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Research Article

Improved Relative-Entropy Method for Eccentricity Filtering in Roundness Measurement Based on Information Optimization

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Abstract: In this study, we propose the improved relative-entropy of the ideal circle function to the measured information of the radius error of the workpiece surface to make an eccentricity filtering in roundness measurement. Along with a correct assessment for the parameters of the eccentricity filtering, the extracted information from the measured information is obtained by the minimization of the improved relative-entropy. The case studies show that the information optimization is characterized by decreasing the improved relative-entropy, the extracted information almost coincides with the real information, the improved relative-entropy has a strong immunity to the stochastic disturbance of the rough work piece-surface and the increase of the minimum of the improved relative-entropy counteracts the effect of the stochastic disturbance on the assessment for parameters in eccentricity filtering.

Keywords: Filtering, roundness measurement, eccentricity, information optimization, relative entropy

INTRODUCTION

The information entropy is a measure for the uncertainty of an information source (Woodbury and Ulrich, 1998; Xia *et al.*, 2010). The relative entropy as an important concept in the information entropy theory is a measure for the relation of two information sources (Woodbury and Ulrich, 1998; Aviyente and Williams, 2005; Zhang *et al.*, 2008; Molnár and Szokol, 2010; Saravanan *et al.*, 2012; Guan *et al.*, 2012).

So far the relative entropy is widely applied to many areas in science and technology, such as time-frequency analysis (Aviyente and Williams, 2005), fault diagnosis (Guan *et al.*, 2012), models election (Techakesari and Ford, 2012), data processing (Sharma, 2012) and phase retrieval (Soldovieria *et al.*, 2010) and so on.

The minimum relative-entropy theory is one of the popular relative-entropy theories. According to this theory, the smaller the relative-entropy is, the closer the relation of two information sources is; and vice versa. But, the use of the relative entropy relies on the known probability density function of a population studied (Woodbury and Ulrich, 1998; Woodbury, 2004; Molnár and Szokol, 2010). If the probability density function is unknown in advance, the relative entropy becomes ineffective. For this reason, taking the eccentricity filtering in roundness measurement as an example, this study proposes an improved relative-entropy method without any requirement for prior information about probability density functions and the characteristic of the improved relative-entropy function is investigated with the help of information optimization.

In this study, we propose the improved relative-entropy of the ideal circle function to the measured information of the radius error of the workpiece surface to make an eccentricity filtering in roundness measurement. Along with a correct assessment for the parameters of the eccentricity filtering, the extracted information from the measured information is obtained by the minimization of the improved relative-entropy. The case studies show that the information optimization is characterized by decreasing the improved relative-entropy, the extracted information almost coincides with the real information, the improved relative-entropy has a strong immunity to the stochastic disturbance of the rough work piece-surface and the increase of the minimum of the improved relative-entropy counteracts the effect of the stochastic disturbance on the assessment for parameters in eccentricity filtering.

MATHEMATICAL MODEL OF INFORMATION OPTIMIZATION

Assume the real information of the radius error of a workpiece surface is expressed as:

$$r = r(t) = R_0 + F_j \sin(jt + t_0) \tag{1}$$

where,

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r = The real information of the radius error of the workpiece surface
t = The angle variable
R_0 = The information of a basic circle produced by the pressed deformation of the sensor, j = The harmonic order
F_j = The amplitude of the jth harmonic and t_0 is the initial phase

The measured information of the radius error of the workpiece surface can be obtained by a roundness measuring instrument, as follows:

$$r_1 = r_1(t) = \sqrt{x_1^2 + y_1^2} \tag{2}$$

with

$$x_1 = x_1(t) = a + r\cos(t) \tag{3}$$

and

$$y_1 = y_1(t) = b + r\sin(t) \tag{4}$$

where, r_1 is the measured information of the radius error of the workpiece surface and a and b are two eccentricity components.

The eccentricity components a and b are of the positioning errors (Adam *et al*., 2012; Garcia-Plaza *et al*., 2012) and must be filtered out from the measured information r_1 so that the real information r is extracted and the roundness of the workpiece surface can then be evaluated. For this reason, the improved relative-entropy is defined as:

$$E = E(\mathbf{x}) = \left\| r_{00} \left| \ln\left(\frac{r_{00}}{r_1}\right) \right| \right\|_2 \tag{5}$$

with

$$r_{00} = r_{00}(\mathbf{x}, t) = \sqrt{(x(1) + x(3)\cos(t))^2 + (x(2) + x(3)\sin(t))^2} \tag{6}$$

and

$$\mathbf{x} = [x(1), x(2), x(3)]^{\mathrm{T}} \tag{7}$$

where E is the improved relative-entropy of the ideal circle function to the measured information of the radius error of the workpiece surface, r_{00} is the ideal circle function, $x(1)$, $x(2)$ and $x(3)$ are three parameters to be solved and $\mathbf{x}$ is the parameter vector.

In equations (6) and (7), three parameters, x (1), x (2) and x (3), correspond to a, b and R_0, respectively. According to the minimum relative-entropy theory, if the minimum of the improved relative-entropy, $E_{\min}$, is gained at:

$$E_{\min} = \min_{\mathbf{x}} E \tag{8}$$

then the values of $x(1)$, $x(2)$ and $x(3)$, are, respectively, the estimated values of a, b and R_0, as follows:

$$\mathbf{x}^* = [x^*(1), x^*(2), x^*(3)]^{\mathrm{T}} = [a^*, b^*, R_0^*]^{\mathrm{T}} \tag{9}$$

where, $\mathbf{x}^*$ is the optimum value vector of the three parameters and a^*, b^*, and R_0^* are, respectively, the optimum values of a, b and R_0, which satisfy Eq. (8).

The information of estimating for the real information of the radius error of the workpiece surface is given by:

$$r_0 = r_0(t) = \sqrt{(-a^* + x_1)^2 + (-b^* + y_1)^2} \tag{10}$$

where, r_0 is the estimated information of the real information of the radius error of the work piece surface, that is, r_0 is the information extracted from the measured information of the radius error of the workpiece surface by means of eccentricity filtering and can be employed to evaluate the roundness of the workpiece surface.

The roundness of the workpiece surface can be obtained as:

$$\delta r = \max_t r_0 - \min_t r_0 \tag{11}$$

where, δr is the roundness of the workpiece surface.

From Eq. (8) and (11) the obtained roundness relies on the information extracted from the measured information with the help of the minimum relative-entropy. Therefore, it is a characteristic parameter of the improved relative-entropy based on information optimization for eccentricity filtering. It follows that the roundness evaluating process is a process of decreasing the relative entropy, along with an optimization of the measured information, without any requirement for prior information about probability density functions.

CASE STUDIES AND DISCUSSION

Roundness evaluation of of smooth workpiece-surface with change of relative-entropy: This is Case 1. This case studies the change of the relative-entropy with three parameters, $x(1)$, $x(2)$ and $x(3)$, by a simulation of the roundness evaluating for the measured information of the radius error of the smooth workpiece-surface with the second harmonic interfered

Table 1: Imitated parameter in case 1

Parameter	Value
a/mm	0.00628
b/mm	0.0998
R_0/mm	0.1
j	2
F_2/mm	0.002
t_0/rad	0.5235983
δr/mm	0.004

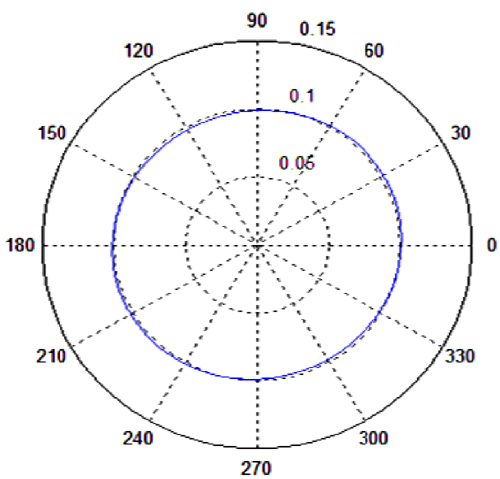

Fig. 1: Real information r in Case 1

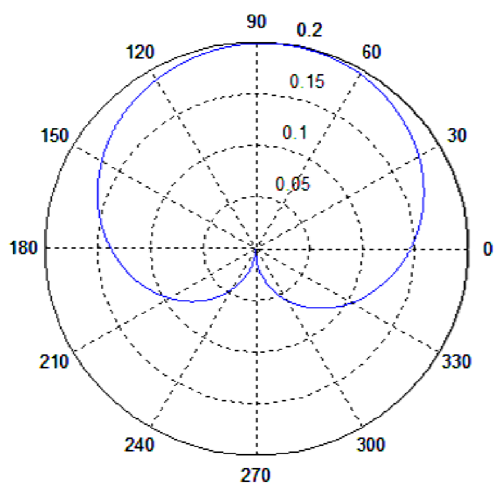

Fig. 2: Measured information r_1 in Case 1

by an eccentricity. The imitated parameters are shown in Table 1.

From Eq. (1) and (2) and Table 1, the real information and the measured information of the radius error of the workpiece surface are imitated, as shown in Fig. 1 and 2, respectively. Obviously, in polar coordinates (t = 0~360°) the real information r is an ellipse and the measured information r_1 is a cardioid,

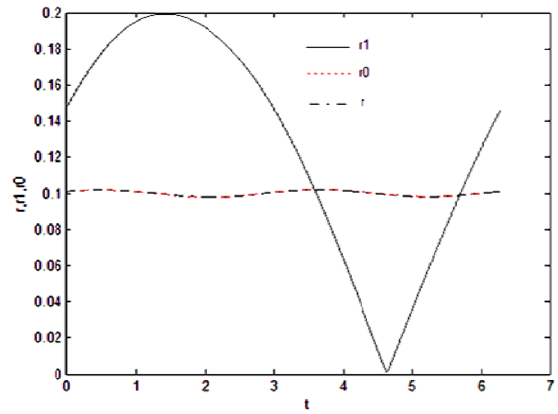

Fig. 3: Comparison among real information r, measured information r_1, and extracted information r_0 in Case 1

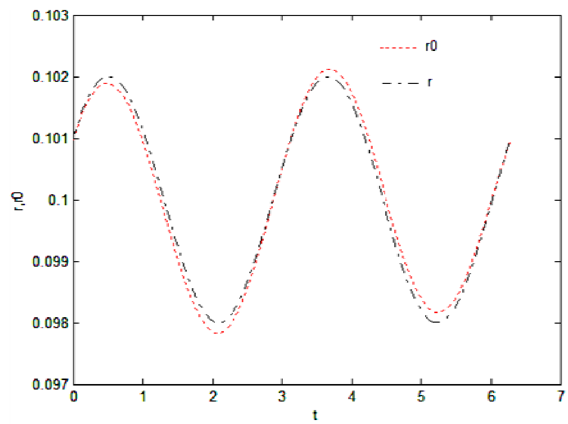

Fig. 4: Comparison between real information r and extracted information r_0 inase 1

with a large difference between the two due to the eccentricity interference.

In Cartesian coordinates as shown in Fig. 3 and 4 (t = 0~2 π), the difference between the two can directly be compared and the good result obtained using the method proposed in this study can clearly be found. It can be seen from Fig. 3 and 4 that although the difference between the real information r and the measured information r_1 is very significant, the extracted information r_0 still is perfect, which almost coincides with the real information r. This is because the decreasing relative-entropy based on information optimization for eccentricity filtering makes a correct assessment for the parameters a, b, R_0, F_2 and δr , as shown in Table 2.

It is easy to see from Table 2 that the relative errors between the estimated values and the truth values are very small; indicating that the improved relative-entropy based on information optimization is good at eccentricity filtering in roundness measurement,

Table 2: Result of assessment for parameter in Case

Parameter	Truth value	Estimated value	Relative error%
a/mm	0.00628	0.006298	0.29
b/mm	0.0998	0.10000	0.2
R_0/mm	0.1	0.09979	0.21
F_2/mm	0.002	0.00215	7.5
δr /mm	0.004	0.0043	7.5
E_{min}	-	0.0453	-

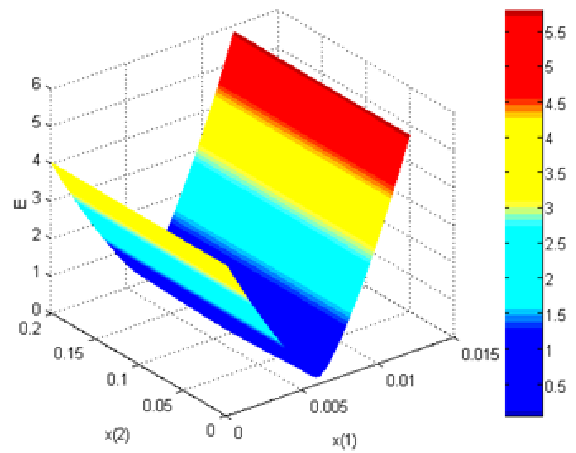

Fig. 5: Change of E with $x(1)$ and $x(2)$ (let $x(3)=R^*_0$ in Case 1

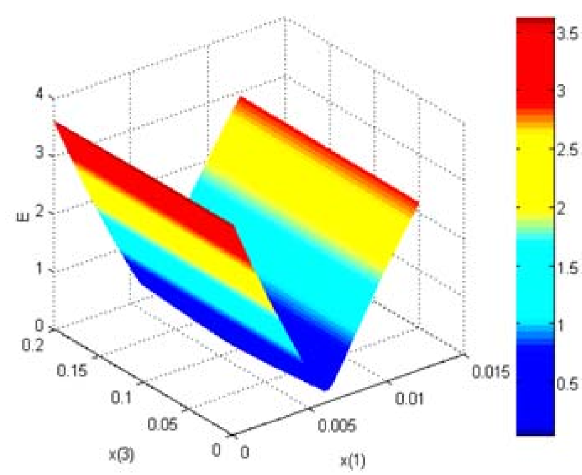

Fig. 6: Change of E with $x(1)$ and $x(3)$ (let $x(2)$ = b^*) in Case 1

without any requirement for prior information about probability density functions.

Figure 5 to 7 present the change of the improved relative-entropy E with three parameters, $x(1)$, $x(2)$ and $x(3)$. It is found from Fig. 5 to 7 that the improved relative-entropy E proposed in this study is a complex function of three parameters, $x(1)$, $x(2)$ and $x(3)$ and information optimization is characterized by decreasing the improved relative-entropy E. When the improved relative-entropy E is approaching to the minimum E_{min}, the parameter vector x = $[x(1), x(2), x(3)]^T$takes the optimum value vector $x^* = [x^*(1), x^*(2),, x^*(3),]^T = (a^*, b^*, R^*_0)^T$and the information of eccentricity filtering is optimized in roundness measurement.

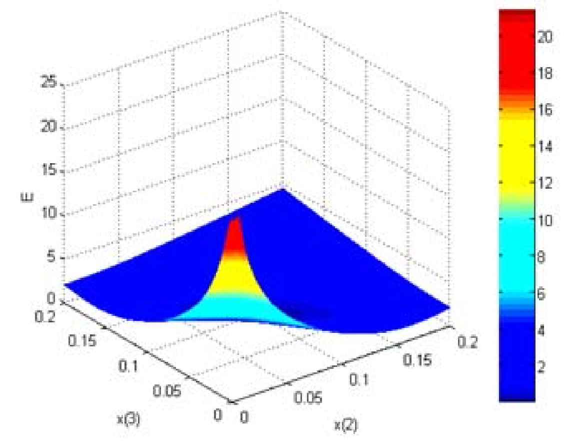

Fig. 7: Change of E with x (2) and x (3) (let x (1) = a^*) in Case 1

Roundness evaluation of of rough workpiece-surface with change of relative-entropy: This is Case 2. This case studies the change of the relative-entropy with three parameters, $x(1)$, $x(2)$ and $x(3)$, by a simulation of the roundness evaluating for the measured information of the radius error of the rough workpiece-surface with the second harmonic interfered by an eccentricity.

The real information is simulated by:

$$r = r(t) = R_0 + F_j \sin(jt + t_0) + N(0, s^2) \tag{12}$$

where, $N(0,\ s^2)$ stands for the normal distribution function with the 0 mean and the s standard deviation, which is employed for the expression of the rough workpiece-surface.

The imitated parameters are shown in Table 3.

The real information and the measured information of the radius error of the rough workpiece-surface are shown in Fig. 8 and 9, respectively. The difference between the two can directly be compared and the good result obtained using the method proposed in this study can clearly be found. It can be seen from Fig. 8 to 10 that although the difference between the real information r and the measured information r1 is very significant, the extracted information r0 still is perfect, which almost coincides with the real information r. This is because the decreasing relative-entropy based on

Table 3: Imitated parameter in Cases 2 and 3

Parameter	Value
a/mm	0.00628
b/mm	0.0998
R_0/mm	0.1
j	2
F_2/mm	0.002
t_0/rad	0.5235983
s/mm	0.0001, 0.0002, 0.0003, 0.0005, 0.0009, 0.002

Table 4: Result of assessment for parameter in Case 2 (s = 0.0001 mm)

Parameter	Truth value	Estimated value	Relative error /%
a/mm	0.00628	0.006325	0.72
b/mm	0.0998	0.10000	0.20
R_0/mm	0.1	0.09976	0.24
δr /mm	0.0046	0.0048	4.35
E_{min}	-	0.0454	-

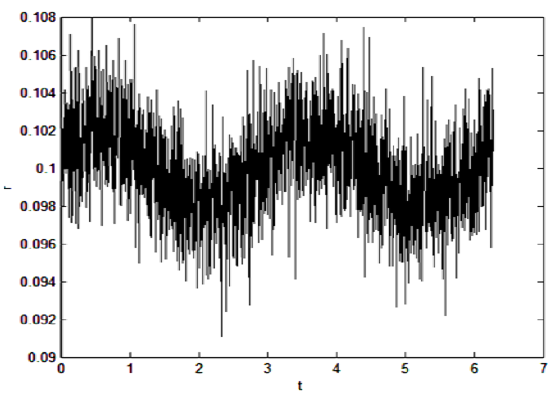

Fig. 8: Real information r in Case 2 (s = 0.0001 mm)

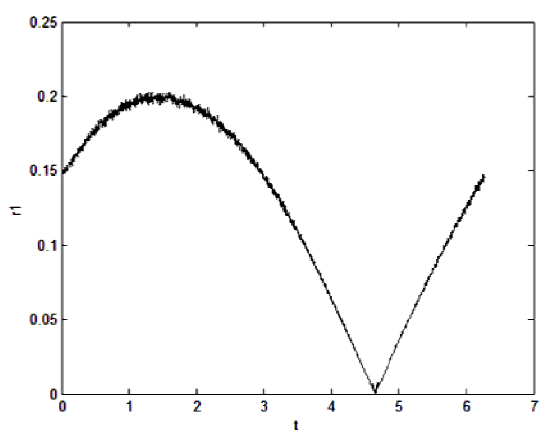

Fig. 9: Measured information r_1 in Case 2 (s = 0.0001 mm)

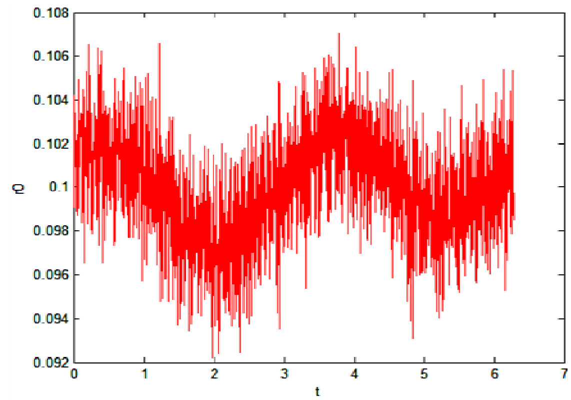

Fig. 10: Extracted information r_0 in Case 2 (s = 0.0001 mm)

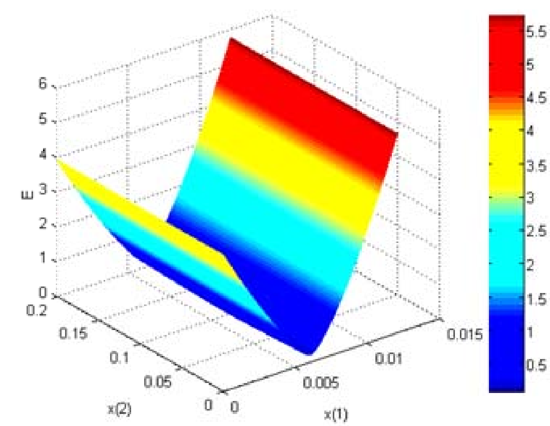

Fig. 11: Change of E with x (1) and x (2) (let x (3) $=R^*_0$) in Case 2 (s = 0.0001 mm)

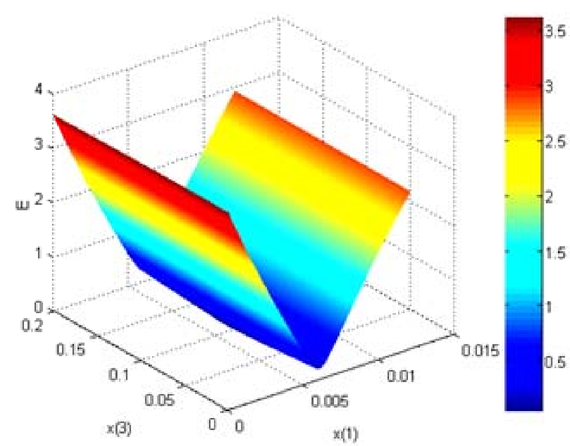

Fig. 12: Change of E with x(1) and x(3) (let x(2) = b^*) in Case 2 (s = 0.0001 mm)

information optimization for eccentricity filtering makes a correct assessment for the parameters a, b, R_0, F_2 and δr , as shown in Table 4.

It is easy to see from Table 4 that the relative errors between the estimated values and the truth values are very small, indicating that the improved relative-entropy based on information optimization is good at

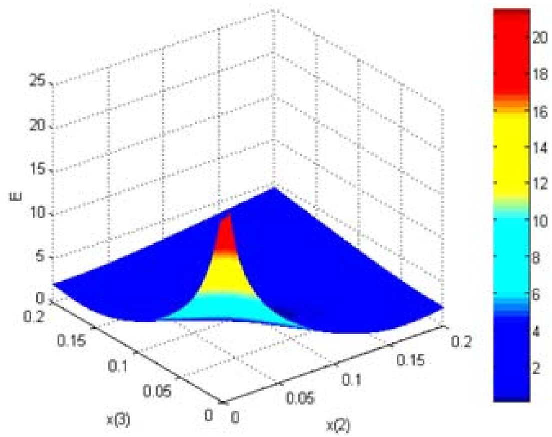

Fig. 13: Change of E with x (2) and x (3) (let x (1) = a^*) in Case 2 (s = 0.0001 mm)

Table 5: Result of assessment for parameter in Case 3 (s = 0.0002 mm)

Parameter	Truth value	Estimated value	Relative error%
a/mm	0.00628	0.006165	1.83
b/mm	0.0998	0.099348	0.45
R_0/mm	0.1	0.100491	0.49
δr/mm	0.0052	0.0055	5.77
E_{min}	-	0.0457	-

Table 6: Result of assessment for parameter in Case 3 (s = 0.0005 mm)

Parameter	Truth value	Estimated value	Relative error /%
a/mm	0.00628	0.006381	1.61
b/mm	0.0998	0.09982	0.20
R_0/mm	0.1	0.10017	0.02
δr /mm	0.007	0.0068	2.86
E_{min}	-	0.0492	-

Table 7: Result of assessment for parameter in Case 3 (s = 0.0009 mm)

Parameter	Truth value	Estimated value	Relative error /%
a/mm	0.00628	0.006086	3.09
b/mm	0.0998	0.099002	0.80
R_0/mm	0.1	0.101037	1.04
δr /mm	0.0094	0.0101	7.45
E_{min}	-	0.0539	-

Table 8: Result of assessment for parameter in Case 3 (s = 0.002 mm)

Parameter	Truth value	Estimated value	Relative error /%
a/mm	0.00628	0.006455	2.79
b/mm	0.0998	0.100110	0.31
R_0/mm	0.1	0.099362	0.64
δr /mm	0.016	0.0159	0.62
E_{min}	-	0.0788	-

eccentricity filtering in roundness measurement for the rough workpiece-surface, just like the smooth workpiece-surface in Case 1.

The change law of the improved relative-entropy is presented in Fig. 11 to 13 in this Case and, overall, it is

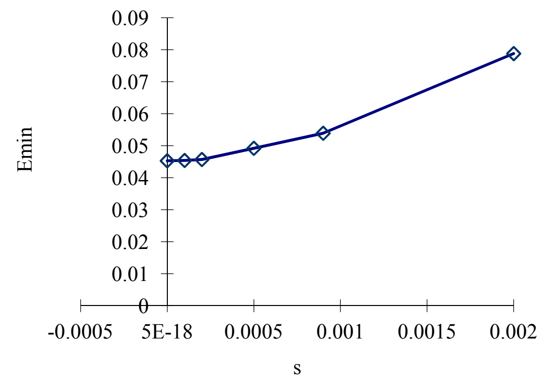

Fig. 14: Change of minimum of improved relative-entropy with stochastic disturbance of rough work piece-surface in Case 3

similar to that in Fig. 5 to 7 in Case 1, revealing a strong immunity of the improved relative-entropy to the stochastic disturbance of the rough workpiece-surface. This shows that the improved relative-entropy is of high robustness that is discussed below.

Robustness of improved relative-entropy with stochastic disturbance of rough workpiece-surface in roundness evaluation: This is Case 3. This case studies the robustness of the improved relative-entropy with the stochastic disturbance of the rough workpiece-surface in roundness evaluation. The imitated parameters are shown in Table 3 and the results are shown in Tables 5 to 8.

Clearly, although the standard deviation s takes the value in the range from 0.0001mm to 0.002mm, the relative errors between the estimated values of the parameters, a, b, R_0 and δr and their truth values still are very small. The price of such a result is the increase of the minimum of the improved relative-entropy with the stochastic disturbance of the rough workpiece-surface in roundness evaluation, as shown in Fig. 14, that is, the increase of the minimum of the improved relative-entropy counteracts the effect of the stochastic disturbance of the rough workpiece-surface on the assessment for parameters, a, b, R_0 and δr and the real information can soundly be extracted.

CONCLUSION

This study proposes the improved relative-entropy method for the eccentricity filtering in roundness measurement based on information optimization, without any requirement for prior information about probability density functions.

The improved relative-entropy is a complex function of the parameters of eccentricity filtering and information optimization is characterized by decreasing the improved relative-entropy. When the improved relative-entropy is approaching to the minimum, the parameter vector takes the optimum value vector and the information of eccentricity filtering is optimized in roundness measurement.

The extracted information from the measured information almost coincides with the real information. This is because the decreasing relative-entropy based on information optimization for eccentricity filtering makes a correct assessment for the parameters.

The improved relative-entropy has a strong immunity to the stochastic disturbance of the rough workpiece-surface. The increase of the minimum of the improved relative-entropy counteracts the effect of the stochastic disturbance on the assessment for parameters in eccentricity filtering and the real information can soundly be extracted.

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