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Research Article

Study of Mechanical Model of Sports Ball'S Flight Trajectory and Its Application

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Abstract: In order to resolve the controversy of ball games the placement problem fined on the other hand, to promote the teaching and training ball flight trajectory. Tennis is the main object of study, use of literature, mathematical statistics and experimental research methods and to establish an appropriate mathematical model, using MATLAB software to accurately determine the trajectory and impact point of the ball's flight, used in teaching, training and competition in. The conclusions show that the law, accurate data analysis using MATLAB software can simulate the flight of the ball moving on sports development has a very good help provide a new way of thinking for the in-depth study the pattern of winning ball games and similar sports teaching, training and selection of athletes is an important complementary and valuable.

Keywords: Ball, mechanical model, sports, trajectory

INTRODUCTION

"Sports biomechanics", written by Li Shuping, explains the generation and trajectory of spinning ball, base on fluid mechanics. He proposes, if spinning exists, the ball will conduct curving motion. "University Physics", written by Liu Dawei from Harbin Industry University, discusses strange ball flight trajectory (Min, 2009). He holds, the reason for the parabolic motion caused by spinning excluding the curving motion in the horizontal level is the spinning relates to the air. Air is fluid. Some changes must happen when the football crossing the fluid (Zhang, 2003).

The most valuable study on curving ball is that of Xiong Zhifeng, who is from Jiangxi Normal College, from his studies on biomechanical features and competitive abilities of Backham's curving ball, explores its technical characters and principles and provides some references for developments of advanced curving ball technologies. Therefore, the curving ball includes many inspects and it is obviously important. We will analyze and explain briefly in the following (Haung, 2008).

In 1976, Peter Bearman and colleagues, from London Loyal College, once found, with the same spinning velocity, the spinning force of faster ball will be smaller. That is to say, the spinning force of a ball will stop on the end could not be ignored. The spinning ball with curving trajectory is gradually noticed by people, which becomes a special way to goal. Qianlai and Zhou Yuling (Money, 2007), in their book "effect of eagle eye on tennis ball", proposes it is good for improving players' technology and enhance the tactics. Its pictures are clear, which can be valuable papers for players and coaches after competitions to observe the actions, to analyze players' tactics and provides important references. After Nadar wins Fedeler many times, Nadar's principles of spinning ball and running can be analyzed by papers from eagle eye. It can help Fedeler cultivate new tactics. Meanwhile, it can stimulate players to continuously improve, improve their tactics, finally bringing in great improvements in whole level. Eagle eye gets three dimensional coordinates, velocity and directions in the motion to calculate its flight trajectory and make sure the falling point.

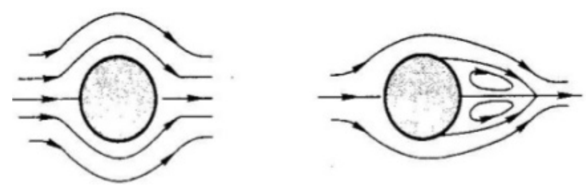
(a) (b)

Fig. 1: Forces of non-spinning ball

Magnus Effect: In ideal state, imagine the ball not spinning, in reality, a ball has to spin without absolute level motion. It is forced by gravity, buoyancy, extra mass and air resistance, etc. the resistance is opposite to motion's direction (Fig. 1).

Conduct up spinning force on the ball, it will spin intensely and Magnus effect generates. That is, when the spinning angle velocity doesn't coincide with flight velocity, a cross force will generate vertical to the plane consisting of spinning angle velocity vector and translational velocity vector. The flight trajectory will deflect under that force. In the physical perspective, the cross force is generated because the spinning can stimulate the surrounding fluid spinning, making side's

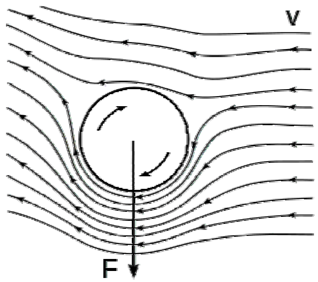

Fig. 2: Forces of spinning ball

fluid velocity to increase, the other side's velocity to reduce(Sun, 2003). The essence is a viscous effect, which is generated by motion in the viscous fluid.

According to Bernoulli theorem, the increasing fluid velocity leads to decreasing pressure, which results in the cross pressure difference and cross force is generated. Meanwhile, cross force is vertical to motion direction (Wang, 2006). Therefore, it only changes the motion direction, which is the centripetal force. It changes the motion direction. Magnus force is positive proportion to fluid spinning velocity, motion velocity and cubic of diameter. Therefore, the bigger the volume is, the faster it is and the bigger Magnus force is. The following formula can express the Magnus force. Magnus force is vertical to motion's direction (Fig. 2). The calculation formula is:

V represents wind velocity and direction F is strength leading to low pressure.

The formula is:

$$F = 1/2\rho\upsilon^2 AC_L$$

F = Lift force
ρ = Density of the fluid
r = Radius of the ball
v = Velocity of the ball
A = Cross-sectional area of ball
C_L = Lift coefficient

Study purpose and significance: Eagle eye has been accepted by sports area. It is called as instant replay system, capturing basic data of flight trajectory from different directions. Then generate three-dimensional image in those data. Finally, in constant forming image technology, demonstrating clear motion path and falling point. It is a powerful tool for judgers. It can avoid the conflicts. The falling point is a shadow, which is not taken, but the calculation results. In order to resolve the controversy of ball games the placement problem fined on the other hand, to promote the teaching and training ball flight trajectory. Tennis is the main object of study, use of literature, mathematical statistics and experimental research methods and to establish an appropriate mathematical model, using MATLAB software to accurately determine the trajectory and impact point of the ball's flight, used in teaching, training and competition in. The conclusions show that the law, accurate data analysis using MATLAB software can simulate the flight of the ball moving on sports development itself very good and provides a new way of thinking for the in-depth study the pattern of winning ball games (Wu, 2007).

MATHEMATICAL MODEL FOR BALL FLIGHT TRAJECTORY

Task and hypothesis: Set middle point of field as coordinate origin, x axis parallel to bottom line, y axis parallel to sides line, z axis is vertically upward and build three-dimensional coordinate system. Unit length is m. If the velocity on direction (p, q, 1) at point (x_0, y_0, z_0) is v. Build the model and calculate the falling point of tennis.

Due to spinning existing, due to Bernoulli principle, spinning in a certain direction will exist. If the ball in last task still go through point (x_1, y_1, z_1), try to judge the falling point.

Problem analysis: As a kind of ball games, tennis has general features of ball games. If ignoring air resistance, on condition of no spinning, only forced by gravity, the trajectory is standard parabola. But when it is in the air, due to its viscosity, the object is forced by resistance. It will have big effect on horizontal displacement. So it should be considered in calculating range and falling point.

According to fluid mechanics, air resistance can be classified into friction and pressure difference. To tennis, its surface is not fine, so friction exists. Also due to the fast motion and big Reynolds number, pressure difference resistance also exists. They become the air resistance f opposite to motion velocity, which is:

$$f = \frac{1}{2} c\rho A v^2$$

Resistance relates to velocity, whose quantity and direction changes every time. It leads to inconvenient calculations. Therefore, the motion should be disposed. With calculus thoughts, solve its value and direction of velocity, horizontal displacement and vertical displacement at some time, so as to get the mathematical formula. To the third item, adding up the spinning, the surface of tennis is not fine. While spinning, the surface stimulates the surrounding air's motion, then the air separation will be decreased while it progressing, which leads to pressure difference resistance decreasing. Meanwhile, the velocity of surrounding air changes, according to Bernoulli formula, in Matlab software, modeling the parabola trajectory in vertical direction and bias trajectory in horizontal distance, the falling point can be calculated.

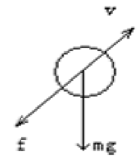

Fig. 3: Forces of ball in the air

Model hypothesis:

- Imagine the air stationary, with fine density. Its density is ρ = 1.2g/L
- Imagine the tennis is rigid, no transforming in the motion
- Imagine falling point without any area
- Imagine the gravity acceleration constant, 9.8m/s^2
- Imagine on effects of tennis net

Symbol rule:

- (x_0, y_0, z_0) ：coordinate of a point
- (p, q, 1) ：velocity direction of a point
- m: mass of tennis (200 g)
- k: equals to $\frac{1}{2}c\rho A$, to simplify the coefficient of air resistance
- t: Time
- v: Velocity
- c: Air friction coefficient
- A: Maximum cross sectional area
- ρ：Air density (1.2 g/L)
- h: Initial height
- g: Gravity acceleration (9.8m/s^2)

Build model for non-spinning ball: At first, set the three-dimensional coordinate as (x_0, y_0, z_0). The value of velocity is v_0 and direction is (p, q, 1). For calculating conveniently, rule such point as the origin, the horizontal projecting of velocity is x axis, y axis replacing z axis. Then the velocity direction is $\left(\sqrt{p^2+q^2},\ 0,\ 1\right)$. Then the motion trajectory is two-dimensional, in 0-xz plane (Fig. 3). Dispose the velocity. f is positive proportion to square of v, which is short as $f = kv^2$. The equation in the following exists:

X direction:

$$m\frac{\partial^2 x}{\partial t^2} = -k\left(\frac{\partial x}{\partial t}\right)^2 \tag{1}$$

Y direction:

$$m\frac{\partial^2 y}{\partial t^2} = -k\left(\frac{\partial y}{\partial t}\right)^2 - mg \qquad \left(\frac{\partial y}{\partial t}\right) \geq 0 \tag{2}$$

$$m\frac{\partial^2 y}{\partial t^2} = k\left(\frac{\partial y}{\partial t}\right)^2 - mg \qquad \left(\frac{\partial y}{\partial t}\right) \leq 0 \tag{3}$$

In that formula y, the air resistance should changes direction when the ball begins moving upwards or downwards, therefore the symbol before the air resistance should change:

Bring $v_y = \frac{\partial y}{\partial t}$ into formula (1):

$$m\frac{\partial v_x}{\partial t} = -kv_x^{\ 2} \tag{4}$$

Separate variance on formula (4) and conduct calculus on two sides. Apply initial marginal conditions $v_x\big|_{t=0} = v_{0x}$, the result of partial differential equation is:

$$v_x = \frac{mv_{0x}}{m + kv_{0x}t} \tag{5}$$

Go on using such method and applying initial condition x $\big|_{t=0} = 0$, then:

$$x = \frac{m}{k}\ln\left(\frac{kv_{0x}t}{m} + 1\right) \tag{6}$$

Bring $v_y = \frac{\partial y}{\partial t}$ into formula (2), then:

$$m\frac{\partial v_y}{\partial t} = -kv_y^{\ 2} - mg \tag{7}$$

In the same way and apply initial condition $v_y\big|_{t=0} = v_{0y}$, then:

$$v_y = \sqrt{\frac{mg}{k}}\tan\left[\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right) - t\sqrt{\frac{gk}{m}}\right] \tag{8}$$

Due to $v_y = \frac{\partial y}{\partial t}$, go on using such method and apply initial condition y $\big|_{t=0} = h$, then:

$$y = h - \frac{m}{k}\ln\left|\frac{\cos\left[\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right)\right]}{\cos\left[\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right) - t\sqrt{\frac{gk}{m}}\right]}\right| \tag{9}$$

When $v_y = 0$ in formula (8), formula (9) gets maximum y_{max}, which is:

$$t = t_1 = \sqrt{\frac{m}{gk}}\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right) \tag{10}$$

$$y_{\max} = h - \frac{m}{k}\ln\left\{\cos\left[\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right)\right]\right\} \tag{11}$$

At t_1, Eq. (9) gets maximum value, which is the peak. So its time area for t is $t \in [t_1, \infty]$.

Bring $v_y = \frac{\partial y}{\partial t}$ into formula (3), using separate variance method, adding initial condition $y\big|_{t=t_1} = 0$, then:

$$v_y = \frac{\sqrt{\frac{mg}{k}}\left(1 - e^{\sqrt{\frac{kg}{m}}(t-t_1)}\right)}{1 + e^{\sqrt{\frac{kg}{m}}(t-t_1)}} \tag{12}$$

Make such transformation:

$$a = e^{\sqrt{\frac{kg}{m}}(t-t_1)} \quad a > 0 \tag{13}$$

Using $v_y = \frac{\partial y}{\partial t}$, then formula (13) and Eq. (12) can become:

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial a}\frac{\partial a}{\partial t} = \frac{\partial y}{\partial a}\bullet a\sqrt{\frac{kg}{m}} = \frac{\sqrt{\frac{kg}{m}}(1-a)}{1+a} \tag{14}$$

Go on using such method and apply initial condition: $y\big|_{t=t_1} = y_{max}$, $a\big|_{t=t_1} = 1$. Conduct calculus on formula (14) and take variances back, getting:

$$y = y_{\max} + \frac{m}{k}\ln\left\{\frac{4e^{\sqrt{\frac{kg}{m}}(t-t_1)}}{\left[1 + e^{\sqrt{\frac{kg}{m}}(t-t_1)}\right]^2}\right\} \tag{15}$$

t area in both Eq.(12) and (15) is $t \in [t_1, \infty]$. When the ball falls on the ground, y = 0. Bring that into formula (15), the total time T can be calculated (select reasonable result):

$$T = t_1 + \sqrt{\frac{m}{kg}}\ln\left(\frac{2 - e^{-\frac{k}{m}y_{\max}} + 2\sqrt{1 - e^{-\frac{k}{m}y_{\max}}}}{e^{-\frac{k}{m}y_{\max}}}\right) \tag{16}$$

Bring T into formula (6) and get the range.

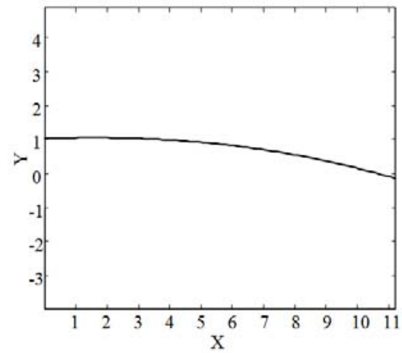

Fig. 4: Flight trajectory of ball

The coordinate of falling point (x , y , z) can be known.

Due to the transformation above, the coordinate should be turned back. Then, the coordinate of falling point is (x_t, y_t, z):

$$x_t = x_0 + x\cdot\frac{p_0}{\sqrt{p_0^{\ 2} + q_0^{\ 2}}}, \quad y_t = y_0 + x\cdot\frac{q_0}{\sqrt{p_0^{\ 2} + q_0^{\ 2}}}, \quad z = 0$$

Next, if specifically quantify the initial data, the coordinate of falling point should also be quantified specifically. In the same way, with Matlab, we can describe the flight trajectory of tennis in the air. In the following, bring the value known h, m, k = 1/2cρA = 3.982×10^{-3}, g = 9.8m/s^2, v_{0x}, v_{0y} into formula (6) and (8), get: (x, y).

It is the flight trajectory in $t \in [0, t_1]$. (The coordinate is still that after transformation, so the image is two-dimensional. The original one is three-dimensional, so the ordination should be transformed back).

Bring into formula (10), get t_1

Bring into formula (11), get y_{max}

Bring into formula (6) (15), get (x, y)

That is the flight trajectory of tennis in $t \in [t_1, T]$.

Combine the images in two sections, the result shown as Fig. 4:

Model for spinning ball: In the hitting process, due to horizontal displacement, usually conduct spinning. Then, Magnus force is increased, which is vertical to velocity direction and angle velocity. The force direction changes, the trajectory changes. The path is not just in a plane, but a spatial curve. The Magnus force is positive proportion to velocity. For calculating conveniently, it can be written as $F_M = G_v$. G relates to air density, tennis diameter and spinning velocity. its value is unknown. So it needs another coordinate on the trajectory to solve.

Imagine spinning axis of the tennis is parallel to z axis, in the studying process, the air resistance, velocity and Magnus are in the same plane. The force situation is shown as Fig. 5:

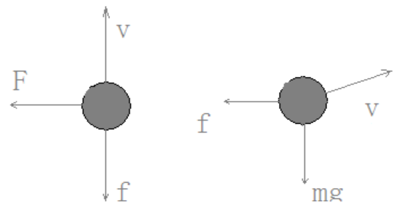

Fig. 5: Forces of tennis

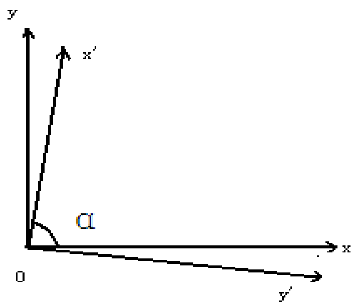

Fig. 6: Coordinate transformation

To calculate conveniently, the axis should be transformed. Consider horizontal shadowing direction of velocity as x axis, displacing direction as y axis and z is unchanged. The effect is shown as Fig. 6 (z axis isn't given).
Some relations exist:

$$x' = x\cos\alpha + y\sin\alpha$$
$$y' = x\sin\alpha - y\cos\alpha$$

$$\cos\alpha = \frac{2}{\sqrt{260}}$$

$$\sin\alpha = \frac{16}{\sqrt{2^2+16^2}} = \frac{16}{\sqrt{260}}$$

Then the tennis trajectory can be described.

Vertical direction:

$$m\frac{d^2z}{dt^2} = -mg \tag{17}$$

Horizontal section:

$$m\frac{dv}{dt} = -kv^2 \tag{18}$$

Horizontal normal direction:

$$m\frac{v^2}{\rho} = Gv \tag{19}$$

Calculus on formula (18) and bring in initial condition $t_0 = 0$, $v\big|_{t=0} = v_0$, then

$$v = \frac{mv_0}{m+kv_0t} \tag{20}$$

Due to $ds = vdt$:

$$s = \int_0^t \frac{mv_0}{m+kv_0t}dt = \frac{m}{k}\ln\frac{m+kv_0t}{m} \tag{21}$$

Bring $\rho = \frac{ds}{d\theta} = \frac{ds}{dt}\frac{dt}{d\theta} = v\frac{dt}{d\theta}$ into formula (19):

$$\frac{d\theta}{dt} = \frac{G}{m} \tag{22}$$

So:

$$\theta = \int_0^\theta d\theta = \int_0^t \frac{G}{m}dt = \frac{G}{m}t \tag{23}$$

Therefore,

$$v_{x'} = v\cos\theta = \frac{mv_o}{m+kv_0t}\cos\frac{G}{m}t \tag{24}$$

$$v_{y'} = v\sin\theta = \frac{mv_0}{m+kv_0t}\sin\frac{G}{m}t \tag{25}$$

Considering small kv_0t comparing to m, it can be ignored. It almost:

$$v_{x'} = v_0\cos\frac{G}{m}t \tag{26}$$

$$v_{y'} = v_0\sin\frac{G}{m}t \tag{27}$$

Then:

$$x' = \frac{mv_0}{G}\sin\frac{G}{m}t \tag{28}$$

$$y' = \frac{mv_0}{G}\left(1-\cos\frac{Gt}{m}\right) \tag{29}$$

Due to formula (27) (28), eliminate t, then:

$$y'^2 - \frac{2mv_0}{G}y' + x'^2 = 0 \tag{30}$$

According to a point (x_1, y_1, z_1) on the trajectory, bring into formula (29) (after coordinate transformation), G can be solved. Bring that result into formula (27), solve the flight time t. Draw the flight trajectory in t area in Matlab.

CONCLUSION AND APPLICANT ANALYSIS

Conclusion: With calculus principles, we describe the velocity changes and trajectory in the flight, avoid the errors from the stable velocity and other hypotheses. Besides, we use the coordinate transformation method to reduce the complexity of calculations. And, with that method, transform three-dimensional image into two-dimensional one, enhancing the straightness of path to some degree. But increase the steps for transforming it back. It needs more rigid logics. Our results have reached the standards, comparing to other papers, it is more specific and accurate. Matlab for calculation and figuring makes it more direct.

Applicant analysis:

- According to air dynamics and fluid mechanics, build air flight motion equation and conduct numerical simulation, prove its correct in object.
- It reveals the flight principles in a deeper way. The accurate data is helpful for future development and provides some ideas for new tactics.
- It is the basis for motion robot dynamics.

REFERENCES

Haung, D.M., 2008. Effect of hawkeye on tennis competition. Bull. Sport Sci. Technol., 16(8).

Min, Y.L., 2009. Two air resistance model the flight path of the projectile research. Manuf. Equipment Technol., Vol. 12.

Money, Z.Y., 2007. On the study on the effects of Hawk-eye on development of tennis. J. Guangzhou Inst., Physical Educ., Vol. 27.

Sun, C., 2003. Rotating ball with the non-rotating ball movement mechanics principles. Xiaogan Univ., Bull., Vol. 5.

Wang, G., 2006. Hawkeye tennis match. North Univ., China, 23(5): 32-33.

Wu, Z., 2007. Such as Aerodynamics. Tsinghua Univ., Press, Beijing, pp: 65-71.

Zhang, X., 2003. Modern Tennis user Manual. People's Sports Publishing House, Beijing, 47: 166.