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Research Article

Optimization Design of a Gear Profile Based on Governing Equations

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Abstract: Aimed at some questions of the rotated gear in the conventional steam turbine center, such as the uneven force and local stress concentration along the radial direction of the gear, based on governing equations and Stodola's mathematical model, this study made use of the idea of partial differential equations to make axisymmetric and uniform thickness gear modeling under constraint conditions of specified gear mass and moment of inertia and optimization algorithm is used to obtain the thickness profile that results in a radial stress distribution that is as even as possible. The maximum radial stress via the optimization design of the gear decreases by about 25% in contrast with the maximum radial stress of the gear with the even thickness profile.

Keywords: Governing equation, optimization design, radial stress, thickness distribution

INTRODUCTION

The weight of the gear, the contour shape and the stability of the gear affect the performance most of the machine with gears. Many of the past studies are focused on the way to make the gear lighter and meet the gear speed level adjustment same time and the way of research is using the empirical formula to design. This way lacks of the scientific accuracy and have not taken into account the gears' radial stress distribution affect to the gears strength and the fatigue life when the gears are working (Cheng-Hong *et al*., 2011; Chunquan *et al*., 2011; Gongxun *et al*., 2012; Haidong and Zhaoyao, 2010; Hongwei, 2010). The fast development of the computational science and the simulation technology gives a new way to make a reasonable science design of gears.

When the traditional ax symmetrically and uniform thickness gears are working, there is a peak of radial stress components in the vicinity of the inner diameter (Jozef *et al*., 2011; Jun and Gelan, 2011; Nelson and Mc Vaugh, 1976; Qi *et al*., 2011; Shilong *et al*., 2012). The closer the gear outer rim, the lighter the gear radial stress. At the gear outer edge, the radial stress is zero (Tiejun and Leina, 2012; Qifeng *et al*., 2011; Tao and Jianping, 2009; Umezawa, 1984; Yang *et al*., 2011). This uneven distribution of the stress also reflected in the angular component (Xiaohua, 2011; Xurong and Zheng, 2007; Xilong *et al*., 2011; Xiaoning and Peng, 2012; Yongfeng and Jianjun, 2012). It shows that this gear designed has not made the full use of existing materials (Zhiming *et al*., 2012; Zhong-Hua *et al*., 2003).

In this study, we make a comprehensive analysis which based on the conventional mathematical model and the control equation and Stowe Daura mathematical model. In the case of giving constraint condition of the mass and moment of inertia of the gear, we use the finite element techniques to optimize design the unevenness of traditional gear design and local stress concentration, control by partial differential controlling equations module and the optimization of the design module to establish the mathematical model of the optimized design of the gear profile. Through the finite element software COMSOL Multiphysics built-in optimization algorithm to obtain uniform radial stress distribution profile thickness distribution.

OPTIMUM MODEL OF GEAR SECTION PROFILE

In this study, we are about to establish the model which is used for designing the gear radial thickness to make the gear radial stress distribution in the case of a given mass and moment of inertia of the gear. Before the gear optimization, we need analyzed and summarized some of the kinetic equation on disc gear and derive representation equation in differential form. The optimization model created in this study need the use of one-dimensional generalized partial differential equations interface and optimization modules in the software COMSOL Multiphysics multi-physics finite element.

Stowe Daura mathematical model: Stowe Daura once had the Turbine center disc gear optimization designed,

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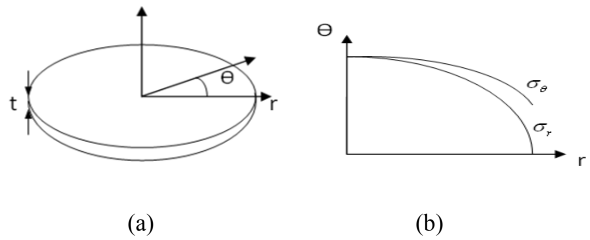

Fig. 1: Turbine center uniform thickness disc gear diagram and gear radial stress and angular distribution of stress; (a): Cylindrical coordinates of gears; (b): Stress distribution of uniform thickness gear

in the case of considering the change of the thickness of the disc gear, derived a generalized equation of motion based on gears radial force conservation. Eq. (1) are described below:

$$\frac{d}{dr}\sigma_r H + \frac{H}{r}(\sigma_r - \sigma_\theta) + Hr\omega^2\rho = 0 \tag{1}$$

Radial stress σ_r in the rim must meet the natural boundary conditions, from Fig. 1, the angle stress σ_θ is stronger than the radial stress. Two stress curve presents a monotonically increasing trend with the reducing of the distance to gear center and show a zero slope at the center of the gear. Therefore, the design of the gears should make radial stress and angle stress equal and constant, as:

$$\sigma_r = \sigma_\theta = \sigma \tag{2}$$

Therefore, disc gear generalized movement balance equation can be simplified to a thickness t as the dependent variable differential equations; differential Eq. (3) is described as follows:

$$\frac{dH}{dr} + \frac{\rho}{\sigma}H\omega^2 r = 0 \tag{3}$$

So we can derive the mathematical expression for the radially gear cross-sectional contour of the stress distribution:

$$H = H_0 e^{-\frac{\rho}{2\sigma}\omega^2 r^2} \tag{4}$$

However, Stowe Daura mathematical model, a sufficient condition for the establishment of a one-dimensional model based on generalized equations of motion gear radial force conservation is the gear initial (reference) the value of the thickness is much less than the gear outer diameter. Only this condition is satisfied, Stowe Daura mathematical model designed gear will meet the accuracy requirements.

Table 1: Optimum model parameter values of gear profile

Model parameters	Parameter values
Inner diameter of gear	0.02 m
Outside diameter of gear	0.8 m
Initial thickness of gear	0.02 m
Young modulus	2.1×10^{11}N/m^2
Poisson ratio	0.3
Rotational velocity	$2\pi\cdot50$ rad/s
Normalization constant - stress terms	10^7 Pa
Normalization constant- the smoothness of items	1

Optimum model of gear profile based on objective function: The traditional steam turbine disc gear is made from a homogeneous isotropic elastic material, Table 1 lists the values of the input parameters of the model and material properties. In the initial gear design, its thickness is uniform and constant. The material properties is the ordinary steel.

The stresses caused by the gear due to the rotational movement is much stronger than gear weight stress. Therefore, this model ignores gravity stress and we use elastic modulus E (N/m^2) and Poisson ratio indicate the radial stresses of the gear and the angular stress as follows:

$$\sigma_r = \frac{E}{1-\nu^2}\left[r\frac{dU}{dr} + (1+\nu)U\right] \tag{5}$$

$$\sigma_\varphi = \frac{E}{1-\nu^2}\left[\nu r\frac{dU}{dr} + (1+\nu)U\right] \tag{6}$$

and u is local radial displacement, r is radial coordinate and substitute it into the equation based on the equations of motion of the infinitesimal mass element, to derive the optimized gear profile second-order Ordinary Differential Equations （ODE） as :

$$-r^2\frac{d^2U}{dr^2} - (3+\phi)r\frac{dU}{dr} + (1-(1+\nu)\phi)U = \frac{1-\nu^2}{E}\rho\omega^2 r^2$$
$$r_0 < r < r_1 \tag{7}$$

This equation also applies to the optimization calculation of the center punch gear which inner diameter is r_0, outside diameter is r_1 and rotation angular velocity is ω. In this equation, Gear thickness H is a function of the radial coordinate, represent as dimensionless function description as follows:

$$\phi = \frac{r}{H}\frac{dH}{dr} = r\frac{d}{dr}\log\left(\frac{H}{H_0}\right) \tag{8}$$

At the inner radius of the gear, its displacement is zero boundary conditions and its corresponding boundary conditions of the control equation is as followed:

$$U\Big|_{r=r_0} = 0 \quad (9)$$

At the outside diameter of the gear, its radial stress component is zero and the control equation of the corresponding boundary conditions is followed:

$$r\frac{dU}{dr} + (1+\nu)U\Big|_{r=r_1} = 0 \quad (10)$$

In the case of a given function ∅, according to the equation we can derive the posedness of ordinary differential equations. If we solve $U = U(r)$, then we can use the generalized equations of motion (7) to determine the radial stress component.

For a uniform thickness and axisymmetric gear, its radial thickness function is a constant as $H(r) = H_0$, H_0 is the gears initial thickness and the function ∅ is identically zero. The radial stress of a uniform thickness gear is uneven distribution and at the inner radius of the gear, $r = r_0$, the radial stress is maximum. That the model in this study is used to solve the problem which is in the case of a given quality of a gear and the moment of inertia, how to make a full use of the existing materials to design the contour shape of the gear, then make the gear radial stress along the radial uniform distribute. In this study, the optimization of design abstracts for mathematical problems, using mathematical formulas to introduce objective function to optimize the design of a cross-sectional contour of the gear, its objective function describes as below:

$$Q_{stress}(H) = \int_{r_0}^{r_1} \frac{(\sigma_r - \sigma_{r,mean})^2}{\sigma_0^2} dr \quad (11)$$

In this function, $\sigma_{r,\,mean}$ is gear radial average stress, σ_0 is normalization constant-stress terms, so that the integrand function becomes dimensionless function by introducing a normalization constant. According to σ_r magnitude to determine the value of σ_0. Under normal circumstances, σ_0 magnitude is slightly smaller than σ_r, so conferred Q_s an appropriate order of magnitude. The optimal design problem turns into gear mass and moment of inertia of a given constraint solving mathematical formula gear profile $H(r) = H_0$ in order to make the objective function Q_{stress} Value of the minimum gear quality constraints Using mathematical formulas are described below:

$$m = 2\pi\int_{r_0}^{r_1} Hrdr = m_0 \quad (12)$$

The gears moment of inertia constraints is using the mathematical formula described below:

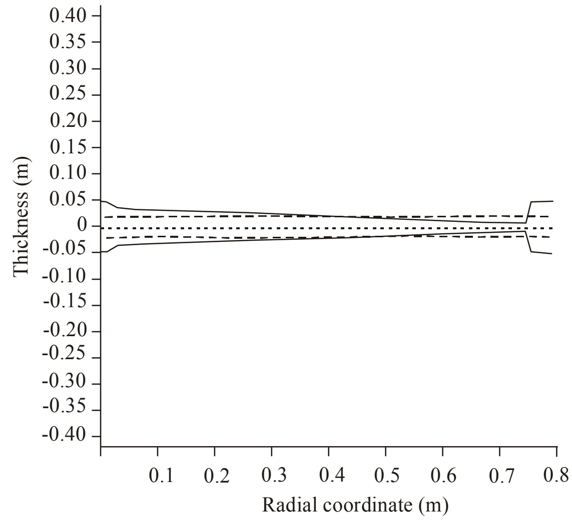

Fig. 2: Optimal profile design for gears which have same quality and inertia moment constraint condition

$$I = \pi\rho\int_{r_0}^{r_1} Hr^3 dr = I_0 \quad (13)$$

In this function, m_0 and I_0 are the mass and moment of inertia of the given gear. However, by the Eq. (9) is unable to obtain an accurate result, the gear profile of the obtained curve is not smooth, so the objective function need to insert a dH/dr Smooth integral function, the smooth integral function is described as follows:

$$Q_{smooth}(H) = A\int_{r_0}^{r_1} \left(\frac{dH}{dr}\right)^2 dr \quad (14)$$

Among this, A is the normalization constant - the smoothness of items is based on Q_{stress} and Q_{smooth}. If Q_{stress} and Q_{smooth} is in the same order of magnitude, once this condition is met, the value of the model normalization constant A is not sensitive. The ultimate goal of this model function in accordance with Eq. (9) and (12) obtained as follows:

$$Q = Q_{stress}(H) + Q_{smooth}(H) = \int_{r_0}^{r_1} \frac{(\sigma_r - \sigma_{r,mean})^2}{\sigma_0^2} dr + A\int_{r_0}^{r_1} \left(\frac{dH}{dr}\right)^2 dr \quad (15)$$

THE RESULTS OF OPTIMAL PROFILE DESIGN FOR GEARS

Figure 2 shows in the constraint condition to have the initial gear of the same mass and moment of inertia, the use of the control equations to optimize the design of the gear profile (in the figure the initial contour of the green curve represents gear, representative of the black curve gear optimization contour).

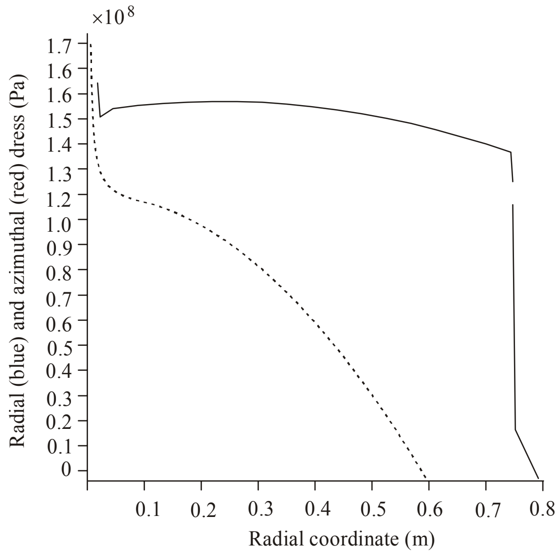

Fig. 3: The variation curve of radial stress or crankshaft fillets stress of initial profile design and optimal profile design for gears

Figure 3 shows the outline of the design optimization for the initial outline of the design and optimization of gear radial stress or angle contour design to stress curve (the green curve represents the initial outline of the design of the gear radial stress curve, the black curve represents the gear to optimize the outline of the design of the radial stress curve). Optimize the contours of the design of radial stress from Fig. 2 gear change curve can be seen in the gear radial cross-section the radial stress and angular stresses are almost constant and radially uniform distribution; Its maximum radial stress value appears at the inner diameter of the gear and its value is about 160 MPa about 210 MPa, maximum stress value was reduced by about 25%, compared to the maximum radial stress in the in gear initial contour design.

CONCLUSION

Traditional steam turbine rotates the gear along the radial the uneven force evenly and local stress concentration problems. In this study, given the quality and the moment of inertia of the gear prerequisite established based on the control the equation and Stowe Daura mathematical model optimized design model of the disk-line gear, obtained through the optimization algorithm to obtain the optimal thickness of uniform cross-section of the radial stress distribution distribution. The contour optimization design gear radial cross-section of the radial stress and angular stress near uniform distribution and its maximum radial stress compared to the initial outline design of wheel gear maximum radial stress decreased by about 25%.

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