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Research Article

Based on the Force Deployment Model of Unascertained Expectation

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Abstract: In this study, we utilize the unascertained mathematics method to give the unascertained number of countermeasure of anti-terrorism strategic force deployment and unknown event. It has been defined the situation sets of force deployment, condition density and mathematical expectation of density model. It has been given the unascertained parameters Cij which decide and direct the force deployment. Find out the condition density matrix of force deployment, further get the conditional density of single target force deployment, using the maximum density mathematical expectation in order to get the optimal mathematical model of multiple target force deployment. Analyzing the coefficient of model and provide two kinds of discussed computing method. The model overcomes the limitation of past deterministic thinking method which study the force deployment and provide the decision maker a relative substantial theory evidence.

Keywords: Conditional density, force deployment, mathematical expectation, model, situation, unascertained number

INTRODUCTION

The problem of forces 'deployment is a fundamental problem in military decision-making, in the past decision we can usually meet optimal result in a positive background and a result in stochastic condition .the practical anti-terrorism strategic forces' deployment problem is usually anfractuosity., because it's not a doubtless problem but not a Stochastic Process. It has a preordained fate relationship, it also has some complicated indeterminacy factors. In practice the system of anti-terrorism strategic forces' deployment is a complicated "indeterminacy system. Thus it's practical for us to use th**e** Unascertained Mathematical ideal method to study the problem forces' deployment. Bi and Yan (2010) study the research of military strategy thought in the countemporaryera. Li (2005) study the military strategy learns lectures. Shen *et al*. (2005) study the targets being misty to make policy troops an allotment model. Ren *et al*. (2007) have a research of the troops of the rate deployment model. Liu *et al*. (2010) analyze the troop disposition assessment Reconnaissance-Attack system.

In this study, we utilize the unascertained mathematics method to give the unascertained number of countermeasure of anti-terrorism strategic force deployment and unknown event. It has been defined the situation sets of force deployment, condition density and mathematical expectation of density model. It has been given the unascertained parameters Cij which decide and direct the force deployment. Find out the condition density matrix of force deployment; further get the conditional density of single target force deployment, using the maximum density mathematical expectation in order to get the optimal mathematical model of multiple target force deployment. Analysing the coefficient of model and provide two kinds of discussed computing method. The model overcomes the limitation of past deterministic thinking method which study the force deployment and provide the decision maker a relative substantial theory evidence.

DESCRIPTION OF THE PROBLEM

At present, the research of forces'deployment model consist of ascertain model and stochastic model. the problem of forces'optimized deployment in Military Operations Research is practically a ascertain designating problem**.** The model is expressed 0-1 planning:

$$\max p(s) = \sum_{i=1}^{m} a_{ij} x_{ij} \quad \text{s.t.} \sum_{i=1}^{m} x_{ij} = 1 \sum_{j=1}^{n} x_{ij} = 1$$
$$x_{ij} = 0.1;\ (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$$

The 0-1 variable x_{ij} is defined as:
when A_i is disposed B_j, $x_{ij=1}$
when A_i isn't disposed B_j $x_{ij=0}$

Although we can use parallelism method in parallelism to solve problem in order to get the result of forces 'deployment (it is a 0-1 matrix in practice) to guide the military decision-making. Actually, it is based on the premise that the above-mentioned model is specific ascertain assumption. But in the practical application, the eruption of hostilities, even in the forces' deployment process is full of many uncertain factors, thus many scholars use the probability method to study forces' deployment so as to make forces' deployment model transform studying ascertain problem to study stochastic problem:

$$\max p(S) = \sum_{j=1}^{n} p(B_j) p(S / B_j) = \sum_{i=1}^{m} p_{ij} p(A_{ij} B_j)$$

There, S is represented where we gain the victory For the problem of studying probability is a stochastic process at the same time we need a larger specimen space, the eruption of hostilities, even in the forces' deployment process are not only a stochastic progress. It has a preordained fate relationship ,it also has some complicated indeterminacy factors. Thus it's practical for us to use the Unascertained Mathematical ideal method to study the problem forces' deployment.

Definition1: All troops countermeasures of deployment problem research of troops are called countermeasures collection. Called $A = (A_1, A_2, \ldots, A_m)$, Among them, $A_i(i = 1, 2, \ldots, m)$ is the i-th deployment countermeasure troops, If mean the x_i grow the troops number of troops deployment countermeasures (when $i<j$, have $x_i<x_j$), if mean $\varphi(x_i) = \frac{x_i}{\sum_{i=1}^{m} x_i}$,

Then call $[[x_1, x_m], \varphi(x)]$ for the unascertained number of countermeasures.

Definition 2: All events that may occur of deployment problem of troops are called events collection. Called $B = (B_1, B_2, \ldots, B_n)$, among them, $B_j(j = 1, 2, \ldots, n)$ is the j-th event. The b_j is the possibility size of the occurrence of the event B_j when $x = j$, $h(x) = b_j$,

We called $[[1,n], h(x)]$ as the unascertained number of the event.

Definition 3: Cartesian product of countermeasures collection $A = (A_1, A_2, \ldots, A_m)$ and events collection $B = (B_1, B_2, \ldots, B_n)$, $A\times B = \{(A_i, B_j) | A_i \in A, B_j \in B\}$ are called the situation in collection, called $S = A\times B$. For arbitrarily $A_i \in A$, $B_j \in B$, we called (A_i, B_j) as situation. Make $S_{ij} = (A_i, B_j)$, Then event B_j occur and A_i deploy troops and carry out a task with B_j.

Definition 4: For situation (A_i, B_j), Use C_{ij} to mean the reliability of a defence B_j for countermeasures A_i, then we called C_{ij} as the unascertained density of situation.

Definition 5: For situation (A_i, B_j), Use b_{ij} to mean the reliability of performance the task and obtaining a victorious while using countermeasure A_i when event B_j occur. Then we called b_{ij} as the unascertained factor density of situation. Then we called:

$$\left(b_{ij}\right)_{m\times n} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m2} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

as the unascertained factor density array.

Definition 6: Set $[[a, b], \varphi(x)]$ as an unascertained number, $\varphi(x)$ is an factor density, Then we called $E = \int_a^b x\varphi(x)dx$ as the unascertained density expectation.If $[[x_1, x_m], \varphi(x)]$ is an unascertained racional,then $E = \sum_1^m x_i \varphi(x_i)$.

Definition 7: Suppose there are R experts analyse the density of situation (A_i, B_j). order $b_{ij}^{(k)}$ to be density b_j generalized residual result which the k th expert deal with situation (A_i, B_j), thus $(b_j^{(1)}, b_j^{(2)}, \ldots, b_j^{(R)})$ are b_j generalized residual vector. If the weight of k th expert is w_k $0<w_k \le w_k \le 1$ $(k = 1, 2, \ldots, R)$ that $\sum_{k=1}^{R} w_k b_j^{(k)}$ is the generalized residual vector $b_j^{(*)}$ of b_j which is density of (A_i, B_j) $b_j^{(*)} \sum_{k=1}^{R} w_k b_j^{(k)}$.

Definition 8: Order $b_{ij}^{(k)}$ be density b_j generalized residual result which the k th expert deal with situation (A_i, B_j), thus:

$$\left(b_{ij}^{(k)}\right)_{m\times n} = \begin{bmatrix} b_{11}^{(k)} & b_{12}^{(k)} & \cdots & b_{1n}^{(k)} \\ b_{21}^{(k)} & b_{22}^{(k)} & \cdots & b_{2n}^{(k)} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m2}^{(k)} & b_{m2}^{(k)} & \cdots & b_{mn}^{(k)} \end{bmatrix}$$

The generalized residual matrix of b_{ij} which is made by the k the expert $(i = 1, 2\ldots m, j = 1, 2 \ldots n)$ If the weight of the k th expert is w_k $0< w_k \le 1(k = 1, 2, \ldots, R)$ thus $\sum_{k=1}^{R} w_k b_{ij}^{(k)}$.

Is be density $b_{ij}^{(*)}$ generalized residual result which is the unascertained condition density b_{ij} of situation (A_i, B_j), thus $b_{ij}^{(*)} \sum_{k=1}^{R} w_k b_{ij}^{(k)}$

The troops deployment problem can describe for our armies have the troops of total amount M, Constitute m battle groups $A_1, A_2, \ldots, A_m$ of independence according to the amount organic combination of the weapon material, true war technique and the personnel's amount, probably produce the point region of the riot activity of the terror raid as $B_1, B_2, \ldots, B_n$.

At some a particular period, the region of B_j takes place the possibility of terrible activity as b_j and regard as war the group A_i to set up defense at the B_j, setback terror assault riot activity of the credibility is b_{ij}, how to set up defense from the overall situation, namely with how of the situation set up defense formation situation to gather, or choice which c_{ij} to make the possibility of setback enemy biggest for our army, so we can obtain warlike victory.

THE MODEL OF TROOPS DEPLOYED

The uncertain number of troops deployed: Suppose that independent battle group number is m that is as A_1, A_2, …, A_m , the independent battle group A_i consist of x_i members, Suppose $x_1<x_2<\ldots<x_m$. The main defiance areas to have possible terrorism war are B_1, B_2, …, B_n. The possibility that B_i take place terrible riot activity is b_i, Then we can get following 2 uncertain number:

$$[[x_1, x_m], \varphi(x)] \text{ where } \varphi(x_i) = \frac{x_i}{\sum_1^m x_i} \; [[1, n], h(y)] \text{ where } h(y=j) = b_j \tag{1}$$

The b_j means that the possibility size of the war to happen in B_j region.

The condition density of troops deployment:

$$\text{Suppose } b_{ij} = b_j(x, x = x_i) \tag{2}$$
$$(i = 1, 2, \ldots t; j = 1, 2, \ldots n)$$
$$0 \le b_{ij} \le 1$$

b_{ij} is defined to the condition density of troops deployment, The b_{ij} means the credibility to obtain overall victory, which happen in B_j region, when A_i troops is organized to defiance B_j region to carry out the task of battling .

so get the uncertain condition density matrix of troops deployment:

$$\left(b_{ij}\right)_{m\times n} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m2} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \tag{3}$$

The jth list $(b_{1j}, b_{2j}, \ldots, b_{mj})^T$ of in the matrix show the credibility distribution that when battle group A_1, A_2, …, A_m is organized the defence to B_j, respectively, A_i defeated the enemy to get overall victory. The c_{ij} Mean the credibility of the battling task to carry out when war happens in B_j region , the A_i troops is deployed in B_j. So then$(c_{1j}, c_{2j}, \ldots, c_{mj})$ means respectively the possibility size that battling group $A_1, A_2, \ldots, A_m$ is organized a defiance to B_j.

The density model of troops deployment: Make

$$\left[c_{1j}, c_{2j}, \cdots c_{mj}\right]\left[b_{1j}, b_{2j}, \cdots b_{mj}\right]^T = \sum_{i=1}^{m} b_{ij}c_{ij} = g(b_j) \tag{4}$$

The $g(b_j)$ means the credibility that $A_1, A_2, \ldots, A_m$ defeated of enemy war that happen in B_j region, when B_j being defended with $A_1, A_2, \ldots, A_m$ $b_p = \min\{b_j\} b_q = -\max\{b_j\}$(j = 1, 2, …, n) Combine to line up a preface according to the size of b_j, Get a condition density type uncertain number:

$$\left[\left[b_p, b_q\right], g(b)\right]$$

Define:

$$E(b) = \sum_1^n b_j g(b_j) \tag{5}$$

as uncertain mathematics expectation of density type.

The $E(b)$ means the uncertain mathematics expectation that our army acquires overall victory. The different troops deployment gets the different mathematics expectation. Comprehensive analysis on the above to know: troops deployment problem of the counter-terrorism strategic is look for reasonable troops deployment, To obtain reasonable cij (i = 1, 2, …, t; j = 1, 2, …, n)

Make the $E(b)$ maximize, namely:

$$\max E(b) = \sum_{j=1}^{n} b_j g(b_j) = \sum_{j=1}^{n} b_j \sum_{i=1}^{m} b_{ij} c_{ij} \tag{6}$$

That means under Guiding of the target to look for reasonable uncertain density distribution, make the biggest possibility obtain the victory of war, this c_{ij} is a comprehensive information characteristic that mean the credibility when A_i is deployed to B_j. All of b_j and b_{ij} are uncertain condition density, which relies on politics, culture, diplomacy and military...etc of nation and region of terrible influence place, they are analyzed and calculated by the various factors that induct war.

The model (6) is also abstract into a problem as follows:

$$\max f(x) = \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij} x_{ij} \tag{7}$$
$$\sum_{i=1}^{m} x_{ij} = 1 \;, 0 \le x_{ij} \le 1$$
$$(i = 1, 2, \ldots t; j = 1, 2, \ldots n)$$

Solve the problem and then we can get a set of decision variables x_{ij} (i = 1, 2, …, t; j = 1, 2, …, n), which can guide troops deployment.

THE ANALYSIS AND ASCERTAINMENT OF MODEL COEFFICIENT

The coefficient a_{ij} in the model (7) and coefficient b_j b_{ij} in the model (6) are the promise of optimizing deployment. Next, we carry on analysis and discussion.

Both b_j and b_{ij} are unknown condition density and depending on the comprehensive situation which constituted by the politics, culture, diplomacy, military strength of terrible nation and region.

Suppose these comprehensive situations can be divided into s kind of state $z_1, z_2, \dots, z_s$ and form into comprehensive situation SZ:

$$zi \cap zj = \Phi, (i \neq j)$$

Regard SB as the appearance space of war occurrence. Obviously the SB has two elements. (Occur war or don't). Regard B as the event of war occurrence. and suppose an induct of riot a factor for $F_1, F_1, \dots, F_t$, Only being causing the inducement appear just may cause war. Make (B_j, F_i) as war situation caused by remote cause F_i. f_{ij} is under F_i remote cause, cause B_j war inducement of possibility size, so b_j $\sum_{i=1}^{t} f_{ij}$ = 1, here f_{ij} have two methods to ascertainment.

Method one: The result has been given through the unascertained measure of expert estimation, under the predisposing of F_i that caused the event occurs in war situation B_j, this constitute the predisposing factors matrix:

$$\left[(B_j, F_i)\right]_{n\times t} = \begin{bmatrix} (B_1,F_1) & (B_1,F_2) & \dots & (B_1,F_t) \\ (B_2,F_1) & (B_2,F_2) & \dots & (B_2,F_t) \\ \dots & \dots & \dots & \dots \\ (B_n,F_1) & (B_n,F_2) & \dots & (B_n,F_t) \end{bmatrix}$$

The reason that a war occurs is able to be caused by one factor, or also can be caused by some factors at mean time; here we only discuss the circumstance of single factor independence that caused the war.

Suppose there are R experts, the K experts think under the predisposing of F_i , the possibility which a war occurs in the place of B_j is $f_{ij}{}_{ij}^{(k)}$, the evaluation set is consist of predisposing which caused the war from R experts:

$$\left[b_{ij}^{(1)}, \ b_{ij}^{(2)}, \ \dots \ b_{ij}^{(R)} \right]^T$$

The weight vectors of K experts is $[w_1, w_2, \dots w_R]$, thus get the $f_{ij}^{(*)}$ which is the prior estimate value of f_{ij}:

$$f_{ij}^{(*)} = \left[w_1 \quad w_2 \quad \dots \quad w_R \right] \begin{bmatrix} \left(b_{ij}^{(1)}\right) \\ \left(b_{ij}^{(2)}\right) \\ \dots \\ \left(b_{ij}^{(R)}\right) \end{bmatrix}_{R\times 1} = \sum_{k=1}^{R} w_k f_{ij}^{(k)}$$

$$(i = 1, 2, \dots t; j = 1, 2, \dots n)$$

It is the same for the b_{ij} which is able to be find out through the past experience and expert forecast method.

Method two: Use the probability, because $z_1, z_2, \dots, z_s$ and it has became general trend SZ.

$$zi \cap zj = \Phi, (i \neq j)$$

thus:

$$p(F_i) = \sum_{k=1}^{s} p(F_i / Z_k) p(Z_k), \ (i = 1, 2, \dots t)$$

thus: $b_j = p(B_j)$ should be comprehend as probability:

$$b_j = \sum_{i=1}^{t} p(B_j / F_i)$$

while:

$$p(B_j / F_i) = \frac{P(B_j F_i)}{P(F_i)} = \frac{p(F_i / B_j) P(B_j)}{\sum_{k=1}^{s} p(F_i / Z_k) p(Z_k)}$$

$$(i = 1, 2, \dots m, j = 1, 2, \dots n)$$

here:

$$p(B_j) = \sum_{k=1}^{s} p(B_j / Z_k) p(Z_k), \ (j = 1, 2, \dots n)$$

DISCUSSION

In fact, Still return to produce one some problems which stay at the further study to comprehension or useful property from the research of the b_j and the b_{ij}, Thus produce the model canning and solve method to be advantageous to analysis more and solve. If we can understand $\sum_{i=1}^{m} b_{ij} \leq 1$, under the some condition or not. Here"=" establish to mean inevitable occurrence war feeling, "<" establish to express uncertain occurrence war feeling:

$$0 \leq \sum_{i=1}^{m} b_{ij} \leq 1, \ (i = 1, 2, \dots m) \ 0 \leq \sum_{j=1}^{n} b_{ij} \leq 1, \ (j = 1, 2, \dots n)$$

$$\sum_{i=1}^{m} b_i \le 1, \ \sum_{j=1}^{n} b_j \le 1,$$

Moreover for c_{ij} to speak, when area B_j has war feeling, -1 battle group must defend the war feeling. Namely $\sum_{i=1}^{m} c_{ij} = 1$, ($j$ = 1, 2, …, n), But for any deployment, each war area's hasing war feeling is an indetermination affairs, namely:

$$\sum_{j=1}^{n} c_{ij} \le 1, \cdot (i = 1, 2, ... m)$$

Then we can give model's (6) stipulation condition of increment then get new model thus.

CONCLUSION

According to unascertained condition density deployment model of troops, use unascertained thought method to analysis deploy environment, lead to deploy into the troops of unascertained number and condition density matrix. We can get the best deployment model of troops though the unascertained number expect of biggest worthy, avoided using the limit of the dispose troops for battle of the thought method of assurance before. The key of this model is indeed to make sure that 3 unascertained numbers and condition density matrix.

Along with the development of war, the deployment model of troops by all means faces a new topic and the deployment model of troops also needs a further research.

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