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Research Article

Random Scrap Rate Effect on Multi-Item Finite Production Rate Model with Multi-Shipment Policy

[1]Singa Wang Chiu, [2]Yi-Yu Kuo, [2]Chao-Chih Huang and [2]Yuan-Shyi Peter Chiu
[1]Department of Business Administration,
[2]Department of Industrial Engineering and Management, Chaoyang University of Technology, Taichung 413, Taiwan

Abstract: This study investigates the effect of random scrap rate on multi-item Finite Production Rate (FPR) model with multi-shipment policy. The classic FPR model considers production planning for single product, a perfect condition during the production run and a continuous inventory issuing policy. However, in real life manufacturing environments, in order to maximize machine utilization, vendors often make plan to produce *m* products in turn on a single machine. Also, in any given production run due to various different factors, generation of nonconforming items is inevitable. In this study, it is assumed that these defective items cannot be repaired, thus they must be scrapped with an additional cost and delivery of finished products is under a practical multiple shipment policy. Our objective is to determine an optimal common production cycle time that minimizes the long-run average cost per unit time and to investigate the effect of random scrap rate on the optimal common cycle time. Mathematical modeling is employed and renewal reward theorem is used to cope with the variable cycle length. The expected system cost for the proposed multi-item FPR model is derived and its convexity is proved. A closed-form optimal common production cycle time is obtained. A numerical example and sensitivity analysis is provided to demonstrate the practical use of the obtained results.

Keywords: Common cycle time, finite production rate, inventory, manufacturing, multi-shipment, scrap

INTRODUCTION

The classic Finite Production Rate (FPR) model (Hadley and Whitin, 1963; Hillier and Lieberman, 2001; Nahmias, 2009) considers the production planning for single product. However, in real life manufacturing sector in order to maximize machine utilization, most vendors often make plan to produce *n* products in turn on a single machine. Bergstrom and Smith (1970) used the Linear Decision Rules (LDR) to a Multi- Item Formulation (MDR) which solves directly for the optimum sales, production and inventory levels for individual items in future periods. It is shown that the MDR can seek a solution to maximize profit for the firm over the time horizon by an application in a firm producing a line of electric motors. Dixon and Silver (1981) considered determination of lot-sizes for a group of products which are produced at a single work center. It was assumed that the requirements for each product are known (period by period), out to the end of some common time horizon. For each product there is a fixed setup cost incurred each time production takes place. Unit production and holding costs are linear. Machine set up time is assumed to be negligible.

All costs and production rates can vary from product to product. In each period there is a finite amount of machine time available that can vary from period to period. Their objective is to determine lot-sizes so that:

- Costs are minimized
- No backlogging occurs
- Capacity is not exceeded

A simple heuristic was developed for deriving the feasible solution. Results of a large number of test problems indicate that their heuristic can usually generate a very good solution with a relatively small amount of computational effort. Aggarwal (1984) indicated that multi-item inventory control may be simplified by grouping items into subgroups with a common order cycle for all the items in each group. The methods provided in the literature for determination of the order cycle values are either suboptimal or computationally inefficient. He proposed a procedure which finds the optimal values and is also computationally efficient. Studies related to common cycle time have been extensively conducted to address various aspects of multi-item production planning and

Corresponding Author: Yuan-Shyi Peter Chiu, Department of Industrial Engineering and Management, Chaoyang University of Technology, Taichung 413, Taiwan

optimization issues (Ware and Keown Sr, 1987; Banerjee and Banerjee, 1992; Gupta, 1992; Hahm and Arai Yano, 1995; Khouja, 2000; Clausen and Ju, 2006; Lin *et al*., 2006; Ma *et al*., 2010; Yao *et al*., 2012).

Another unrealistic assumption in classic FPR model is that all items produced are of perfect quality. However, in real life production systems due to many unpredictable factors, generation of defective items seems to be inevitable. Studies have been carried out to address various aspects of imperfect quality issues in production (Schneider, 1979; Bielecki and Kumar, 1988; Chern and Yang, 1999; Teunter and Flapper, 2003; Ojha *et al*., 2007; Lin *et al*., 2008; Chiu *et al*., 2009a, b; Lee *et al*., 2011; Chiu *et al*., 2011; Chen, 2011; Chiu *et al*., 2012a). Unlike the classic FPR model considers a continuous inventory issuing policy to satisfy product demand, in real world situations the multiple or periodic deliveries of finished products are often used. Goyal (1977) studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier and an example was provided to illustrate his proposed method. Banerjee (1986) studied a joint economic lot-size model for purchaser and vendor, with the focus on minimizing the joint total relevant cost. He concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either party. Additional studies dealing with different aspects of vendor- buyer supply chain optimization issues can also be found (see for example: Parija and Sarker, 1999; Yao and Chiou, 2004; Hoque, 2008; Chiu *et al*., 2011; Chiu *et al*., 2012b, c).

This study investigates the effect of random scrap rate on the multi-item FPR model with multi- shipment policy. The objective of this study is to determine an optimal common production cycle time that minimizes the long-run average cost per unit time and to investigate the effect of random scrap rate on the optimal common cycle time in the multi-item FPR model.

MATHEMATICAL MODELLING AND ANALYSIS

This study considers a multi-item Finite Production Rate (FPR) model with random scrap rate and multi-shipment policy. It is assumed that a manufacturing process produces m products in turn on a single machine. All items made are screened and unit inspection cost is included in unit production cost C_i. During the production process for product i (where i = 1, 2 … m) a x_i portion of nonconforming items is produced randomly at a rate d_i. All nonconforming items cannot be repaired and thus they must be scrapped in the end of production with an additional cost C_{Si}. Under the normal operation, to avoid shortages from occurring, the constant production rate for product i, P_i satisfies $(P_i-d_i-\lambda_i)>0$, where λ_i is the annual demand rate for product i and d_i can be expressed as $d_i = x_iP_i$. Unlike the classic FPR model assuming a continuous issuing policy for meeting demand, this research considers a multi-shipment policy. It is also assumed that the finished items for each product i can only be delivered to customers if the whole production lot is quality assured at the end of production of product i. Fixed quantity n installments of the finished batch are delivered at a fixed interval of time during delivery time t_{2i} (Fig. 1). Other cost parameters used in this study include: the production setup cost K_i, unit holding cost h_i, the fixed delivery cost K_{1i} per shipment for product i and unit shipping cost C_{Ti} for product i. Additional notation is listed as follows:

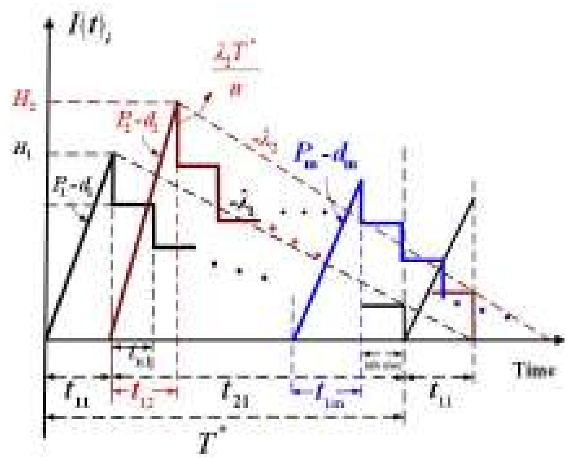

Fig. 1: On-hand inventory of perfect quality items for product i in a cycle

T	=	Common production cycle length, a decision variable (to be determined)
Q_i	=	Production lot size per cycle for product i
n	=	Number of fixed quantity installments of the finished batch to be delivered to customers in each cycle, it is assumed to be a constant for all products
t_{1i}	=	The production uptime for product i in the proposed system
H_i	=	Maximum level of on-hand inventory in units for product i when regular production process ends
t_{ni}	=	A fixed interval of time between each installment of finished products delivered during t_{2i}, for product i
$I(t)_i$	=	On-hand inventory of perfect quality items for product i at time t
$I_c(t)_i$	=	On-hand inventory of scrap items for product i at time t

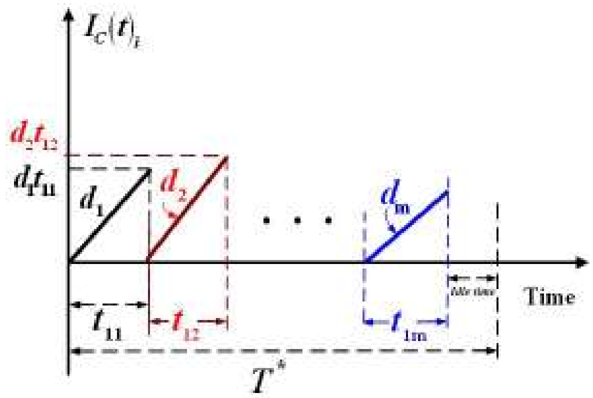

Fig. 2: On-hand inventory of scrap items for product *i* in a cycle

$TC(Q_i)$ = Total production-inventory-delivery costs per cycle for product *i* in the proposed system

E $[TCU(Q)]$ = Total expected production-inventory-delivery costs per unit time for *m* products in the proposed system.

E $[TCU(T)]$ = Total expected production-inventory-delivery costs per unit time for *m* products in the proposed system using common production cycle time as the decision variable.

From Fig. 1, one directly observes the following equations:

$$T = t_{1i} + t_{2i} \tag{1}$$

$$t_{1i} = \frac{Q_i}{P_i} = \frac{H_i}{P_i - d_i} \tag{2}$$

$$t_{2i} = nt_{ni} = Q_i\left(\frac{(1-x_i)}{\lambda_i} - \frac{1}{P_i}\right) \tag{3}$$

$$H_i = (P_i - d_i)t_{1i} = (P_i - d_i)\frac{Q_i}{P_i} = (1-x_i)Q_i \tag{4}$$

$$T = \frac{Q_i}{\lambda_i}(1-x_i) \tag{5}$$

The on-hand inventory of scrap items during production uptime t_{1i} is (Fig. 2)

$$d_i t_{1i} = P_i x_i t_{1i} = x_i Q_i \tag{6}$$

Total delivery costs for product *i* (*n* shipments) in a cycle are:

$$nK_{1i} + C_{Ti}Q_i(1-x_i) \tag{7}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_{2i} where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows (Chiu *et al*. (2009b)).

$$h\left(\frac{n-1}{2n}\right)H_i t_{2i} \tag{8}$$

Total production-inventory-delivery cost per cycle *TC* (Q_i) for *m* products, consists of the variable production cost, setup cost, disposal cost, fixed and variable delivery cost, holding cost during production uptime t_{1i} and holding cost for finished goods kept during the delivery time t_2. Therefore, total *TC* (Q_i) for or *m* products are:

$$\sum_{i=1}^{m} TC(Q_i) = \sum_{i=1}^{m}\left\{\begin{aligned} &C_iQ_i + K_i + C_{Si}x_iQ_i + nK_{1i} + C_{Ti}Q_i(1-x_i) \\ &+h_i\left[t_{1i}\left(\frac{H_i}{2} + \frac{x_iQ_i}{2}\right) + \frac{n-1}{2n}(H_it_{2i})\right]\end{aligned}\right\} \tag{9}$$

Because scrap rate *x* is assumed to be a random variable with a known probability density function. In order to take the randomness of *x* into account, the expected values of *x* can be used in the cost analysis. Substituting all parameters from Eq. (1) to (8) in Eq. (9) and applying the renewal reward theorem and with further derivations the expected E [*TCU* (*Q*)] can be obtained as follows:

$$\begin{aligned} E[TCU(Q)] &= E\left[\sum_{i=1}^{m} TC(Q_i)\right]\frac{1}{E[T]} \\ &= \sum_{i=1}^{m}\left\{\begin{aligned} &C_i\lambda_i\frac{1}{[1-E(x_i)]} + C_{Si}\lambda_i\frac{E(x_i)}{[1-E(x_i)]} + \frac{K_i\lambda_i}{Q_i}\frac{1}{[1-E(x_i)]} + C_{Ti}\lambda_i + \frac{nK_{1i}\lambda_i}{Q_i}\frac{1}{[1-E(x_i)]} \\ &+\frac{h_iQ_i}{2_i}\left[\frac{\lambda_i}{P_i}\frac{E(x_i)}{[1-E(x_i)]} + 1 - E(x_i) - \frac{1}{n}\left(1 - E(x_i) - \frac{\lambda_i}{P_i}\right)\right]\end{aligned}\right\}\end{aligned} \tag{10}$$

where, $E[T] = Q_i(1-E[x_i])\frac{1}{\lambda_i}$.

Applying Eq. (5) one can converting Eq. (10) into E [*TCU* (*T*)] as follows:

$$E[TCU(T)] = \sum_{i=1}^{m}\left\{\begin{aligned} &C_i\lambda_i\cdot\frac{1}{[1-E(x_i)]} + C_{Si}\lambda_i\frac{E(x_i)}{[1-E(x_i)]} + \frac{K_i}{T} + C_{Ti}\lambda_i + \frac{nK_{1i}}{T} \\ &+\frac{h_iT\lambda_i}{2}\left[\frac{\lambda_i}{P_i}\frac{E(x_i)}{[1-E(x_i)]}\cdot\frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n}\left(1 - \frac{\lambda_i}{P_i}\frac{1}{[1-E(x_i)]}\right)\right]\end{aligned}\right\} \tag{11}$$

DERIVATION OF THE OPTIMAL COMMON CYCLE TIME

Convexity of E [TCU (T)]: The optimal common production cycle time can be obtained by minimizing expected cost function E [TCU (T)]. Differentiating E [TCU (T)] with respect to T gives first and second derivative as

$$\frac{\partial E[TCU(T)]}{\partial T}=\sum_{i=1}^{m}\left\{\begin{array}{l}-\dfrac{(K_i+nK_{1i})}{T^2}\\+\dfrac{h_i\lambda_i}{2}\left[\dfrac{\lambda_i}{P_i}\dfrac{E(x_i)}{[1-E(x_i)]^2}+1-\dfrac{1}{n}+\dfrac{1}{n}\left(\dfrac{\lambda_i}{P_i}\dfrac{1}{[1-E(x_i)]}\right)\right]\end{array}\right\} \tag{12}$$

$$\frac{\partial^2 E[TCU(T)]}{\partial T^2}=\sum_{i=1}^{m}\left\{\frac{2(K_i+nK_{1i})}{T^3}\right\} \tag{13}$$

Eq. (13) is resulting positive for K_i, n, K_{1i} and T are all positive. Second derivative of E [TCU (T)] with respect to T [Eq. (13)] is greater than zero and hence E [TCU (T)] is a convex function for all T different from zero.

Derivation of T*: The optimal common production cycle time T^* can be obtained by setting first derivative [Eq. (12)] of E [TCU (T)] equal to zero. Let $E_0 = [1-E(x_i)]^{-1}$ and $E_1 = E(x_i)\,[1-E(x_i)]^{-1}$, then:

$$\frac{\partial E[TCU(T)]}{\partial T}=\sum_{i=1}^{m}\left\{\begin{array}{l}-\dfrac{(K_i+nK_{1i})}{T^2}\\+\dfrac{h_i\lambda_i}{2}\left[1+\dfrac{\lambda_i}{P_i}E_0E_1-\dfrac{1}{n}\left(1-\left(\dfrac{\lambda_i}{P_i}E_0\right)\right)\right]\end{array}\right\}=0 \tag{14}$$

NUMERICAL EXAMPLE

Consider a manufacturer has a routine production plan that is to produce five products in turn on a single machine using a common cycle policy. Annual demands λ_i for 5 different products are 3000, 3200, 3400, 3600 and 3800 respectively (total annual demand is 17000). Each product has its own production rate P_i and they are 58000, 59000, 60000, 61000 and 62000, respectively. Random defective rates during production uptime for each product follow the uniform distribution over the intervals of [0, 0.05], [0, 010], [0, 0.15], [0, 020] and [0, 0.25], respectively. All defective items cannot be repaired or reworked, thus they must be scrapped at additional scrap costs of 20, 25, 30, 35 and $40 per item, respectively. Values for other parameters are:

K_i = Production set up costs are 3800, 3900, 4000, 4100 and $4200, respectively.
C_i = Unit manufacturing costs are 80, 90, 100, 110 and $120, respectively.
h_i = Unit holding costs are 10, 15, 20, 25 and 30, respectively. K_{1in} = the fixed delivery costs per shipment are $1800, $1900, $2000, $2100 and $2200.
C_{Ti} = Unit transportation costs are 0.1, 0.2, 0.3, 0.4 and $0.5, respectively.
n = Number of shipments per cycle, in this study it is assumed to be a constant 4.

Applying Eq. (15) and (11), one obtains the optimal common production cycle time T^* = 0.6662 (years) and total expected production-inventory-delivery costs per unit time for m products in the proposed system E [TCU (T^* = 0.6662)] = $2, 113, 194.

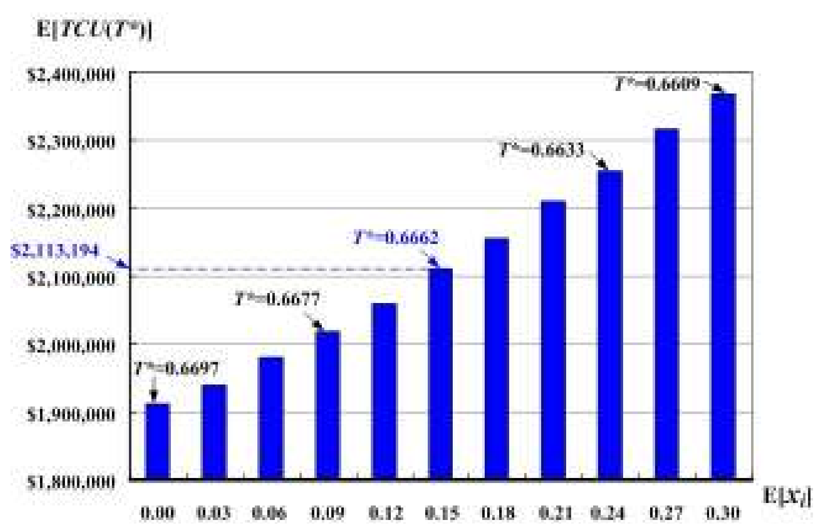

Fig. 3: Variation of average random scrap rate effects on E [TCU (T^*)] and T^*

Variation of average random scrap rate effects on the optimal cycle time T^* and on expected system cost E [TCU (T^*)] are depicted in Fig. 3. One notes that as the average random scrap rate E[x_i] increases, optimal cycle time T^* decreases, but the expected system cost E [TCU (T^*)] increases significantly.

CONCLUDING REMARKS

The classic FPR model considers lot sizing for single product with perfect production and continuous inventory issuing policy. However, in real world business environments, in order to maximize machine utilization, producer often make plan to produce m products in turn on a single machine. It is also inevitable to randomly produce some imperfect quality items during the production process and delivery of finished products to outside clients is commonly under a practical multiple shipment policy. It is important to management to be able to know the effects of random scrap rate and multi-shipment policy on the multi-item finite production rate system. The objective of this study is to determine an optimal common production cycle time that minimizes the long-run average cost per unit time and to investigate the effect of random scrap rate on the optimal common production cycle time and on the expected system cost.

The results of this study are intended to assist management in the fields to better understand, plan and control such a realistic production system. One interesting topic for future research will be to investigate the effects of rework and continuous inventory issuing policy on the optimal common production cycle time for the same production system.

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