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Research Article

Approach to Identification of a Second-Order Volterra Kernel of Nonlinear Systems by Tchebyshev Polynomials Method

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Abstract: In this study, we investigate the Tchebyshev polynomials expansion method for the kernels identification of nonlinear systems. In aerodynamic systems, all the output data to an arbitrary input may be obtained by executing the Computational Fluid Dynamic (CFD) program code. This calculation process may take more than several hours or days to complete. In comparison with the indicial or impulse methods our method is efficient, which does not need more output data for the identification of the second-order kernel by running CFD code repeatedly. This new approach may be applied to the aeroelastic problems. Two examples illustrate the whole process.

Keywords: Kernels identification, tchebyshev polynomials expansion volterra series

INTRODUCTION

Nonlinear systems are systems whose outputs are a nonlinear function of their inputs. There are many practical examples of nonlinear systems. They occur in diverse areas such as biological systems, communication systems and aerodynamic systems. Nonlinear unsteady aerodynamic phenomena can have a significant effect on the performance and stability of a flight vehicle, for an example, at transonic speeds where detrimental aero-elastic phenomena are most likely to occur (Edwards and Malone, 1992).

During the early development of mathematical models of unsteady aerodynamic responses, several analytically-derived unsteady aerodynamic responses, such as Wagner's function, Kussner's function, Theodorsen's function and Sear's function are studied (Herbert, 1925; Theodore, 1935; Küssner, 1936; Dowell *et al*., 2005).

Nowadays, the most powerful and sophisticated tools for predicting nonlinear unsteady aerodynamic characteristics are being developed in the field of computational fluid dynamics (CFD) (Edward and Thomas, 1989). However, sometimes the computational costs become prohibitively expensive. This can be on the order of days, depending on the user demand for a particular computer. In order to develop mathematical models that completely characterize the aerodynamic system of interest and use these models in various analyses without costly re-execution of the CFD code, more efficient approaches are needed (Silva, 1997).

As The Volterra theory of nonlinear systems provides a mathematically rigorous approximation technique to describe these unsteady aerodynamic effects. This theory was first applied by Wiener (1942). In modern digital signal processing fields, the truncated Volterra series model is widely used for nonlinear system representations. In discrete-time signal processing, once the unit sample function, quite different from the unit impulse function, or Dirac delta function, is well defined, Silva (1997) firstly applied the discrete-time Volterra series to a second-order truncated, time-invariant, CFD model.

However, these methods require large numbers of CFD computations to determine Volterra second-order kernel.

In our study, the first-order kernel is computed by the impulse method. The second-order kernel can be expressed by the linear combination of Tchebyshev orthonormal basis set. The unknown coefficients will be solved by the method of least squares. The advantage of this approach is that a few of CFD computations are needed.

LITRATURE REVIEW

Impulse method: In digital filter design, there exist mathematical concepts that are quite different from their continuous time counterparts. The unit impulse function is defined (Silva, 1997) in discrete systems as:

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$$u[n]=\begin{cases}1.0 & n=n_0\\ 0.0 & n\neq n_0\end{cases} \quad (1)$$

where,
n = Discrete-time variable

For a class of weekly nonlinear time-invariable systems, the discrete-time second-order truncated Volterra series has the form:

$$\begin{aligned} y[n] &= h_0+\sum_{k=n-N}^{n} h_1[n-k]u[k] \\ &+\sum_{k_1=n-N}^{n}\cdot\sum_{k_2=n-N}^{n} h_2[n-k_1,n-k_2]u[k_1]u[k_2] \end{aligned} \quad (2)$$

where y is the response of this system to u, an arbitrary input; h_n the n-order kernel; and N the memory length (Stephen and Leon, 1985).

Clancy and Rugh (Steven and Wilson, 1979) proved that the values of the kernels can be determined from the responses to a set of inputs:

$$\begin{aligned} &u_0(k)=\delta(k),\quad u_{i_1\cdot i_m}=\delta(k-i_1)+\cdots+\delta(k-i_1-\cdots-i_m) \\ &m=1,2...N-1,\quad i_1,...,i_m=1,2\cdots \end{aligned} \quad (3)$$

Using the impulse sample function (1), Silva (1997) in his dissertation derived the formulas of first-and-second order kernels in the similar ways.

Obviously the first-order kernel is easily determined by impulse method; however the identification of the second-order kernel needs more output data by executing CFD program code repeatedly, for an instance, sometimes at least hundreds of times. This limits the use of the impulse method to identify higher kernels.

Tchebyshev polynomials expansion method: By using a time transform, the second-order Volterra kernel can be expressed in terms of the orthonormal basis set as:

$$h_2[t,s]=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}a[i,j].T_i[t].T_j[s]S,t\in(-1,1) \quad (4)$$

where, $\alpha[i,j]$ are the coefficients; $T_n[x]$ Tchebyshev polynomials which can be obtained by solving Tchebyshev differential equation $(1-x^2)y'''-xy'+n^2y=0$.

The Tchebyshev polynomials $T_n[x]$ (Ogunfunmi, 2007) can be expressed:

$$T_n[x]=\frac{\sqrt{1-x^2}}{(-1)^n(2n-1)(2n-3)\cdots 1}\frac{d^n}{dx^n}(1-x^2)^{n-0.5} \quad (5)$$

where, n = 1, 2,.. . All the Tchebyshev polynomials form a complete orthogonal set on the interval with respect to the weighting function. It can be shown that:

$$\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}T_m[x]\cdot T_n[x]dx=\begin{cases}0 & m\neq n\\ \pi & m=n=0\\ \pi/2 & m=n=1,2,3...\end{cases} \quad (6)$$

Once all the coefficients $\alpha[i,j]$ have been determined, the second-order kernel will be obtained.

If we know the second-order kernel function in advance, we can determine all the unknown coefficients. This enables us to evaluate whether the identification model is good or not.

IDENTIFICATION

Riccati nonlinear equation: System representations using differential equation models are very popular. Riccati equation can be used to model a class nonlinear circuit system. It has the form below:

$$h_2[t,s]=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}a[i,j]\cdot T_i[t]\cdot T_j[s]$$

S, t ∈ (-1, 1) (7)

where, i(t) is the current around the circuit; v(t) the input voltage; and ε the nonlinear parameter. Here we assumed ε = 0.1.

Modeling this system by second-order truncated Volterra Series, the kernels (Wang *et al*., 2010, 2011) computed by the impulse are the following forms:

$$h_1(t)=\frac{2\cdot e^{-t}}{1.001-0.001\cdot e^{-t}}-\frac{e^{-t}}{1.002-0.002\cdot e^{-t}}\qquad t\in(0,T) \quad (8)$$

$$h_2(s,t)=\frac{A}{B}\frac{50\cdot e^{-t}}{1.001-0.001\cdot e^{-t}}-\frac{50\cdot e^{s-t}}{1.001-0.001\cdot e^{s-t}}\qquad t>s \quad (9)$$

where T is a sufficient large constant which guarantees at $t>T$; and

$$A=(50+\frac{50\cdot e^{-s}}{1.001-0.001\cdot e^{-s}})\cdot e^{s-t},$$

$$B=1.001+\frac{0.001\cdot e^{-s}}{1.001-0.001\cdot e^{-s}}-(0.001+\frac{0.001\cdot e^{-s}}{1.001-0.001\cdot e^{-s}})\cdot e^{s-t}$$

Figure 1 and 2 show the first and second-order kernels respectively which have the decay properties: down to zero as time goes by.

Utilizing Tchebyshev polynomial's orthogonality, all the coefficients in expression (4) can be determined by the following formula:

$$a[i,j]=t_i\cdot s_j\cdot\int_{-1}^{1}\int_{-1}^{1}\frac{h_2(t,s)}{\sqrt{1-t^2}\sqrt{1-s^2}}\cdot T_i[t]T_j[s]dt\cdot ds \quad (10)$$

where,

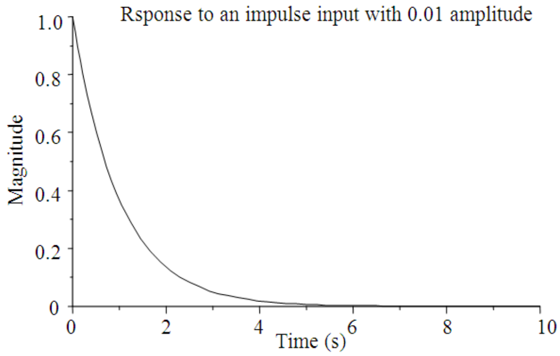

Fig. 1: First-kernel of Riccati equation

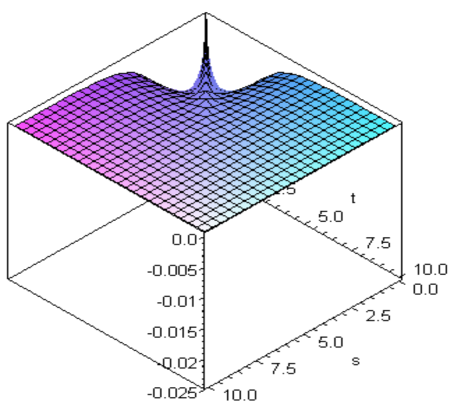

Fig. 2: Second-kernel of Riccati equation

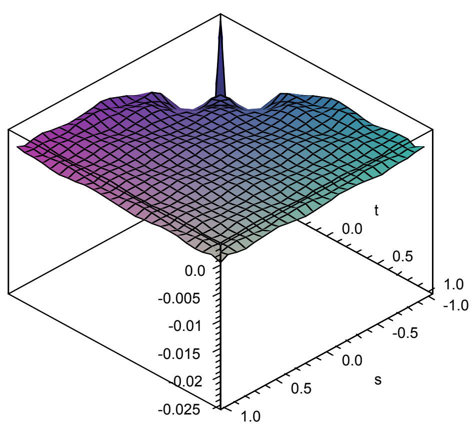

Fig. 3: Second-kernel identified by Tchebyshev polynomials method

$$t_i = \begin{cases} \frac{1}{\pi} & i=0 \\ \frac{2}{\pi} & i \neq 0 \end{cases}, \quad s_j = \begin{cases} \frac{1}{\pi} & j=0 \\ \frac{2}{\pi} & j \neq 0 \end{cases}$$

are constants.

We choose ten terms of Tchebyshev polynomials and work out total the coefficients. Indeed only fifty-five numbers need to be calculated from (10) for the symmetric second-order kernel function.

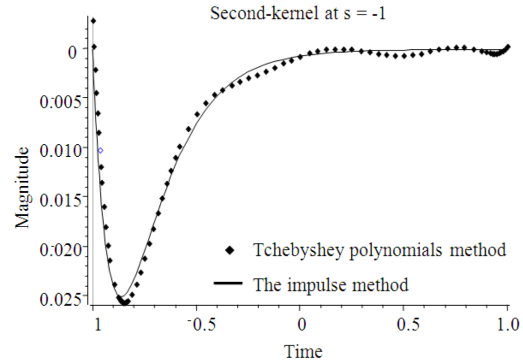

Fig. 4: Comparison of the second-kernel curve by the Tchebyshev method and the impulse method at s=-1

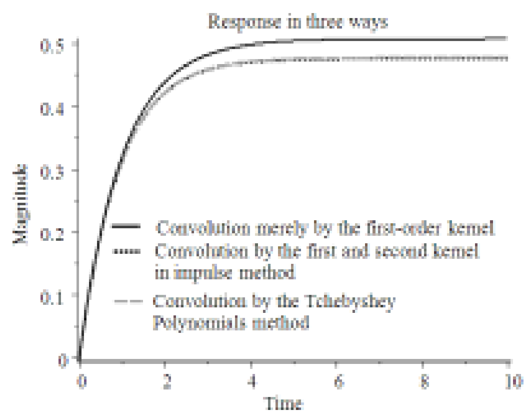

Fig. 5: Convolutions with kernels computed by impulse method and Tchebyshev polynomials method respectively to an indicial excitation with 0.5 amplitudes

We draw the identified three dimensional graph of the second-order kernel below (Fig. 3)

With comparison of Fig. 3 and 2, the result of identification is a reasonable approximation. Let s =-1, a very good approximation result shows in the Fig. 4.

To see the effect of identification clearly, we substitute the first-order kernel made by impulse method and the second-order kernel identified by Tchebyshev polynomials method into expression (2). The response to an indicial input with 0.5 amplitudes in comparison with those responses to the same input by convolution with the kernels obtained in the impulse method shows in the Fig. 5.

From Fig. 5, it shows this nonlinear system's second-order kernel can't be ignored. The second-order kernel identified by using Tchebyshev polynomials method is quite good and can be applied to the convolution formula (2). This indicates that the Tchebyshev polynomial method is really feasible for identification of the nonlinear system's Volterra second-order kernel.

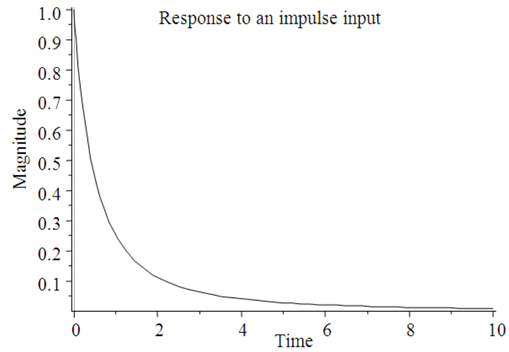

Fig. 6: Response to an impulse excitation

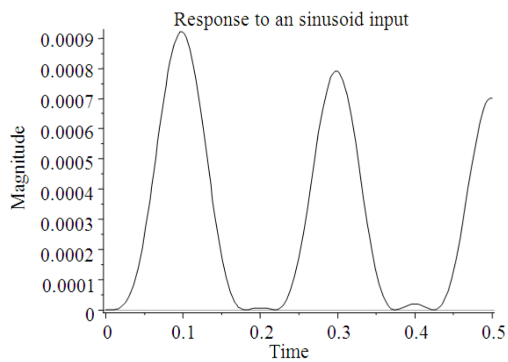

Fig. 7: Response to an sinusoid input

Tchebyshev polynomials method: In the section, we will show how to identify the second-order kernel by Tchebyshev polynomials method.

In aerodynamic system, the responses to an arbitrary input, such as airfoil pitch movement, can be computed by the CFD method. It really needs an hour or more. Here we assume that the nonlinear system of interest can be represented by the Volterra Series which only has a second-order kernel function and has the form below:

$$h_2(\tau_1,\tau_2)=\frac{1}{(1+\tau_1)\cdot(1+\tau_2)} \tag{11}$$

Our aim is to identify this unknown kernel by Tchebyshev polynomials method. We rewrite the unknown kernel in $g(t,s), t,s\in[0,T]$.

The nonlinear time-invariant system's output $y(t)$ to an arbitrary input $u(t)$ can be expressed:

$$y(t)=\int_0^t\int_0^t g(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \tag{12}$$

Note that the output can be obtained to a given input. We first execute an impulse excitation to this nonlinear system and draw its response curve below (Fig. 6).

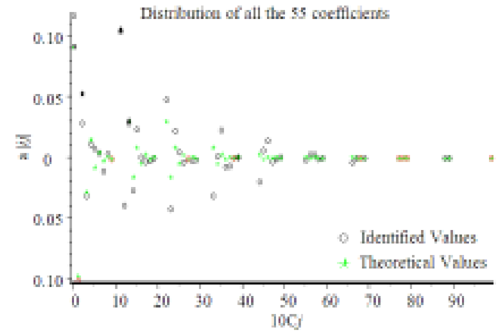

Fig. 8: Distribution of the coefficients for identification of second-order kernel

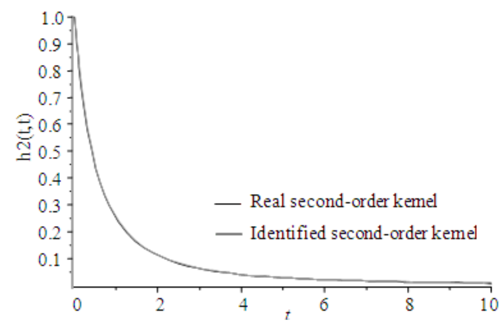

Fig. 9: Effect of identification for the $h_2(t, s)$ at t = s

From Fig. 6, we can see that the response soon decays near to zero after a finite time. We take the length of time $T = 10$.
By a time transform, we have:

$$g(t,s)=g\left(\frac{(m+1).T}{2},\frac{(n+1).T}{2}\right)=f(m,n)m,n\,(-1,1) \tag{13}$$

The new function f(m,n) by Tchebyshev polynomials can be expressed below:

$$f[m,n]=\sum_{i=0}^{N}\sum_{j=0}^{N}a[i,j]T_i[m]T_j[n] \quad m,n\in(-1,+1) \tag{14}$$

Here we choose N = 10. Rewrite (14) in matrix form below:

$$f(m,n)=g(t,s)= (T_0[m]T_0[n]\;T_0[m]T_1[n]+T_1[m]T_0[n] \cdots T_9[m]T_9[n])(a[0,0]\;a[0,1]...a[9,9])^T \tag{15}$$

Substituting expression (13-15) into (12), we yield:

$$y(t)=(\int_0^t\int_0^t T_0[m]T_0[n]u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2... =\int_0^t\int_0^t T_9[m]T_9[n]u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2)(a[0,0]\quad a[0,1]...a[9,9])^T \tag{16}$$

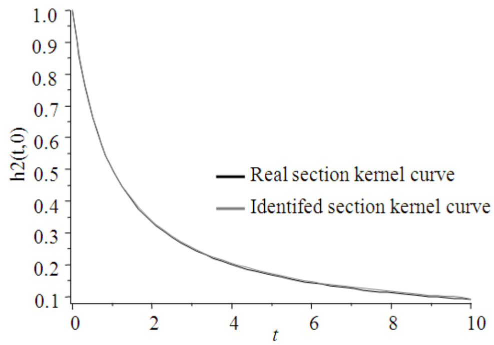

Fig. 10: Effect of identification for the h 2(t, s) at s = 0

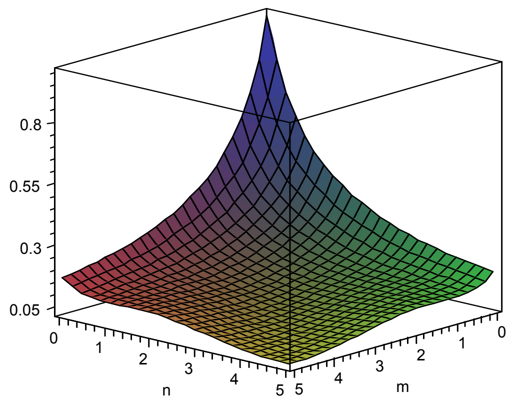

Fig. 11: Three-dimensional graph of second-order kernel at (0, 5) × (0, 5)

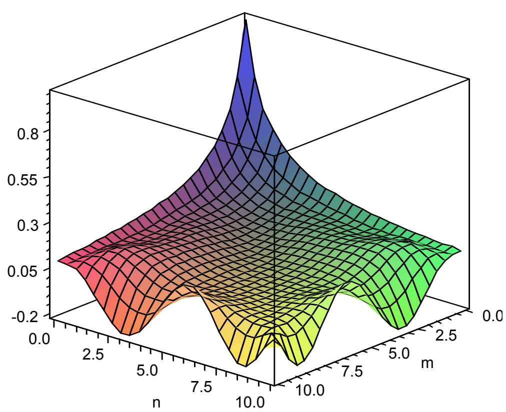

Fig. 12: Three-dimensional graph of second-order kernel at (0, 10) × (0, 10)

where,

$m = \frac{2\tau_1}{T} - 1, \quad n = \frac{2\tau_2}{T} - 1.$ are constants.

We discretize the data in the expression (16) and then can work out all the coefficients by the method of least square.

In the identification process, input is in the form $u(t) = 0.5 \cdot \sin(10 \cdot \pi \cdot t)$ and the output given by (Fig. 7).

To discretize the output data, we adopt the discrete time step 0.01sec. All the fifty-five coefficients can be computed by least square estimate algorithm. The distribution of those coefficients shows in the Fig. 8.

From Fig. 8, the coefficients identified are an acceptable approximation to the theoretical values.

From Fig. 9 to 10, the effects of identification to the second-order kernel in different cases are perfect. Three-dimensional graphs are also given in Fig. 11-12.

CONCLUSION

In this study, we discuss the second-order kernel's identification technique by Tchebyshev polynomials expansion method. In the nonlinear Riccati equation model, the results obtained by different methods preserve the consistency. In an aerodynamic system, output data come from executing the particular CFD code which needs a long time, for an example, several hours or days. Our method does not need more output data in comparison with the impulse method. Therefore, Tchebyshev polynomials expansion method is more efficient than the traditional method: indicial method and impulse method.

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