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Research Article

Diagonal Loading of Robust General-Rank Beamformer for Direction of Arrival Mismatch

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Abstract: This study presents a technique which utilizes the movement of the peak of the main beam towards the presumed signal direction with negative diagonal loading for robust general-rank beamformer. The main beam symmetry along presumed signal direction is improved by this movement. When desired signal is contained in the data snapshots, the conventional beamformers face the problem of performance degradation even if there is a small mismatch between the presumed and the actual signal direction. Diagonal loading is a popular technique to mitigate this problem. There is no definite criterion to find diagonal loading level. A new diagonal loading method has been proposed in the literature which utilizes the movement of the peak of main beam towards the presumed signal direction with positive diagonal loading. The proposed technique works iteratively for the selection of negative diagonal loading level to move the main beam at a position to get the beam symmetry at desired level and hence the desired robustness. The mismatched signal will not be cancelled as long as it is within the half of the width of the main beam. But there is the tradeoff between this robustness and interference cancelling capability.

Keywords: Adaptive beamforming, Minimum Variance Distortionless Response (MVDR) beamformer, robust adaptive beamforming

INTRODUCTION

Adaptive beamforming is a popular spatial filtering technique which utilizes antenna arrays for signal estimation from desired direction (Zaman *et al*., 2012a, b and c) and placing nulls in the direction of undesired signals (Khan *et al*., 2011). The weights of the beamformer are optimized according to some specific criteria, such as minimum variance, maximum entropy and maximum Signal to Interference-plus-Noise Ratio (SINR). Beamforming finds its applications in the fields of radar, sonar, medical imaging and wireless communications (Synnevag *et al*., 2009; Sharma *et al*., 2008; Roy *et al*., 2009). Minimum Variance Distortionless Response (MVDR), Sample Matrix Inversion (SMI), Linearly Constrained Minimum Variance (LCMV) and Generalized Sidelobe Canceller (GSC) are the popular adaptive beamformers. When desired signal is present in data snapshots and there is a mismatch between the presumed and actual signal direction, the desired signal is taken as interference. In such situations, the desired signal is cancelled and the performance of the beamformer degrades severely. Efforts are in progress to develop robust algorithms for such mismatches.

Diagonal loading (Carlson, 1988) is a popular technique robust against direction of arrival mismatch but has no suitable way to find the diagonal loading factor. Another attractive approach is robust adaptive beamforming using worst case performance optimization (Vorobyov *et al*., 2003). But its performance is quite close to the simple algorithm known as diagonal Loading of the Sample Matrix Inversion (LSMI) algorithm. These beamformers utilize positive diagonal loading and their performance depends upon the proper selection of diagonal loading factor. A new diagonal loading technique appears in (Wang and Wu, 2011) which finds diagonal loading level by utilizing the fact that the peak of the main beam moves towards the presumed direction by increasing the diagonal loading level. This technique is effective if the mismatched signal appears within the half of the width of the main beam. There is a tradeoff between robustness against signal look direction error (controlled by the movement of the peak of the main beam) and the interference suppression capability. Robust general-rank beamformer (Shahbazpanahi *et al*., 2003) utilizes negative and positive diagonal loading for presumed and received signal covariance matrices respectively. Negative diagonal loading is meant for the robustness against direction of arrival mismatch. If this loading level exceeds beyond a certain limit, the presumed signal covariance matrix no longer remains positive semi-definite and becomes useless.

This study utilizes the movement of the peak of the main beam towards the presumed signal direction by

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negative diagonal loading and the expression for maximum value of the diagonal loading level, that maintains the positive definite property of the negative diagonal loaded matrix, is given. Diagonal loading level is increased from minimum towards maximum iteratively to achieve the beam symmetry up to the desired level for robust general-rank beamformer. The technique given in (Chen and Vaidyanathan, 2007) utilize angular region of mismatch for the robustness of the beamformer. Same diagonal loading level is applied to the signals regardless of small or large mismatch in that region i.e., same compromise for null depth regardless of small or large DOA mismatch. On the other hand this technique utilizes beam symmetry to select diagonal loading level. If mismatch is small, small value of diagonal loading level will give desired beam symmetry and large diagonal loading level will be required to achieve desired beam symmetry for large mismatch i.e., compromise on null depth is different for different situations.

LITERATURE REVIEW

Mathematical model: Consider a uniform linear array of M antenna elements with inter-element spacing λ/2, where λ is the wavelength of incoming narrow band signal of interest. Let the array receives signals from K far-field sources. The output of the i^{th} antenna element i.e.,$\{y_i(n)\}_{i=1}^{i=M}$ is given by:

$$y_i(n) = \sum_{l=1}^{K} e^{j(i-1)\pi sin\theta_l} s_l(n) + v_i(n)$$

In the above expression, $s_l(n)$ represents the signal amplitude received from l^{th} source and $v_i(n)$ is the additive white noise added at the output of i^{th} sensor. The output vector $\mathbf{y}(n)$ contains the individual outputs of all the elements and is given as:

$$\boldsymbol{y}(n) = [y_1(n)y_2(n) \dots y_M(n)]^T \quad (1)$$

The source signal vector $\mathbf{s}$(n) representing signal amplitudes from K sources and is given as:

$$\boldsymbol{s}(n) = [s_1(n)s_2(n) \dots . s_K(n)]^T$$

These sources are considered to be uncorrelated to each other and their correlation matrix $\mathbf{R_s}$ is given by:

$$\boldsymbol{R_s} = \begin{bmatrix} \sigma_{s1}^2 & 0 \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{sk}^2 \end{bmatrix}$$

In the above expression, $\{\sigma_{sl}^2\}_{l=1}^{l=k}$ represents the power of the signal received from l^{th} source. A set of steering vectors $\boldsymbol{a}(\theta_l): l = 1, \dots K$can be defined as:

$$\boldsymbol{a}(\theta_l) = [1 e^{j\phi l} e^{j2\phi l} \dots e^{j(M-1)\phi l}]^T$$

where, $\emptyset_l = \pi sin\theta_l$

These vectors can be placed in a single matrix $\mathbf{A}$ given as:

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_2) \dots \boldsymbol{a}(\theta_k)]$$

$\mathbf{y}$(n) Can be expressed as:

$$\mathbf{y}\,(n) = \mathbf{As}\,(n) + \mathbf{v}\,(n)$$

where, $\mathbf{v}$(n) is the noise vector having uncorrelated components and hence its correlation matrix is given as:

$$\boldsymbol{R_v} = E[\boldsymbol{v}(n)\boldsymbol{v}^H(n)] = \sigma_v^2 I_M$$

The correlation matrix $\mathbf{R_y}$ of received signal is given by:

$$\boldsymbol{R_y} = E[\boldsymbol{y}(n)\boldsymbol{y}^H(n)] = \boldsymbol{A}\boldsymbol{R_s}\boldsymbol{A^H} + \sigma_V^2 \boldsymbol{I_M} \quad (2)$$

MVDR Beamformer: This beamformer utilizes second order statistics of the array output. It minimizes the variance (average output power) of the beamformer and maintains the distortionless response in the desired signal direction. Let $\boldsymbol{a}(\theta_s)$ be the steering vector in the desired signal direction and $\mathbf{R}$ is the received signal covariance matrix, the optimization problem and its solution in terms of beamformer weight vector $\mathbf{w}_{Mv}$ are given in (3) and (4) respectively (Liu *et al*., 2003):

$$\min_{\mathrm{w}} \boldsymbol{w^H R w} \text{ subject to } \boldsymbol{w^H a}(\theta_s) = 1 \quad (3)$$

$$\boldsymbol{w}_{Mv} = \frac{\boldsymbol{R^{-1}a}(\theta_s)}{\boldsymbol{a}^H(\theta_s)\boldsymbol{R^{-1}a}(\theta_s)} \quad (4)$$

SMI Beamformer: This beamformer minimizes Signal to Interference-plus-Noise Ratio (SINR) of the array output. Let $\mathbf{R_s}$ be the signal covariance matrix, the optimization problem for this beamformer is stated as:

$$\min_{\mathbf{w}} \mathbf{w^H R w} \text{ subject to } \mathbf{w^H R_s w} = 1 \quad (5)$$

The solution to this optimization problem as given in (Shahbazpanahi *et al*., 2003), is $\mathbf{w}_{opt}$ = P{$\mathbf{R}^{-1}\mathbf{R_s}$}, where P{.} is the Eigen vector corresponding to the maximal Eigen value and $\mathbf{w}_{opt}$ is the optimized weight vector.

ROBUST ADAPTIVE BEAMFORMERS

The performance of traditional beamformers degrades severely due to errors in the signal look direction. Robust algorithms have been developed to mitigate this problem. In this section, two robust beamforming algorithms i.e., Loaded SMI and General-Rank beamformers are being discussed.

Loaded SMI beamformer: The key idea for this beamformer is to add some quadratic penalty to regularize the solution for optimum weight vector. The optimization problem for this beamformer is defined as:

$$\min_{\mathrm{w}} \boldsymbol{w}^H(\boldsymbol{R}+\gamma\boldsymbol{I})\boldsymbol{w}\ subject\ to\ \boldsymbol{w}^H\boldsymbol{R_s}\boldsymbol{w}=1$$

The robust weight vector for this beamformer comes out to be as:

$$\boldsymbol{w}_{LSMI}=P\{(\boldsymbol{R}+\gamma\boldsymbol{I})^{-1}\boldsymbol{R_s}$$

In the above expression, $\boldsymbol{R}+\gamma\boldsymbol{I}$ and $\boldsymbol{I}$ are the diagonally loaded sample covariance matrix and identity matrix respectively. The variance of artificial noise is increased by an amount γ in this method. This approach puts more effort to suppress white noise rather than interference. Due to above mentioned modification; LSMI improves the performance of Sample Matrix Inversion (SMI) method in the presence of an arbitrary steering vector mismatch. But this improvement is not so significant in case of look direction error vector with large norm (Song *et al*., 2006). Moreover, another serious shortcoming of this approach is that there is no reliable way to choose proper value for the loading factor γ, as the optimal choice of γ is dependent on unknown parameters of signal and interference (Shahbazpanahi *et al*., 2003). However recommended loading factor is $\sigma_n^2 \leq \sigma_L^2 < 10\sigma_n^2$ where σ_L^2 is the diagonal loading level and σ_n^2 is the noise power (Jeyali and Sukanesh, 2011) so, the minimum loading level must be equal to noise power σ_n^2 i.e., minimal Eigen value of $\mathbf{R}$.

Robust general-rank beamformer: This beamformer assumes that the mismatch in desired signal direction of arrival causes an error matrix Δ in $\mathbf{R_s}$. Let Δ be bounded by some known positive constant ε i.e., $||\Delta|| \leq \varepsilon$. Where $||.||$ denotes Frobenius norm of a matrix. In (Shahbazpanahi *et al*., 2003), the SINR maximization problem has been modified for the robustness of the beamformer against DOA mismatch as given below:

$$\min_{\mathrm{w}} \boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}\ \text{subject to}\ \boldsymbol{w}^H(\boldsymbol{R_s}+\Delta)\boldsymbol{w}\geq 1 \quad \text{for all}\ ||\Delta||\leq\varepsilon \tag{6}$$

For the worst case performance, Δ can be found by solving the following optimization problem.

$$\min_{\Delta} \boldsymbol{w}^H(\boldsymbol{R_s}+\Delta)\boldsymbol{w}\ \text{ subject to } ||\Delta||\leq\varepsilon$$

For this problem Δ, as given in (Shahbazpanahi *et al*., 2003) comes out to be:

$$\Delta=-\varepsilon\frac{\boldsymbol{w}\boldsymbol{w}^H}{\|\boldsymbol{w}\|^2} \tag{7}$$

By putting the value of Δ, the optimization problem (5) becomes as:

$$\min_{\mathrm{w}} \boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}\ \text{subject to}\ \boldsymbol{w}^H(\mathrm{R_s}-\varepsilon\boldsymbol{I})\boldsymbol{w}=1$$

The optimum weight vector for the robust beamformer comes out to be:

$$\boldsymbol{w}_{rob}=P\{\boldsymbol{R}^{-1}(\boldsymbol{R_s}-\varepsilon\boldsymbol{I})\} \tag{8}$$

To overcome other array imperfections, another similar mismatch matrix Δ_1 is considered in R, with the condition $||\Delta_1|| \leq \gamma$. The robust weight vector, as given in (Shahbazpanahi *et al*., 2003) comes out to be:

$$\boldsymbol{w}_{rob.s}=P\{(\boldsymbol{R}+\gamma\boldsymbol{I})^{-1}(\boldsymbol{R}_s-\varepsilon\boldsymbol{I})\} \tag{9}$$

If we put $\varepsilon=0$, $\boldsymbol{w}_{rob.s}=\boldsymbol{w}_{LSMI}$. This shows that LSMI beamformer is the special case of General-Rank beamformer.

PROPOSED DIAGONAL LOADING ALGORITHM

In this section, we will discuss the flow chart with parameters and steps of the proposed algorithm for optimum diagonal loading level. The proposed algorithm presents iterative approach to find diagonal loading level from the range of ε and γ to get desired beam symmetry. Since LSMI beamformer is the special case of General-Rank beamformer, therefore the proposed algorithm is developed for General-Rank beamformer. The parameters used in the proposed algorithm are discussed below.

In case of direction of arrival mismatch if the diagonal loading level is zero, the General-Rank robust beamformer becomes SMI beamformer and the desired signal is cancelled. As the diagonal loading level is increased, the peak of the main beam moves gradually towards the presumed signal direction and the desired signal cancellation along with interference suppression capability will be reduced (Wang and Wu, 2011). If the peak of the main beam is moved at the presumed signal direction and the direction of arrival mismatch is within the half of the width of the main beam, the desired signal will not be suppressed heavily. A sufficient condition to guarantee the peak of the main beam at the presumed signal direction is the exact left-right symmetry at the presumed signal direction. For a uniform linear array of M antenna elements with inter-element spacing d and wavelength of incoming narrow band signal equal to λ, the approximate width of the main beam is given as $\theta_{md} = \frac{50.7\lambda}{Md}$ (Wang and Wu, 2011). When the ideal diagonal loading shifts the peak of the main beam at the presumed signal direction, the following condition will be satisfied:

$$10log_{10}\left|w_{rob,s}^{H}\boldsymbol{a}(\theta_s + \delta\theta)\right|^2 - 10log_{10}\left|w_{rob,s}^{H}\boldsymbol{a}(\theta_s - \delta\theta)\right|^2 = 0 \quad (10)$$

where, $\delta\theta \leq 0.5\theta_{md}$. Under this condition beam is ideally symmetric along the presumed signal direction. But this will require diagonal loading level very high (ideally infinity) with very small interference suppression capability. The above expression can be modified as given below (Wang and Wu, 2011):

$$\left|10log_{10}\left|\boldsymbol{w}_{rob,s}^{H}\,\boldsymbol{a}(\theta_s + \delta\theta)\right|^2 - 10log_{10}\left|w_{rob,s}^{H}\boldsymbol{a}(\theta_s - \delta\theta 2 = \mu\right. \quad (11)$$

where, μ is the trade off parameter which we call symmetry of the main beam at a distance $\delta\theta$ from presumed signal direction. This parameter controls robustness and interference suppression capability.

An expression for negative diagonal loading factor ε for robust general-rank beamformer is given below (Chen and Vaidyanathan, 2007):

$$\varepsilon = \max_{\theta_s - \delta\theta \leq \theta \leq \theta_s + \delta\theta} ||\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta) - \boldsymbol{a}(\theta_s)\boldsymbol{a}^H(\theta_s)|| \quad (12)$$

where ||.|| represents Frobenius norm of a matrix and the desired signal mismatch region is represented by $\theta_s - \delta\theta \leq \theta \leq \theta_s + \delta\theta$. This expression limits the value of ε up to the level to guarantee the negative diagonally loaded signal covariance matrix $(\boldsymbol{R_s} - \varepsilon\boldsymbol{I})$ to remain positive definite. The other parameters used in the flowchart are given as:

$$\varepsilon_{min} = \max_{\theta_s - \delta\theta \leq \theta \leq \theta_s + \delta\theta} ||\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta) - \boldsymbol{a}(\theta_s)\boldsymbol{a}^H(\theta_s)||$$

$$\varepsilon_{max} = \max_{\theta_s - \delta\theta - n\Delta\theta \leq \theta \leq \theta_s + \delta\theta + n\Delta\theta} ||\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta) - \boldsymbol{a}(\theta_s)\boldsymbol{a}^H(\theta_s)||$$

It means ε_{max} is sought by expanding the region around θ_s. This expand and seek process continues until ε_{max} is achieved. It is the upper limit of negative diagonal loading factor:

$\delta\gamma$ = The step size for the positive diagonal loading factor γ

$\delta\varepsilon$ = The step size for the negative diagonal loading factor ε

$m_1 = 10\log_{10}\left|\mathbf{w}^H a(\theta_s + \delta\theta)\right|^2$ = output power of beamformer at $\theta_s + \delta\theta$ for certain values of ε and γ which give weight vector **w** during an iteration

$m_2 = 10log_{10}|\boldsymbol{w}^H\boldsymbol{a}(\theta_s - \delta\theta)|^2$ = output power of beamformer at $\theta_s + \delta\theta$ for certain values of ϵ and γ which give weight vector **w** during an iteration

$\boldsymbol{w} = P\{(\boldsymbol{R} + \gamma\boldsymbol{I})^{-1}(\boldsymbol{R_s} - \varepsilon\boldsymbol{I})$ = weight vector for certain values of ε and γ during an iteration

Step 1: Initialization: In this step minimum and maximum values of diagonal loading level are initialized as shown in flow chart.

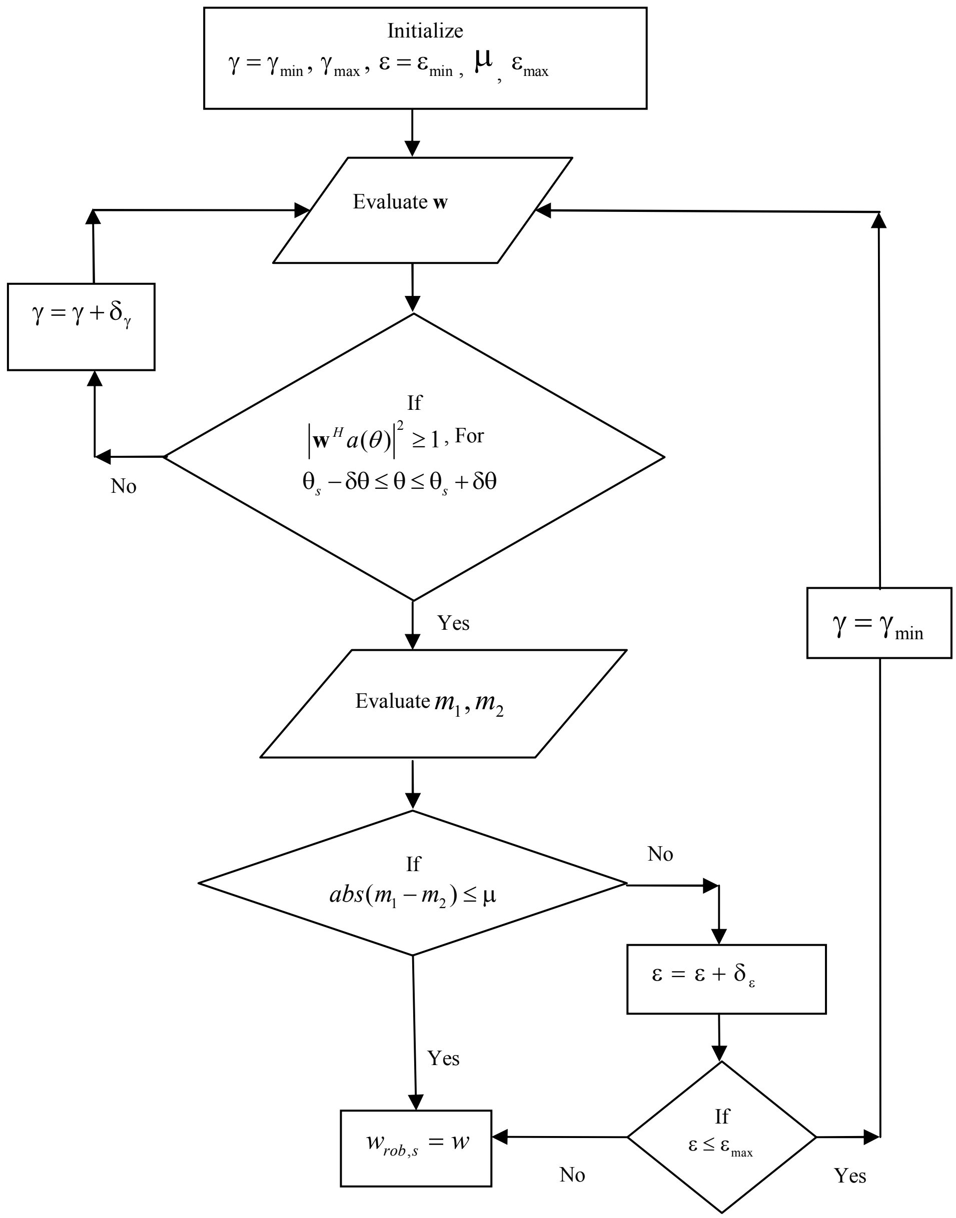
Step 2: Evaluation of **w**: Weight vector **w** with initial values of γ and ε is evaluated.

Step 3: Comparison: Constraint given in (6) is checked using weight vector.

Step 4: Evaluation of m_1 and m_2: If constraint in (6) is satisfied, m_1 and m_2 are evaluated otherwise positive diagonal loading level is increased gradually with step size $\delta\gamma$ and go to step 2 until constraint in (6) is satisfied.
It must be noted that for a certain value of ε there is a particular minimum value of γ which satisfies constraint (6). The algorithm does not converge if γ is below that particular value and converges for all higher values of γ.

Step 5: If desired beam symmetry is achieved, Stop, otherwise increase negative diagonal loading level gradually with step size $\delta\varepsilon$ and go to step 2. Else stop with best available weight vector.

Weight vector for only those minimum values of γ and ε is selected as $\mathbf{w}_{rob,s}$ which gives best symmetry in the half power beam width in the signal mismatch region. Since this weight vector corresponds to minimum values of γ and ε, so gives better null depth in addition to better performance in the signal mismatch region.



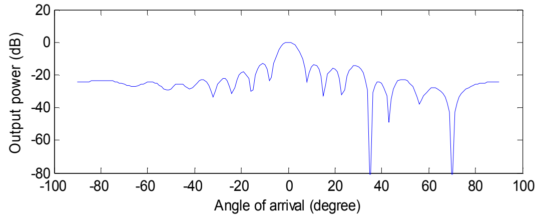
Flow chart for the proposed algorithm

SIMULATION RESULTS FOR ROBUST ADAPTIVE BEAMFORMERS

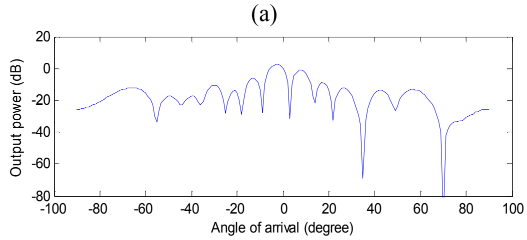
A uniform linear array of 15 antenna elements has been used with inter element spacing $\lambda/2$. One desired signal with presumed direction along 0º and two interferences at 35º and 70º are used. The SNR and INR are 10 dB and 30 dB respectively. Simulations are carried out in MATLAB and all the results are averaged over 500 snapshots.

Performance of MVDR beamformer: In this case, the Performance of MVDR beamformer is discussed for both with and without DOA mismatch. Without DOA mismatch, the output power of the beamformer at 3° is -1.86 dB. While for the DOA mismatch case, the performance of beamformer degraded i.e., the output power of the beamformer at 3° is -31.65 dB. The comparison of both situations can be observed from Fig. 1a and b.

Performance of robust general rank beamformer: In this case, we evaluate the performance of robust general rank beamformer. The actual DOA of the signal is taken along 3°. By using expression (12),



(a)



(b)

Fig. 1: Performance of MVDR beamformer: (a) without mismatch, (b) with mismatch

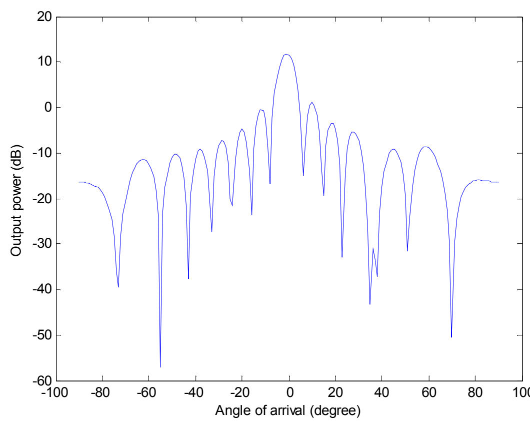


Fig. 2: Performance of robust general-rank beamformer for $\boldsymbol{\varepsilon}$ =4.54, $\boldsymbol{\gamma}$ = 140 and $\boldsymbol{\mu}$ =3.24 dB at ±3°

we get $\varepsilon = 4.54$ and for $\varepsilon = 4.54$, $\mu = 7.33$ dB at ±3°, and the proposed algorithm converges for γ = 39. So clearly, one can observe the improvement by comparing Fig. 1b and 2.

Beam symmetry with ε and γ: In this sub-section, we discuss the beam symmetry which is defined as:

$$|10log_{10}(m_1) - 10log_{10}(m_2)|$$

We use a uniform linear array of 15 elements to observe beam symmetry by varying ε and γ. In Table 1 beam symmetry is elaborated for increasing values of ε. It is clear from Table 1, that as ε increases, beam symmetry improves and the desired signal cancellation in the mismatch region decreases.

In ideal situation the beam symmetry is zero. In Table 2, we discussed the beam symmetry for increasing values of γ. The beam symmetry improves

Table 1: Beam symmetry with increasing values of ϵ

ϵ	γ	Symmetry at ±3°	Symmetry at ±2°	Symmetry at =1°
0	0	36.71dB	11.07 dB	4.66 dB
2	289	2.2891 dB	1.4068 dB	0.6730 dB
4	289	1.9028 dB	1.1720 dB	0.5613 dB
6	289	1.5147 dB	dB	0.4478 dB

Table 2: Beam symmetry with increasing values of γ

ϵ	γ	Symmetry at±3°	Symmetry at ±2°	Symmetry at ±1°
4	30	8.9487 dB	5.1258 dB	2.3795 dB
4	60	6.1032 dB	3.6288 dB	1.7102 dB
4	90	4.6947 dB	2.8324 dB	1.3434 dB
4	120	3.8405 dB	2.3349 dB	1.1114 dB
4	150	3.2509 dB	1.9855 dB	0.9470 dB
4	180	2.8158 dB	1.7248 dB	0.8238 dB
4	210	2.4933 dB	1.5307 dB	0.7319 dB
4	240	2.2291 dB	1.3705 dB	0.6558 dB
4	270	2.0092 dB	1.2368 dB	0.5921 dB

Table 3: Comparison of Null depth with increasing values of ϵ and γ

γ	ϵ	Signal (dB)	Null_1 (dB)	Null_2 (dB)
5	1.0000	-12.04	-69.93	-74.32
5	2.0000	-10.72	-68.60	-72.90
5	3.0000	-8.85	-66.49	-70.01
25	4.0000	1.3	-55.01	-59.47
50	4.0000	4.15	-50.92	-56.27
50	4.5000	4.5	-50.2	-55.87

for the increasing values of γ. From both tables, it is quite clear that the beam symmetry is much improved for small increasing values of ε as compare to γ.

Null depth and diagonal loading level: One can observe from Table 3 the tradeoff between signal strength and null depth. As the value of ε or γ increase the null depth decreases while the strength of signal improves.

CONCLUSION AND FUTURE WORK

In this study, we discussed diagonal loading for robust general rank beamformer. Diagonal loading is selected iteratively on the basis of desired beam symmetry at an angle in the signal mismatch region. In future, we will discuss it for three dimensional arrays.

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