Research Article

Radiation Effects on MHD Stagnation-Point Flow in a Nanofluid

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Abstract: In this study, the two-dimensional Magnetohydrodynamic (MHD) boundary layer of stagnation-point flow in a nanofluid in the presence of thermal radiation is investigated. Using a similarity transform, the Navier-Stokes equations are reduced to a set of nonlinear ordinary differential equations. The similarity equations are solved numerically for three types of nanoparticles, namely copper (Cu), alumina (Al₂O₃) and titania (TiO₂) in water as the base fluid. The skin-friction coefficient and Nusselt number as well as the velocity and temperature profiles for some values of the governing parameters are presented graphically and discussed. Effects of the nanoparticle volume fraction on the flow and heat transfer characteristics are thoroughly examined.

Keywords: MHD, nanofluid nanoparticle volume fraction, stagnation-point flow, thermal radiation

INTRODUCTION

The two-dimensional flow of a fluid near a stagnation-point is a classical problem in fluid mechanics. The steady flow in the neighbourhood of a stagnation-point was first studied by Hiemenz (1911), who used a similarity transformation to reduce the Navier-Stokes equations to nonlinear ordinary differential equations. This problem has been extended by Homann (1936) to the case of axisymmetric stagnation-point flow. Later the problem of stagnation-point flow either in the two or three-dimensional cases has been extended in numerous ways to include various physical effects. The results of these studies are of great technical importance, for example in the prediction of skin friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Mahapatra and Gupta (2002) and Nazar et al (2004) studied the heat transfer in the steady two-dimensional stagnation-point flow of a viscous fluid by taking into account different aspects.

The effect of thermal radiation on flow and heat transfer processes is of major importance in the design of many advanced energy conversion systems operating at high temperature. Thermal radiation within such systems occur because of the emission by the hot walls and working fluid. Many researchers have investigated the radiation effects in their studies, such as Pop et al (2004), Zhu et al (2011) and Bhattacharyya and Layek (2011).

Magnetohydrodynamic (MHD) boundary layer flow is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, high-temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids and (MHD) power generation systems. Sparrow et al (1961) studied the effect of magnetic field on the natural convection heat transfer. Patel and Timol (2011) studied two-dimensional MHD stagnation-point flow of a power law fluid over a stretching surface.

Low thermal conductivity of conventional fluids such as water and oil in convection heat transfer is the main problem to increase the heat transfer rate in many engineering equipments. To overcome this problem, researchers have performed considerable efforts to increase conductivity of working fluid. An innovative way to increase conductivity coefficient of the fluid is to suspend solid nanoparticles in it and make a mixture called nanofluid, having larger thermal conductivity coefficient than that of the base fluid. This higher thermal conductivity enhances the rate of heat transfer in industrial applications. Many researchers have investigated different aspects of nanofluids. Thermophysical properties of nanofluids such as thermal conductivity, thermal diffusivity and viscosity of nanofluids have been studied by Kang et al (2006), Velagapudi et al (2008), Turgut (2009), Rudyak et al (2010), Murugesan and Sivan (2010) and Nayak et al (2010). Bachok et al (2010) studied the flow and heat transfer in an incompressible viscous fluid near the three-dimensional stagnation point of a body that is
placed in a water-based nanofluid containing different types of nanoparticles: copper, alumina and titania. Bachok et al. (2011) also investigated two-dimensional stagnation-point flow of a nanofluid over a stretching/shrinking sheet. They derived the highest values of the skin friction coefficient and the local Nusselt number were obtained for the copper nanoparticles compared with the others. Two-dimensional boundary layer flow near the stagnation-point on a permeable stretching/shrinking sheet in a water-based nanofluid containing two types of nanoparticles: copper and silver, was studied by Arifin et al. (2011). Ahmad and Pop (2010) examined the mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. Nield and Kuznetsov (2009) conducted the classical problems of natural convective boundary layer flow in a porous medium saturated by a nanofluid, known as the Cheng-Minkowycz’s problem. Kuznetsov and Nield (2010) investigated natural convective boundary layer flow of a viscous and incompressible fluid past a vertical semi-infinite flat plate with water-based nanofluids. The two-dimensional boundary layer flow of a nanofluid past a stretching sheet in the presence of magnetic field intensity and the thermal radiation was studied by Gbadeyan et al. (2011). Olanrewaju et al. (2012) examined the boundary layer flow of nanofluids over a moving surface in a flowing fluid in the presence of radiation past a moving semi-infinite flat plate in a uniform free stream. They concluded that radiation has a greater influence on both the thermal boundary layer thickness and the nanoparticle volume fraction profiles. Mirmasoumi and Behzadmehr (2008) investigated the laminar mixed convection of a nanofluid in a horizontal tube using two-phase mixture model. Abu-Nada and Chamkha (2010) conducted a numerical investigation on mixed convection flow in an inclined square enclosure filled with alumina-water nanofluid. Oztop and Abu-Nada (2008) studied natural convection in a rectangular enclosure filled with a nanofluid containing copper, alumina and titania as nanoparticles. They concluded that the highest value of heat transfer is obtained by using copper nanoparticles.

The main subject of the present study is to study the two-dimensional Magnetohydrodynamic (MHD) boundary layer of stagnation-point flow in a nanofluid in the presence of thermal radiation. Using a similarity transform the Navier-Stokes equations have been reduced to a set of nonlinear ordinary differential equations. The resulting nonlinear system has been solved numerically using the Runge-Kutta-Fehlberg method with a shooting technique. Finally, the results are reported for three different types of nanoparticles namely alumina, titania and copper with water as the base fluid.

**Mathematical formulation of problem:** Consider the steady two-dimensional laminar flow of a viscous nanofluid past a plate in the presence of magnetic field and the thermal radiation (Fig. 1). The uniform magnetic field of strength $B_0$ is applied in the positive direction of $y$-axis. The ambient uniform temperature of nanofluid is $T_\infty$, where the body surface is kept at a constant temperature $T_w$. The linear velocity of the flow external to the boundary layer is $U(x)$ as $U(x) = ax$, where $a$ is positive constant. Under these assumptions and following the nanofluid model proposed by Tiwari and Das (2007), the governing equations for the continuity, momentum and energy in boundary layer flow can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
\frac{u \partial u}{\partial x} + \frac{v \partial u}{\partial y} = U \frac{\partial U}{\partial x} + v_nf \frac{\partial u}{\partial y} + \frac{\sigma_{nf}B_0^2}{\rho_{nf}}(U - u)
\]  

\[
\frac{u \partial T}{\partial x} + \frac{v \partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial y^2} - 1 \frac{\partial q_r}{\partial y}
\]

subject to the boundary conditions:

\[
u = 0, v = 0, T = T_w, \text{ at } y = 0
\]

\[
u = U(x) = ax, T = T_\infty, \text{ as } y \rightarrow \infty
\]

where,

\[
u \text{ and } v \text{ = The velocity components along x and y axes, respectively}
\]
\[ T = \text{Temperature} \]
\[ \sigma_{nf} = \text{The electrical conductivity of the nanofluid} \]
\[ q_r = \text{The radiative heat flux} \]
\[ \mu_{nf} = \text{The viscosity of the nanofluid} \]
\[ \alpha_{nf} = \text{The thermal diffusivity of the nanofluid} \]
\[ \rho_{nf} = \text{The density of the nanofluid, which are given by Oztop and Abu-Nada (2008):} \]
\[ \mu_{nf} = \frac{(1 - \varphi) \mu_f + \varphi(\mu_s) + \mu_{nf}}{(\rho c_p)_f(1 - \varphi) + \varphi(\rho c_p)_s} \]
\[ k_{nf} = \frac{k_f + 2k_s - 2\varphi(k_f - k_s)}{(k_f + 2k_s - \varphi(k_f - k_s))}, \]
\[ \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}} \]
where,
\[ \varphi = \text{The nanoparticle volume fraction} \]
\[ (\rho c_p)_nf = \text{The heat capacity of the nanofluid} \]
\[ k_{nf} = \text{The thermal conductivity of the nanofluid} \]
\[ k_f \text{ and } k_s = \text{The thermal conductivities of the fluid and of the solid fractions, respectively} \]
\[ \rho_f \text{ and } \rho_s = \text{The densities of the fluid and of the solid fractions, respectively} \]

The radiative heat flux \( q_r \) is described by Roseland approximation such that (Gbadeyan et al., 2011):
\[ q_r = \frac{4\sigma^* \varepsilon T^4}{3k} \frac{\partial T}{\partial y}, \]
where,
\[ \sigma^* = \text{The Stefan-Boltzmann constant} \]
\[ k = \text{The mean absorption coefficient} \]

We assume that the temperature differences within the flow are sufficiently small so that the \( T^4 \) can be expressed as a linear function after using Taylor series to expand \( T^4 \) about the free stream temperature \( T_{\infty} \) and neglecting higher-order terms. This result is the following approximation:
\[ T^4 \approx 4T_{\infty}^3T - 3T_{\infty}^4, \]

From Eq. (3), (7) and (8) one obtains:
\[ \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{(\rho c_p)_f} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_{\infty}^3}{3(\rho c_p)_f k} \frac{\partial^2 T}{\partial y^2}. \]

To obtain similarity solutions for the system of Eq. (1)-(3), we introduce the following similarity variables:
\[ \eta = \frac{a}{\sqrt{\nu}}, \quad \psi = \sqrt{\frac{\nu}{\sigma}} \frac{\partial \nu}{\partial y}, \quad \theta(\eta) = \frac{T - T_{\infty}}{T_u - T_{\infty}}. \]

where, \( \Psi \) is the stream function defined as \( u = \partial \Psi / \partial y \) and \( v = -\partial \Psi / \partial x \), which identically satisfy Eq. (1).

Using the non-dimensional variables in Eq. (10), Eq. (2) and (3) reduce to the following ordinary differential equations:
\[ \frac{1}{(1-\varphi)^{2\eta}} \left[ 1 + \frac{4}{3} R_d \right] \frac{\partial T^{\eta}}{\partial \eta} = \frac{\partial \theta^{\eta}}{\partial \eta} + Pr \rho \theta^{\eta} = 0 \]
and the boundary conditions (4) and (5) become:
\[ f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1, \]
\[ \theta(0) = 1, \quad \theta(\infty) = 0, \]

where primes denote differentiation with respect to \( \eta \).

The Hartman number \( M \), the Prandtl number \( Pr \) and the radiation parameter \( Rd \) are, respectively, defined as:
\[ M^2 = \frac{\sigma^* B_{\infty}^2}{\rho_{\infty} a}, \quad Pr = \frac{\mu c_p}{\alpha}, \quad R_d = 4\sigma^* T_{\infty}^3 \]

The physical quantities of interest are the skin friction coefficient \( c_f \) and the local Nusselt number \( Nu_x \), which are defined as:
\[ c_f = \frac{\tau_w}{\rho u^2}, \quad Nu_x = \frac{x q_0}{k_f (T_u - T_{\infty})}, \]

where the surface shear stress \( \tau_w \) and the surface heat flux \( q_0 \) are given by:
\[ \tau_w = \mu_k \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_0 = \left( k_f + \frac{16\sigma^* T_{\infty}^3}{3k} \right) \frac{\partial T}{\partial y} \bigg|_{y=0}. \]

Using the non-dimensional variables (10), we get:
\[ Re_x^{1/2} C_f = \frac{1}{(1-\varphi)^{2\eta}} f'(0) \]
\[ Nu_x Re_x^{-1/2} = \left( 1 + \frac{4}{3} R_d \right)^{1/2} \frac{k_{nf}}{k_f} \theta'(0) \]
RESULTS AND DISCUSSION

Equation (11) and (12) subject to the boundary conditions (13) are solved numerically using the Runge-Kutta-Fehlberg method with a shooting technique for some values of the governing parameters. Three types of nanoparticles are considered, namely, copper (Cu), alumina (Al₂O₃) and titania (TiO₂). Following Oztop and Abu-Nada (2008) and Bachok et al. (2010), the value of the Prandtl number Pr is taken as 6.2 (for water) and the volume fraction of nanoparticles is from 0 to 0.2 (0 ≤ φ ≤ 0.2), in which φ = 0 corresponds to the regular Newtonian fluid. The thermophysical properties of the fluid and nanoparticles are given in Table 1. (Oztop and Abu-Nada, 2008).

Figure 2 and 3, respectively show the variations of the velocity profiles \( f'(\eta) \) and temperature profiles \( \theta(\eta) \) for different nanoparticle volume fractions for copper-water nanofluid. It can be seen from Fig. 2 that the velocity components increase with increase in the nanoparticle volume fraction \( \phi \). From Fig. 3 the temperature \( \theta(\eta) \) increases as the nanoparticle volume fraction \( \phi \) increases.

Table 1: Thermophysical properties of the base fluid and the nanoparticles (Oztop and Abu-Nada, 2008)

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (water)</th>
<th>Cu</th>
<th>Al₂O₃</th>
<th>TiO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p(J/kgk) )</td>
<td>4179</td>
<td>385</td>
<td>765</td>
<td>686.2</td>
</tr>
<tr>
<td>( \rho(kg/m^3) )</td>
<td>997.1</td>
<td>8933</td>
<td>3970</td>
<td>4250</td>
</tr>
<tr>
<td>( K(W/mk) )</td>
<td>0.613</td>
<td>400</td>
<td>40</td>
<td>8.9538</td>
</tr>
<tr>
<td>( \alpha \times 10^{-3}(m^2/s) )</td>
<td>1.47</td>
<td>1163.1</td>
<td>131.7</td>
<td>30.7</td>
</tr>
</tbody>
</table>

Figure 4 and 5 respectively illustrate the variation of \( f'(\eta) \) and \( \theta(\eta) \) for different nanoparticles when \( \phi = 0.2 \). Figure 4 shows that the Cu nanoparticle (compared to Al₂O₃ and TiO₂) has the largest velocities. From Fig. 5 it is observed that the Al₂O₃ nanoparticles have the highest value of temperature distribution (compared to Cu and TiO₂).

Figure 6 is prepared to present the effect of the nanoparticle volume fraction \( \phi \) on the skin friction coefficient \( \Re^{-1/2}C_f \) for different types of nanofluids. It is observed that the magnitude of skin friction coefficient increases with the nanoparticle volume.
In addition, it is noted that the highest skin friction coefficient is obtained for the Cu nanoparticle. The influence of the nanoparticle volume fraction $\varphi$ on the local Nusselt number $N_{\text{u}_x}R_e^{-1/2}$ for different types of nanofluids are shown in Fig. 7. It is observed that the local Nusselt number increases with the nanoparticle volume fraction $\varphi$. Moreover, it is noted that the lowest heat transfer rate is obtained for the nanoparticles TiO$_2$ due to domination of conduction mode of heat transfer. This is because TiO$_2$ has the lowest value of thermal conductivity compared to Cu and Al$_2$O$_3$, as seen in Table 1. This behaviour of the local Nusselt number is similar to that reported by Bachok et al. (2010). However, the difference in the values of Cu and Al$_2$O$_3$ is negligible. The thermal conductivity of Al$_2$O$_3$ is approximately one-tenth of that of Cu, as given in Table 1. A unique property of Al$_2$O$_3$ is its slow thermal diffusivity. The reduced value of thermal diffusivity leads to higher temperature gradients and, therefore, higher enhancement in heat
Fig. 9: The effect of the Hartman number $M$ on the temperature profiles $\theta(\eta)$ for copper-water nanofluid, when $\varphi = 0.2$

transfer. The Cu nanoparticles have high values of thermal diffusivity and, therefore, this reduces the temperature gradients, which will affect the performance of Cu nanoparticles.

The effect of $M$ on the velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ for copper-water nanofluid ($\varphi = 0.2$) are illustrated in Fig. 8 and 9 respectively. Figure 8 displays that the velocity $f'(\eta)$ increases as $M$ increases. Moreover, The boundary layer thickness is increased by increasing $M$. From Fig. 9, we can see that the temperature profiles $\theta(\eta)$ decreases as $M$ increases. Figure 10 and 11 respectively show the influence of $R_d$ and $Pr$ on the temperature profiles $\theta(\eta)$ for copper-water nanofluid ($\varphi = 0.2$). It can be seen from Fig. 10 that the temperature profiles $\theta(\eta)$ increases by increasing $R_d$, therefore, the thermal boundary layer increases when $R_d$ increases. From Fig. 11, it is noted that the temperature $\theta(\eta)$ increases as the Prandtl number $Pr$ decreases.

Table 2 shows the values of the skin friction coefficient $Re_x^{1/2}C_f$ and the local Nusselt number $Nu_xRe_x^{-1/2}$ for some values of the nanoparticle volume fraction $\varphi$ using different nanoparticles. It is observed that, the large values of average Nusselt number can be obtained by adding copper.

Table 2: The effect of the various nanoparticle volume fractions on the skin friction coefficient and local Nusselt number for the different nanoparticles, when $M = 1$ and $R_d = 0.5$

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$Cu$</th>
<th>$Al_2O_3$</th>
<th>$TiO_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Re_x^{1/2}C_f$</td>
<td>$Nu_xRe_x^{-1/2}$</td>
<td>$Re_x^{1/2}C_f$</td>
</tr>
<tr>
<td>0.00</td>
<td>1.585400</td>
<td>1.650833</td>
<td>1.585400</td>
</tr>
<tr>
<td>0.05</td>
<td>1.997958</td>
<td>1.847150</td>
<td>1.811292</td>
</tr>
<tr>
<td>0.10</td>
<td>2.405934</td>
<td>2.031578</td>
<td>2.060305</td>
</tr>
<tr>
<td>0.15</td>
<td>2.875792</td>
<td>2.218840</td>
<td>2.336844</td>
</tr>
<tr>
<td>0.20</td>
<td>3.369125</td>
<td>2.402679</td>
<td>2.647819</td>
</tr>
</tbody>
</table>
Table 3: The effect of the various values of $M$ and $R_d$ on the skin friction coefficient and the local Nusselt number for copper-water nanofluid, when $\phi = 0.2$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$R_d$</th>
<th>$Re^{1/2}C_p$</th>
<th>$NuRe^{-1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.5</td>
<td>2.619519</td>
<td>2.295027</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0</td>
<td>3.017294</td>
<td>2.355254</td>
</tr>
<tr>
<td>1.0</td>
<td>1.2</td>
<td>3.650730</td>
<td>2.437593</td>
</tr>
<tr>
<td>1.2</td>
<td>0.0</td>
<td>3.369125</td>
<td>2.928719</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>3.369125</td>
<td>2.402679</td>
</tr>
<tr>
<td>1.0</td>
<td>1.5</td>
<td>3.369125</td>
<td>1.908934</td>
</tr>
</tbody>
</table>

Table 3 is made to give the values of the skin friction coefficient and the local Nusselt number for different values of $M$ and $R_d$ for copper-water nanofluid. The values of the skin friction coefficient and the local Nusselt number increases when $M$ increases. From this table, the magnitude of the local Nusselt number decreases when $R_d$ increases.

**CONCLUSION**

Here, the steady two-dimensional laminar flow of a viscous nanofluid past a plate in the presence of magnetic field and the thermal radiation was investigated. The governing partial differential equations were converted to ordinary differential equations by using a suitable similarity transformation and were then solved numerically using the Runge-Kutta-Fehlberg method with a shooting technique. In this study, three types of nanoparticles, namely copper (Cu), alumina (Al$_2$O$_3$) and titania (TiO$_2$) with water as the base fluid were considered to investigate the effect of the nanoparticle volume fraction $\phi$, the Hartman number $M$ the Prandtl number $Pr$ and the radiation parameter $R_d$ on the flow and heat transfer characteristics. Finally, from the presented analysis, the following observations are noted.

- For all three nanoparticles, the magnitude of the skin friction coefficient and local Nusselt number increases with the nanoparticle volume fraction $\phi$.
- For a fixed value of the nanoparticle volume fraction $\phi$, the velocity of fluid increases by increasing $M$.
- For a fixed value of the nanoparticle volume fraction $\phi$, the temperature increases by decreasing $M$ and $Pr$. A similar effect on the temperature is observed when $R_d$ increases.
- The highest values of the skin friction coefficient and the local Nusselt number were obtained for the Cu nanoparticles compared to Al$_2$O$_3$ and TiO$_2$.
- The type of nanofluid is a key factor for heat transfer enhancement. The highest values are obtained when using Cu nanoparticles.

**REFERENCES**


