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Research Article

Research on Multi-Dimension Cooperative Game of Enterprises

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Abstract: Because of enterprise's own resources can not meet the needs of the innovative activities, more and more enterprises looking forward to cooperative innovation with others. In the cooperation process, for multidimensional, there is a game behavior. Due to the fact that incomplete information exists in the process of cooperation, most multidimensional game belongs to incomplete information game. According to the definition of fuzzy stable set and solution, in this study, the fuzzy mathematics is used in multidimensional game and provides league members with incomplete information state problem decision support. At the same time, provides new ideas to the analysis of dynamic gains among cooperation process.

Keywords: Cooperative innovation, fuzzy stable set, multi-dimension, n-person game

INTRODUCTION

Many scholars do not strange to technology innovation concept. As early as in 1912, famous scholar Schumpeter (Schumpeter, 1912) published his great book "the theory of economic development", in this book, Mr. Schumpeter pointed out that technology innovation is refers to bring a kind of new combination about production elements which never used of in production system. This concept covers the following five cases:

- The introduction of a new good-that is one with which consumers are not yet familiar -or of a new quality of a good.
- The introduction of a new method of production, that is one not yet tested by experience in the branch of manufacture concerned, which need by no means be founded upon discovery scientifically new and can also exist in a new way of handling a commodity commercially.
- The opening of a new market, that is, a market into which the particular branch of manufacture of the country in question has not previously entered, whether or not this market has existed before.
- The conquest of a new source of supply of raw materials or half-manufactured goods, again irrespective of whether this source already exist or whether it has first to be created.
- The carrying out of the new organization of any industry, like the creation of a monopoly position or the breaking up of a monopoly position. In the knowledge economy era, the essence of technologies is knowledge, so many scholars think that technology innovation is equal to knowledge creation and the process of knowledge creation include dynamic, complexity, features such as socialization. Layton (1974) pointed that technical innovation in essence is a kind of new skills and knowledge production and create, produce technology behavior one must contain knowledge. Visible, knowledge is the core of the technology innovation elements.

Because of a single enterprise hard to have all the resources needed for innovation, more and more enterprise want to get cooperation with others, as cooperative innovation. Haussler pointed out those Hausler *et al*. (1994), due to the high cost of innovation and uncertainty, cooperative enterprises do not have independent innovation management ability and not exclusive innovation achievement, so choose cooperation development. Sakabara considered that motivation (Sakakibara, 1997), technical complexity, risk and capital acquisition etc. factors and cooperation innovation activity had close contact. Some scholars had analyzed the cooperation innovation motivation, general from the several aspects of research: reduce transaction costs and improve cooperation members of the complementary skills (Kogut, 1988). Due to the uncertainty of innovation, enterprises need to find companied to share cooperation R&D costs and share cooperation R&D risk (Das and Teng, 2000), Shorten the innovation cycle (Pisano, 1990), improve the innovation efficiency (Hamel, 1991). In addition, still

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include monitoring market, monitoring and control process of organizational learning etc (Hamel, 1991). In fact, according to different types of cooperation, the goal of the parties is different, but for cooperation innovation, the main target usually is to reduce the innovation cost and expand the market size.

The modern game theory originated in the early 20th century, it researched in behavior of main body (decision makers) such as interaction or the balance of decision. Cooperative game and non-cooperative game are two important branches of the game theory. The most important difference is, when game subject interacting, they can reach a strong binding agreement, if any, called the cooperative game (stressed group rationality, efficiency and fairness); If not, called the non-cooperative game (emphasize individual rationality and individual optimal decision). In reality, the vast majority of games belong to non-cooperative game, the participants that emphasizes individual optimal decision in order to get the best interests of the individual, but the optimal decision may be efficient, also may be inefficient. In this study, according to the practice of enterprise cooperative innovation, we are more interested in cooperative game.

N-PERSON MULTI-DIMENSION GAME

Characteristics of multi-dimensional game: The so-called multi-dimension game, generally means participants in so many aspects (or so many areas) at the same time in the game and there are certain relation or inner link among each aspects (or area) of the game. For example, two (or more) enterprises (players) in cooperation in the process of innovation, they game in the capital investment, knowledge resources investment, profit distribution and so on at the same time. Its characteristics are:

- In any one of the stage multidimensional game player (game participants) i choose a m ($m \geq 2$) strategy vector ($S_{i1}, S_{i2}, \ldots, S_{im}$), i = 1, 2, …, n i_m means game dimension, also is the players decision vector dimension (decision-maker to each game field to the decision). n means numbers of players(game participants), S_{ij} means ith player's decision in jth game dimension S_{ij} and S_{ik} ($j \neq k$) each means player i's decision in jth and kth dimension, and there are may be certain relevance between decision vector S_{ij} and decision vector S_{ik}, and ($j \neq k$).
- In any stage if the game, the total payoff function of player i is not only with its own m decision vector correlation ($S_{i1}, S_{i2}, \ldots, S_{im}$), related, but also associated with other player's m decision vector, namely $U_i = \{(S_{11}, S_{12}, \ldots, S_{1m}), \ldots, (S_{i1}, S_{i2}, \ldots, S_{im}), \ldots, (S_{n1}, S_{n2}, \ldots, S_{nm})\}$
- If and only if m=1,multidimensional game degradation for one dimensional game
- We need to pay attention to is: although, n-person game in many fields, but from a field point of view, may be the player's final optimal strategy vector ($S_{i1}, S_{i2}, \ldots, S_{im}$) is not the best. In other words, S_{ij} is not always optimal strategy of player i in field j.

Similar to one dimensional game, multidimensional game also can be divided into four types, it is respectively: complete information static multi-dimension game, complete information dynamic multidimensional game, incomplete information static multi-dimension game and incomplete information dynamic multi-dimension game. Among them, the incomplete information dynamic game is more suitable to the actual situation of the game parties, so this study expounds only in this.

Incomplete information dynamic n-person multi-dimension cooperative game: Hypothesis: $\tilde{v}$ represent n-person multidimensional cooperative game, then n-person cooperative game targets set is OB_i (i = 1, 2, …, m), for this target set, only in one field all $\tilde{v}_i = [\tilde{v}_{i'}, \tilde{v}_{i''}]$ game is $H(\tilde{v}_{i'}) = [H(\tilde{v}_{i'}), H(\tilde{v}_{i''})]$, in all dimension, the n-person game goal set is $H(\tilde{v}) = [H(\tilde{v}_1), H(\tilde{v}_2), \ldots, H(\tilde{v}_m)]$.

Note that, if $\tilde{\boldsymbol{v}}_i = [\tilde{\boldsymbol{v}}_{i'}, \tilde{\boldsymbol{v}}_{i''}]$ meet the ultra additives, the game can be called true game, otherwise called the false game (Dhingra and Rao, 1995).

If for any dimension, the value of game $\tilde{\boldsymbol{v}}_i = [\tilde{\boldsymbol{v}}_{i'}, \tilde{\boldsymbol{v}}_{i''}]$ for OB_i (i = 1, 2, …, m) is 0 or 1 and for any $M \subset Q$, $M, Q \in 2^n$ there is, $\tilde{\boldsymbol{v}}_i(M) \leq \tilde{\boldsymbol{v}}_i(Q) = 1\tilde{\boldsymbol{v}}_i$ ({C}), among them $C \in Q$, We can say the game is simple game for multidimensional target (Branzei *et al*., 2003)

Because the research scope of this study is limited in dynamic cooperative game (which is more suitable to cooperation between enterprise and the actual situation of innovation), so the formation of the alliance has relevance with time, that is, in a certain period, the formation of the alliance is randomness. For example: at the moment t_0, maybe alliance is not formed, but at moment t_1($t_0 \neq t_1$), the alliance can be formed. Therefore, the forming probability of alliance is different in different time,

We use $P_{ij}(T, t_0)$ represent probability of player ibelong to alliance, so for any dimension OB_i(i = 1, 2, …, m), there is:

$$P_{ij}(T, t_0) = \prod_{j \in T} p_{ij}(t_0) \tag{1}$$

INCOMPLETE INFORMATION DYNAMIC N-PERSON MULTI-DIMENSION COOPERATIVE GAME FUZZY STABLE SET

Theorem 1: Assume that $H(\tilde{\boldsymbol{v}}_i)$ to R^n mapping is F_i and it meet the following two axiom at the same, thus we can call F_i as a fuzzy stable set of cooperative game, recorded a $F_i(\tilde{\boldsymbol{v}}_i)$, the effective interval can be expressed as $[F_i(\tilde{\boldsymbol{v}}_i'), F_i(\tilde{\boldsymbol{v}}_i'')]$.

Axiom 1: Assume that $H(\tilde{\boldsymbol{v}}_i)$ to R_n mapping is F_i, for each an ally $j \in \boldsymbol{\Gamma} \subset 2^n - \{\Phi\}$,and also:

$$\sum_{i=1}^{m} \sum_{j \in \Gamma \subset M \subseteq Q} (-1)^{|M|-|\Gamma|} F_{ij}(\tilde{v}_M) = P_{ij}(\Gamma, t_0)$$
$$F_{ij}(\tilde{v}_M) = 0,\ i \notin M \tag{2}$$

Axiom 2: Assume that $\tilde{\boldsymbol{v}}_1 = [\tilde{\boldsymbol{v}}_{1'}, \tilde{\boldsymbol{v}}_{1''}]$ and $\tilde{\boldsymbol{v}}_2 = [\tilde{\boldsymbol{v}}_{2'}, \tilde{\boldsymbol{v}}_{2''}]$ are any two cooperative game in $H(\tilde{\boldsymbol{v}}_1)$ and has the ultra additive, namely $F_{ij}(\tilde{\boldsymbol{v}}_1 + \tilde{\boldsymbol{v}}_2) = F_{ij}(\tilde{\boldsymbol{v}}_1) + F_{ij}(\tilde{\boldsymbol{v}}_2)$

Theorem 2: For any i = 1, 2, … m, has and only has a mapping which can meet the interval fuzzy stable set, namely F_i: $H(\tilde{\boldsymbol{v}}_i) \rightarrow R_n$, and there is $F_{ij}(\tilde{\boldsymbol{v}}_1) = \sum_{i=1}^{m} \sum_{j \in \Gamma \subseteq Q} \boldsymbol{P}_{ij}(\boldsymbol{\Gamma}, t_0)\, (\tilde{\boldsymbol{v}}_i(\boldsymbol{\Gamma}) - \tilde{\boldsymbol{v}}_i(\boldsymbol{\Gamma} - \{\mathbf{j}\}))$ its interval lower limit for expression is: $F_{ij}(\tilde{\boldsymbol{v}}_i') = \sum_{i=1}^{m} \sum_{j \in \Gamma \subseteq Q} \boldsymbol{P}_{ij}(\boldsymbol{\Gamma}, t_0)\, (\tilde{\boldsymbol{v}}_i'(\boldsymbol{\Gamma}) - \tilde{\boldsymbol{v}}_i'(\boldsymbol{\Gamma} - \{\mathbf{j}\}))$, and its upper limit for expression is: $F_{ij}(\tilde{\boldsymbol{v}}_i'') = \sum_{i=1}^{m} \sum_{j \in \Gamma \subseteq Q} \boldsymbol{P}_{ij}(\boldsymbol{\Gamma}, t_0)\, (\tilde{\boldsymbol{v}}_i''(\boldsymbol{\Gamma}) - \tilde{\boldsymbol{v}}_i''(\boldsymbol{\Gamma} - \{\mathbf{j}\}))$, The interval value can be expressed as $F_{ij}(\tilde{\boldsymbol{v}}_i)$, $= [F(\tilde{\boldsymbol{v}}_i'), \boldsymbol{F}(\tilde{\boldsymbol{v}}_i'')]$. For any dimension to the fuzzy set of steady set can be expressed as:

$$F_i(\tilde{v}_i) = \{F_{i1}(\tilde{v}_i), F_{i2}(\tilde{v}_i), \cdots, F_{iQ}(\tilde{v}_i)\} =$$
$$\{[F_{i1}(\tilde{v}_i'), F_{i1}(\tilde{v}_i'')], [F_{i2}(\tilde{v}_i'), F_{i2}(\tilde{v}_i'')], \cdots, [F_{iQ}(\tilde{v}_i'), F_{iQ}(\tilde{v}_i'')]\}$$

Lemma 1: For any an ally $j \in \Gamma \subseteq Q$, there is $\sum_{i=1}^{m} \sum_{j \in \Gamma \subset M \subseteq Q} (-1)^{|M|-|\Gamma|} z_{ijM} = P_{ij}(\Gamma, t_0)$, $j \in \Gamma$, and this equation can be expressed as linear equation $\sum_{M=t}^{n} \binom{n-t}{M-t} = 2^{n-t}$ which z_{ijm} is unknown and this equations has unique solution.

Lemma 2: For $\tilde{\boldsymbol{v}}_i = [\tilde{\boldsymbol{v}}_i', \tilde{\boldsymbol{v}}_i''] \in H\, \tilde{\boldsymbol{v}}_i$, there real number λ_M are 2^n-1 and $M \in 2^n$, $M \neq \Phi$. Equation $\sum_{i=1}^{m} \tilde{\boldsymbol{v}}_i = \sum_{i=1}^{m} \sum_{M \subseteq Q} \lambda_M v_M$ is established and $\lambda_M = \sum_{\Gamma \subset M} (-1)^{|M|-|t|}\, \tilde{\boldsymbol{v}}_i(\Gamma)$.

Theorem 3: Set the weight vector for:

$(\omega_{ij},\ \omega_{1j}, \ldots,\ \omega_{mj})^T$, i = 1, 2, …, m

and meet the requirements $\omega_{ij} \geq 0$, $\sum_{i=1}^{m} \omega_{ij} = 1$.So we can call

$$F_j(\tilde{v}) = \sum_{i=1}^{m} \sum_{j \in \Gamma \subset M \subseteq Q} \omega_{ij} F_{ij}(\tilde{v}_i)$$

as ally j multidimensional cooperative game interval fuzzy stable set:

$$F(\tilde{v}) = \{F_1(\tilde{v}), F_2(\tilde{v}), \cdots F_Q(\tilde{v})\} =$$
$$\{[F_1(\tilde{v}'), F_1(\tilde{v}'')], [F_2(\tilde{v}'), F_2(\tilde{v}'')], \cdots, [F_Q(\tilde{v}'), F_Q(\tilde{v}'')]\}$$

PRACTICAL APPLICATION

Suppose there are three enterprises A, B and C, these enterprises have R&D cooperation for one project. They have four goals: OB_1, OB_2, OB_3, OB_4. Represent respectively, the requirements of the game is: Above four goals, pay configuration of three enterprises are optimal Of course, there is a.

Hypothesis, any one of the three companies can not complete the task alone and they must get alliance in order to achieve common development cooperative innovation.

First we give various targets and league characteristic function value scope, as is shown in Table 1. For the convenience of discussion, above four goals, we give the probability which three company's willing to join each league, such as shown in Table 2.

Table 1: Four dimensional interval range of cooperative game function

Game	{A}	{B}	{C}	{A, B}	{A, C}	{B,C}	{A, B, C}
$\tilde{v}_1$	[16, 20]	[30, 32]	[27, 29]	[58, 64]	[8 4, 90]	[70, 77]	[95, 104]
$\tilde{v}_2$	[10, 13]	[17, 24]	[10, 14]	[50, 57]	[40, 46]	[30, 37]	[80, 87]
$\tilde{v}_3$	[13, 15]	[19, 23]	[20, 26]	[75, 80]	[58, 64]	[80, 86]	[105, 123]
$\tilde{v}_4$	[10, 12]	[20, 25]	[17, 20]	[57, 62]	[45, 51]	[60, 69]	[89, 96]

Table 2: The probabilities of three companies to join in each league

Target	Probability	{A}	{B}	{C}	{A, B}	{A, C}	{B,C}	{A, B, C}
	p_{11}	0.1			0.25	0.25		0.4
OB_1	P_{12}		0.2		0.25		0.35	0.2
	p_{13}			0.2		0.3	0.3	0.2
	P_{21}	0.2			0.25	0.25		0.3
OB_2	P_{22}		0.15		0.25		0.35	0.15
	P_{23}			0.25		0.25	0.25	0.25
	P_{31}	0.3			0.1	0.25		0.35
OB_3	P_{32}		0.25		0.15		0.35	0.25
	P_{33}			0.2		0.15	0.35	0.3
	P_{41}	0.25			0.25	0.35		0.15
OB_4	P_{42}		0.2		0.25		0.25	0.3
	P_{43}			0.2		0.25	0.35	0.2

Table 3: The weight of each target evaluation

Weight	OB_1	OB_2	OB_3	OB_4
ω_{i1} {A}	0.25	0.25	0.2	0.3
ω_{i2} {B}	0.15	0.35	0.25	0.25
ω_{i3} {C}	0.15	0.35	0.35	0.15

Table 4: Membership probability of cooperative game

Probability	{A}	{B}	{C}	{A, B}	{A, C}	{B,C}	{A, B, C}
P_1	0.1	0.15	0.2	0.0375	0.0875	0.0875	0.021
P_2	0.125	0.275	0.15	0.1025	0.1875	0.0875	0.085
P_3	0.0875	0. 175	0.215	0.175	0.0875	0.0125	0.075
P_4	0.175	0.125	0.1875	0.215	0.185	0.175	0.1025

Table 5: Fuzzy stable set interval of each target

Fuzzy stable set	OB_1	OB_2	OB_3	OB_4
$F_{i1}(\tilde{v}_i^{'})$	[8.7545,9.8565]	[8.755,8.865]	[7.012,7.275]	[5.896,8.235]
$F_{i2}(\tilde{v}_i^{'})$	[9.0225,10.3565	[11.2568,12.5696]	[8.2115,11.25]	[11.5685,12.8576]
$F_{i3}(\tilde{v}_i^{'})$	[6.358,8.025]	[8.675,9.258]	[8.25,8.56]	[11.256,12.025]

Three (3) enterprises have weight assessment on the four targets, such as Table 3 shows.

According to the formula $P_{ij}(T, t_0) = \prod_{j\in T} p_{ij}(t_0)$, $p_{ij}(T, t_0)$ can be obtained and the results of calculation can be seen in Table 4.

From the formula (2) computation, we got interval distribution of fuzzy stable set, as shown in Table 5.

Consider the weighting factor, according to theorem 3, we can respectively calculate the fuzzy preference stable set interval which based on the target are:

$$F_1(\tilde{v}) = [7.5785, 8.4257]$$

$$F_2(\tilde{v}) = [10.4568, 12.0256]$$

$$F_3(\tilde{v}) = [9.523, 10.214]$$

so, these 3 enterprises fuzzy stable set interval based on four goals is:

$$F(\tilde{v}) = \{F_1(\tilde{v}), F_2(\tilde{v}), \cdots F_Q(\tilde{v})\} = \{[7.5785, 8.4257], [10.4568, 12.0256], [9.523, 10.214]\}$$

From this formula, it is easy to see fuzzy interval value of B was maximum, which can be concluded that, When the league distribute income, the enterprise B's income is the highest, followed by enterprise C, the least is the enterprise A, namely after this cooperation, three enterprise income distribution is B>C>A.

CONCLUSION

This study researched on multi-dimension game among enterprise cooperation. The main achievement was, using fuzzy mathematics research method, analyzed in fuzzy stability interval of multidimensional targets N-person cooperative game and through calculation got feasible income distribution fuzzy interval. Compared with other solving method of multi-dimension game, our method is much easy, for player in game; we provide decision support when the player only has incomplete information. At the same time, we got a new way to analysis in the process of cooperation dynamic income problem. And this study got a certain extent try for fuzzy mathematics in the application of management problems.

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REFERENCES

Branzei, R., D. Dimitrov and S. Tijs, 2003. Convex fuzzy games and participation monotonic allocation schemes. Fuzzy Set. Syst., 139(2): 267-281.

Das, T.K. and B.S. Teng, 2000. A resource-based theory of strategic alliance. J. Manag., 26(1): 31-60.

Dhingra, A.K. and S.S. Rao, 1995. A cooperative fuzzy game theoretic approach to multiple objective design optimization. Europ. J. Oper. Res., 83(2): 547-567.

Hamel, G., 1991. Competition for competence and inter-partner learning within international strategic alliances. Strat. Manag. J., 12(1): 83-103.

Hausler, J., H.W. Hohn and S. Lutz, 1994. Contingencies of innovative networks: A case study of successful inter-firm R&D cooperation. Res. Pol., 23(1): 47-66.

Kogut, B., 1988. Joint ventures: Theoretical and empirical perspectives. Strat. Manag. J., 9: 319-332.

Layton, E.T., 1974. Technology as knowledge. Technol. Cult., 15(1): 31-41.

Pisano, G.P., 1990. The R&D boundaries of the firm: An empirical analysis. Admin. Sci. Quart., 35(1): 153-176.

Sakakibara, M., 1997. Heterogeneity of firm capabilities and cooperative research and development: An empirical examination of motives. Strat. Manag. J., 18(1): 143-164.

Schumpeter, J.A., 1912/1934. Theory of Economic Development (German). Leipzig: Duncker and Humblot. English Translation Published in 1934 as the Theory of Economic Development. Harvard University Press, Cambridge, MA.