) and exact (line) solutions are compared at the output time 0.25 units: (a) pressure, (b) velocity and (c) density

This is can be proved that the Total Variation Difference (TVD) is reduced in the scheme, need not to select the free parameters and has the space second order accuracy. It can effectively capture the shockwave; it is the combination of the second-order central scheme and a second-order upwind scheme and has fourth-order dissipation of negative coefficient. The scheme also suppress the parity oscillations, therefore the scheme is used widely (Baghlani, 2008). (16), (17) and (22) are the difference method that we established combing improved pass Steger-Warming splitting vector method. The established scheme has the space second order accuracy except extreme point and the scheme can suppress the numerical calculation oscillation effectively.

EXAMPLE

The calculation of the flow field in shock wave Pipe is the common example to test ability in capturing the shock wave. The answer to this problem can compare to the Riemann's analytical solution. Assumed the gas in per unit length shock tube is the same gas with some specific heat capacity in this study x denotes the coordinates of grid and there have totaled 100 grid points. Shock wave tube was divided into two groups (ρ_L,u_L,p_L) and (ρ_R,u_R,p_R) from the initial data at $x = 0.5$, two different initial data in Table 1 were used to

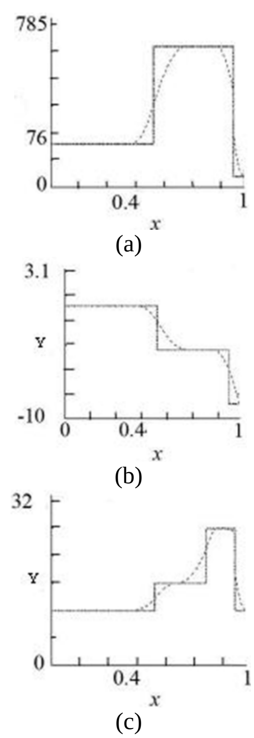

Fig. 2: The method applied to Test 2.Numerical (symbol) and exact (line) solutions are compared at the output time 0.035 units: (a) pressure, (b) velocity and (c) density

experiment. The pressure, velocity and density distribution calculated by the scheme proposed in this study are shown in Fig. 1 and 2.

The computational scheme that constructed by combination of Steger-Warming splitting vector method and the NND scheme has good ability in capturing shock wave for the single shock in test 1 and the dual shock in test 2.The calculated results and the theoretical values are in good agreement and there is no calculated value oscillation appeared near shock wave.

CONCLUSION

The computational scheme that constructed by combination of Steger-Warming splitting vector method and the NND scheme has good ability in capturing shock wave and has a good precision and efficiency. The characteristics that NND scheme can suppress numerical oscillations good and Steger-Warming vector splitting method has high precision were taken advantage of in this study and combined the two, so the scheme not only has high efficiency, but also can suppress the numerical oscillation, this method maintaining the precision and stability, Ensuring to capture the shock wave in flow field effectively.

REFERENCES

Baghlani, A., 2008. A shock-capturing model based on flux-vector splitting method in boundary-fitted curvilinear coordinates. Appl. Math. Mod., 32(3): 249-266.

Baghlani, A., 2011. Simulation of dam-break problem by a robust flux-vector splitting approach in Cartesian grid. Scient. Iran., 18(5): 1061-1068.

Drikakis, D. and S. Tsangaris, 1993. On the solution of the compressible Navier-stokes equations using improved flux vector splitting methods. Appl. Math. Modell., 17: 282-296.

Steger, J.L. and R.F. Warming, 1981. Flux vector splitting of the inviscid gasdynamic equations with application to finite difference method. J. Comput. Phys., 40: 263-293.

Strunk, W.J. and E.B. White, 1979. The Elements of Style. 3rd Edn., Macmillan, New York.

Toro, E.F., 2009. Riemann Solvers and Numerical Methods for Fluid Dynamics: A Practical Introduction. 3nd Ed, Springer, Berlin, pp: 724.

Van der Geer, J., J.A.J. Hanraads and R.A. Lupton, 2000. The art of writing a scientific article. J. Sci. Commun., 163: 51-9.

Yin, Z.F. and X.F. Ge, 2012. The difference method of 2-dimensional Euler equations with flux vector splitting. Lect. Notes Elec. Eng., 135: 485-491.

Zhang, H.X., 1988. Non-oscillatory and non-free-parameter dissipation difference scheme. Acta Aerodyn. Sinica, 6: 143-165.

Zhang, H.X. and F.G. Zhuang, 1992. NND schemes and their application to numerical simulation of two and three dimensional flows. Adv. Appl. Mech., 29: 193-256.