Research Article
Performance Analysis of the Different Null Steering Techniques in the Field of Adaptive Beamforming

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Abstract: In this study, we compare the performance of three null steering techniques using uniform linear array. These techniques include Null Steering without using Phase Shifters, Null Steering by Decoupling the Real Weights and Null Steering by Decoupling the Complex Weights. The evaluation criteria of these techniques is based on the bases of different parameters i.e., null depth, main beam width, side lobe levels, number of steerable nulls, computational complexity and number of sensors used in the array. The validity and effectiveness of these techniques is reflected by the resultant radiation pattern of the array.

Keywords: Adaptive beamforming, independent null steering, linear arrays, null steering

INTRODUCTION

Null steering is a compulsory and important part in adaptive beam forming and it has vast applications in radars, sonar, mobile communications, etc., (Vu, 1984; Ibrahim, 1991; Khan *et al*., 2011, Zaman *et al*., 2012a, b and c). In literature, different methods have been developed for null steering. These methods can be categorized in two classes. The 1st class relies on physical perturbation of the sensors in the array which requires servo motors along with other hardware to achieve the results (Hejres, 2004). Due to physical perturbation, it has not been considered an efficient approach. The 2nd class of algorithms presents methods in which digital attenuators and phase shifters are required to change the weights of an array. In this class two methods are famous to update the coefficients of the array factor. The 1st type of methods updates the weights using phase shift of the current (Steyskal, 1983) but the problem is its complexity and hardware constraints. The 2nd type of methods updates the weights using only the current amplitudes to steer nulls (Vu, 1984).

Null steering in linear adaptive arrays using current amplitudes is the best line of approach to meet the requirements of the field. It reduces computational time by getting symmetrical distribution of the amplitude of the currents about centre of the array. This is an efficient way, but unfortunately it reduces number of steerable nulls i.e., half to the total number of elements in the array. Furthermore, this method is also unable to steer nulls independently. In Ibrahim (1991), the concept of independent null steering is introduced. It achieves this by decoupling the real weights of the array. But, unfortunately, this technique could not increase the number of controllable nulls of the array. Finally, the technique presented in Khan *et al*. (2011), takes over the issue and control has been obtained on all possible nulls of the linear array. Moreover, nulls are steered independently. In other words, the method can steer (N-1) nulls in arbitrary directions using array of N elements with extra feature of independent nulling. Method is structure based and it needs (N-1) sets of complex digital attenuators, in an array of N elements.

In this study, the performance of Independent Null Steering by Decoupling the Complex Weights is evaluated further with different parameters and environments. Two other methods, Null Steering without using Phase Shifters and Independent Null Steering by Decoupling the Real Weights are also discussed with different constraints. Moreover, the performance of Independent Null Steering by Decoupling the Complex Weights is compared with the other two null steering techniques. The evaluation criteria of these techniques is based on the bases of different parameters i.e., null depth, main beam width, side lobe levels, number of steerable nulls, computational complexity, different number of sensors in the array and hardware cost. The two techniques, Independent Null Steering by Decoupling the Complex Weights and Independent Null Steering by Decoupling the Real Weights, are structure based so these required more hardware cost. Lastly, the validity and effectiveness of these techniques is reflected by the resultant radiation pattern of the array.

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PROBLEM FORMULATION

Array of N elements is considered with equal spacing d. All elements are placed in a straight line with first element considered as reference point. For far field observation, angle of arrival is considered θ. A simple case is considered here, where current amplitudes of all elements in the linear array are uniform and α is taken as progressive phase (Applebaum, 1976). The array factor is given as:

$$AF = \sum_{i=1}^{N} e^{j(i-1)\psi} \tag{1}$$

where,

$$\psi = kd\cos\theta + \alpha$$

$$k = \frac{2\pi}{\lambda}$$

Array factor in expanded form is:

$$AF = 1 + e^{j\psi} + e^{2j\psi} + \ldots\ldots\ldots + e^{j(N-1)\psi} \tag{2}$$

After considering, $z = \exp(j\psi)$, AF can be expressed in the following way:

$$AF = 1 + z + z^2 + z^3 + \ldots\ldots\ldots + z^{N-1} \tag{3}$$

Hence, we got a polynomial of order (*N-1*). This is the pioneering work of Schelkunoff (1943) that he related a polynomial to the radiation pattern of the linearly phased array (Schelkunoff, 1943). Polynomial of the array factor has (*N-1*) roots on the unit circle in the complex plane. Zeros of the above polynomial has fixed positions along the unit circle. These positions can be named as $r_1, r_2, r_3, \ldots\ldots\ldots, r_{N-1}$ with their corresponding directions $\psi_1, \psi_2, \psi_3, \ldots\ldots\ldots, \psi_{N-1}$ respectively. Polynomial of the array factor can be expressed in the form of product of the binomials:

$$AF = (z - r_1)(z - r_2)(z - r_3) \ldots\ldots\ldots (z - r_{N-1}) \tag{4}$$

where, $r_1, r_2, r_3, \ldots\ldots\ldots, r_{N-1}$ are roots of the polynomial and these roots corresponds to the nulls of the array factor in the following fixed directions $\theta_1, \theta_2, \theta_3, \ldots\ldots\ldots, \theta_{N-1}$. In simple words, we cannot steer nulls with such kind of uniform linear array. In order to rotate zeros or roots of the polynomial of the array factor on the unit circle of complex plane in arbitrary directions, coefficients of the polynomial should be changed accordingly. To steer nulls in the arbitrary directions, weights of the array elements must be updated (Karim and Viberg, 1996). Therefore, array factor can be expressed in the following way for the array of N elements:

$$AF = A_0 + A_1 z + A_2 z^2 + \ldots\ldots\ldots + A_{N-1} z^{N-1} \tag{5}$$

NULL STEERING TECHNIQUES

Null steering without using phase shifters: The technique Null Steering method using Real Weights (NSRW) uses current amplitudes to steer nulls (Vu, 1984). By varying current amplitudes, arbitrary nulls can be generated towards desired locations. Main beam is steered using phase shifters in the technique. In other words, main beam can be scanned by controlling the progressive phase shift only and jammers or noise sources at different locations can be suppressed by pointing nulls towards their locations and this can be achieved easily by varying only current amplitudes appropriately. Distribution of current amplitudes of the linear array is symmetrical about its center, which drastically reduces computational time to half. If jammers or noise sources are less than half to the total number of elements in the array then extra nulls can be used to suppress side lobe levels of the main beam which definitely improves radiation pattern of array factor for better communication. To achieve null steering without using phase shifters, the technique selects a set of zeros occurring in conjugate pairs on unit circle in complex plane. At first, to simplify our illustrations, we consider the following two cases:

- **Case-1 (odd number of elements):**

$$K = \frac{N-1}{2}$$

where,
N = Number of elements in the array
K = Number of achievable arbitrarily nulls

Array factor for array of odd number of elements will be expressed as follows:

$$AF = (z - r_1)(z - r_1^*)(z - r_2)(z - r_2^*) \ldots\ldots\ldots (z - r_K)(z - r_K^*) \tag{6}$$

- **Case-2 (even number of elements):**

$$K = N/2$$

where,
K-1 = Number of achievable arbitrary nulls.

In this case, pattern of the array factor becomes like this:

$$AF = (z + 1)(z - r_1)(z - r_1^*) \ldots\ldots\ldots (z - r_{K-1})(z - r_{K-1}^*) \tag{7}$$

Here we noticed an extra factor $(z + 1)$ which has no complex conjugate pair and this unpaired factor has

zero at $z = -1$ on the unit circle. Now we will extract some important conclusions from above array factor. Consider one pair from the product of binomials of the array factor:

$$(z - r_1)(z - r_1^*) = z^2 - (r_1 + r_1^*)z + |r_1|^2$$
$$= z^2 - 2Re(r_1)z + 1$$

It is clear from the above result that each pair of factors generates real coefficients and by combining the results of all pairs of factors we get the polynomial of the array factor which has all coefficients real. In other words, phases of the currents are omitted and only amplitudes of the currents are involved in the array pattern. Hence, we have been established a technique in which no phase changes are required for steering the arbitrarily nulls and coefficients of the polynomial of the array factor are solely determined by the current amplitudes. But unfortunately, we are achieving this technique at some cost. And that cost is, the total number of steerable nulls that can be produced is effectively halved to the total number of the elements in array. To illustrate the proposed technique mathematically we consider an array of seven elements combined linearly:

$$N = odd = 2K + 1 = 7$$

Steerable nulls for this array will be $K = 3$ and same number of real coefficients of the polynomial of the array factor will be found, next. Array factor for such an array can be modeled in this way as shown below:

$$AF = (z - r_1)(z - r_1^*)(z - r_2)(z - r_2^*)(z - r_3)$$
$$(z - r_3^*) \tag{8}$$

We know that r_i is root of the array factor for the given array and is calculated as:

$$r_i = \exp(j\psi_i) = e^{j\psi_i} = cos\psi_i + jsin\psi_i$$

where,
$\psi_i = \alpha + kdcos\theta_i$
$\alpha = kdcos\theta_s$
$\theta_s = main\ beam\ direction$
$\theta_i = arbitrary\ directions\ towards\ nulls$

Then symmetrical array factors becomes like this:

$$AF = 1 - (2p)z + (3 + 4q)z^2 - (4p + 8r)z^3 +$$
$$(3 + 4q)z^4 - (2p)z^5 + z^6 \tag{9}$$

where,
$p = cos\psi_1 + cos\psi_2 + cos\psi_3$
$q = cos\psi_1 cos\psi_2 + cos\psi_1 cos\psi_3 + cos\psi_2 cos\psi_3$
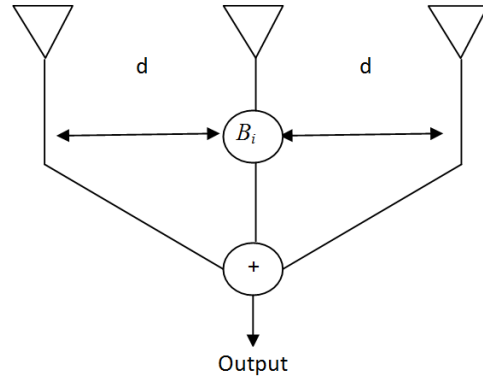$r = cos\psi_1 cos\psi_2 cos\psi_3$

Fig. 1: Basic structure for independent null steering

But the proposed array factor with symmetrical weights for such an array was as follows:

$$AF = 1 + C_1 z + C_2 z^2 + C_3 z^3 + C_2 z^4 + C_1 z^5 + z^6 \tag{10}$$

where,

$$C_1 = -2p, C_2 = 3 + 4q, C_3 = -4p - 8r$$

Independent null steering by decoupling the real weights: Although, Vu (1984) contributed to the field of adaptive null steering very efficiently, but two problems were could not be resolved. Firstly, steerable nulls were effectively reduced to half the total number of elements in the array. Secondly, steerable nulls were coupled to each other. To change one null position, all null positions have to be calculated again even if they were not intended to be steered. In other words, the algorithm was not capable to steer nulls independently. Ibrahim (1991) addressed second problem in deliberately. The technique, Independent Null Steering by decoupling the Real Weights (INSRW), presents a method by which nulls are decoupled from each other and only weight of the desired null will be updated in order to change the corresponding null location. Hence, each null is linked to a certain weight. To change position of one null, there is no need to update (recalculate) all other weights as well, what we need is only to change the linked weight to that null. For this purpose, the array is arranged in combination of sets, where each set consists on three-elements as shown in Fig. 1.

The middle element is multiplied to its weight before adding to other two elements of the set. Array factor of this arrangement can be expressed in the following expression:

$$AF = (z - r_i)(z - r_i^*)$$

where, $r_i = e^{j\psi}$, is root of the array factor lies on unit circle of complex plane:
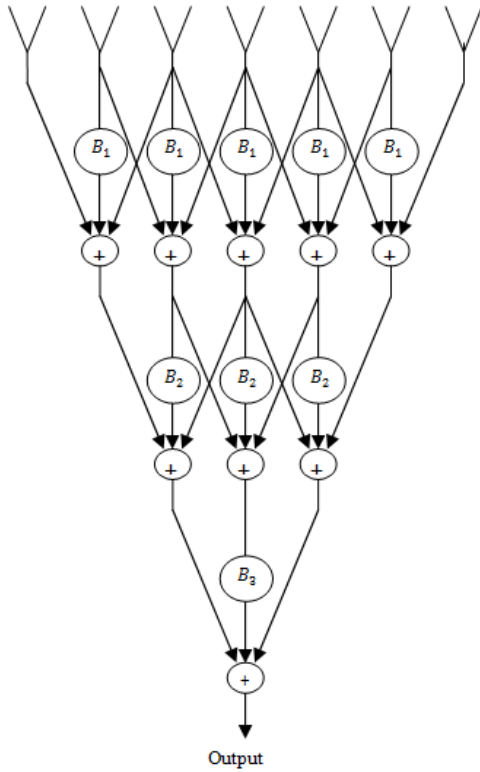
Fig. 2: Structure for the array of seven elements for independent null steering

$$AF = z^2 - (r_i^* + r_i)z + 1 = 1 + B_i z + z^2 \quad (11)$$

where,

$$B_i = -(r_i^* + r_i) = -2cos\psi_i \quad (12)$$

This basic arrangement can steer one null in direction of θ_i. Maximum value of this weight is +2 while minimum value is -2, where negative sign can be achieved by $+\pi$ or $-\pi$ phase bit available in phase shifter of the element. Now we will normalize the radiation pattern of the basic set of elements.

Let, $z = e^{j\psi}$. Then AF becomes:

$$AF = 1 - 2cos\psi_i(cos\psi + jsin\psi) + (cos\psi + jsin\psi)^2$$
$$= 2(cos\psi + jsin\psi)(cos\psi - cos\psi_i)$$

Hence, the proposed normalized array factor is expressed as:

$$AF_n = (cos\psi - cos\psi_i) = cos[kdcos\theta + \alpha] + \frac{B_i}{2} \quad (13)$$

Proposed structure for seven elements is shown in Fig. 5. The proposed structure has nine attenuators with same number of summers. Three sets of attenuators have been used and each set has attenuators of the same value. First set of attenuators is represented by B_1 while second set and third set is named with B_2 and B_3 respectively. Important thing to be noticed in the structure is the number of sets of attenuators, because this number declares the number of independent steerable nulls. In this case, we have three sets of attenuators i.e., B_1, B_2 and B_3. It means independently control-able nulls are also three. And each set or array of attenuators is associated to one independently steerable null. As shown in the Fig. 2 output of each middle element is split into three paths.

Elements at corners have only one path while next elements have two equally paths. And the same process is repeated for next two stages, before to get final output. Our final normalized radiation pattern is as following:

$$AF_n = \prod_{i=1}^{(N-1)/2} F_i(\theta) \quad (14)$$

$$= \prod_{i=1}^{(N-1)/2}[cos\psi - \frac{B_i}{2}] \quad (15)$$

Independent null steering by decoupling the complex weights: The technique Independent Null Steering by decoupling Complex Weights (INSCW) presents a new way to increase number of steerable nulls (Khan *et al.*, 2011). The technique uses structure to decouple the complex weights. The proposed structure needs (N-1) sets of digital attenuators for an array of N elements whereby each set has same value of weights. Moreover, these digital attenuators should be capable of handling of complex weights. In other words, attenuators must be qualified to adjust amplitude and phase of the current accordingly. Two input line summers are used in the structure by which one of two input lines is multiplied by negative of the weight before added in summer. Required number of digital complex attenuators and summers for an array of N elements can be calculated by N (N-1) /2 to elaborate fully the presented technique, two different cases are considered here.

Table 1: Comparison between different algorithms

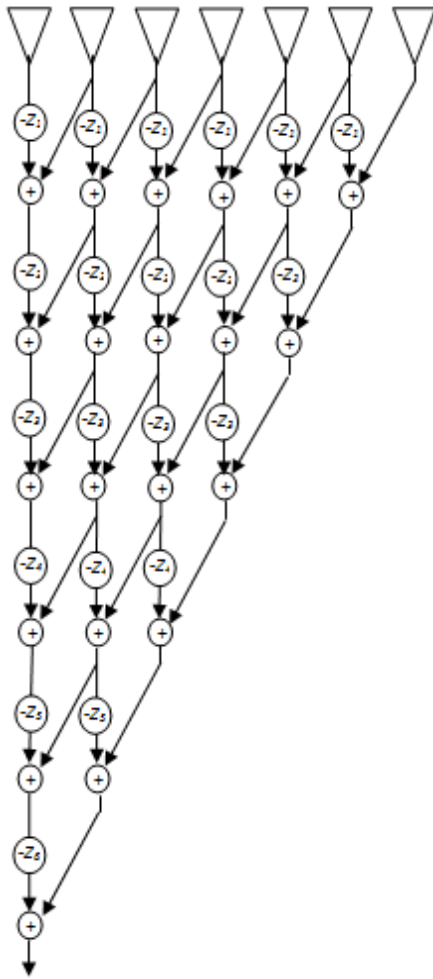| Tech.	Ele.	Nulls	Att.	Sum	Side lobes
NSRW	7	3	3	1	-11 to -17 dB
NSRW	8	3	3	1	-21 to -22 dB
INSRW	7	3	9	9	-7 to -13 dB
INSRW	8	3	9	16	-24 to -25 dB
INSCW	7	6	21	21	-10 to -11 dB
INSCW	8	7	23	28	-17 to -20 dB

Fig. 3: Structure for the array of seven elements for independent null steering

Table 1 shows the required hardware for the array of each case. For illustration of the method an array of seven elements is considered. Proposed structure for the array of seven elements is shown in Fig. 3.

Six sets of complex digital attenuators are required and same number of stages is involved. Each stage uses attenuators of the same set. And in a set all attenuators have same values (weights). For this array twenty-one attenuators and same number of summers are used. Two input line summers are used in structure. First input line is multiplied with weight before to add with second input line. General expression can modeled for all stages of the structure for the array of N-elements which are uniform linearly phased. There are (N-1) stages for this array:

$$y_{i,j} = z^{j-1}(z - z_1)(z - z_2) \ldots \ldots \ldots (z - z_k)$$

where,
$$i = 1,2, \ldots \ldots \ldots N - 1$$

$$j = 1,2, \ldots \ldots \ldots N - 2$$
$$k = 1,2, \ldots \ldots \ldots i$$

and final output for such an array can also be modeled as the following way:

$$y_{N-1,1} = (z - z_1)(z - z_2)(z - z_3) \ldots \ldots \ldots (z - z_{N-1})$$

Hence,

$$AF = (z - z_1)(z - z_2)(z - z_3)(z - z_4)$$
$$(z - z_5)(z - z_6)$$

where,

$$z_i = \exp(j\psi_i)$$

RESULTS AND SIMULATION

In this section results and simulations have been presented. Two cases are considered with the help of four graphs. Required hardware for each case is shown in Table 1. For simplicity, distance between the adjacent elements of each array is taken λ/2 There are a few parameters that should be considered to estimate the performance of the algorithm such as: number of elements in the array, depth of each null, side lobe levels, width of the main lobe, computational time, number of steerable nulls, system implementation complexity, hardware cost and the technique reliability. We will focus on all such parameters in the following simulation of each case to check the worth of the proposed solution.

Case-1: For simulation of this case, we considered an array of seven elements. Using this array, Vu (1984) method is able to produce three arbitrary nulls in the direction of jammers, while Ibrahim (1991) method can handle three arbitrary nulls independently but at the cost of nine attenuators and nine summers. And Khan *et al.* (2011) method can steer six nulls independently in arbitrary directions. In other words, last method makes this array fully functioned as all possible nulls are taken under control with the help of twenty-one digital complex attenuators and two input line summers and complex weights are used to steer nulls while main beam is controlled by progressive phase shift. Calculated coefficients of the array are shown in the Table 1. For this case, interfering jammers are considered in the following directions:

$$\theta_T = \{30^o, 60^o, 130^o\}$$

$$\theta_H = \{30^o, 60^o, 120^o\}$$

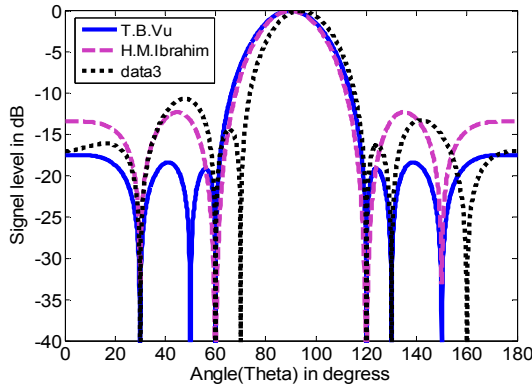$$\theta_Z = \{30^o, 60^o, 70^o, 120^o, 130^o, 160^o\}$$

Fig. 4: Comparison of techniques for seven elements

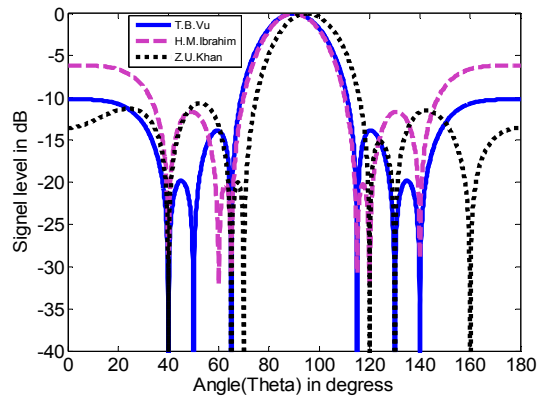Fig. 5: Comparison of techniques for seven elements

Main beam direction is taken in the direction, $\theta_s = 90^o$. By referring Fig. 4, we can see that jammers directions are nulled appropriately while main beam is pointing towards its desired direction. Estimated weights for the method of NSRW, INSRW and INSCW are shown below, respectively:

Z_T = {1: 3.58: 6.86: 8.41: 6.86: 3.58: 1}

Z_H = {1.99: 0.10: 1.14}

Z_z = {0.38 - 0.92i: 0.90 - 1.63i: 1.87 - 1.91: 2.75 - 1.83: 2.48 - 0.99i: 1.85 - 0.20i: 1}

For further investigation on the technique, a change is made with two interfering jammer directions while positions of other interferers are kept on their respective previous locations as shown below:

$$\theta_1 = 30^o \xrightarrow{direction\ is\ changed\ to} \theta_{new1} = 40^o$$

$$\theta_2 = 60^o \xrightarrow{direction\ is\ changed\ to} \theta_{new2} = 65^o$$

Figure 5 shows this alteration appropriately and we can confirm the required variations accordingly while calculated weights of the array with altered null locations are shown below for each method, orderly:
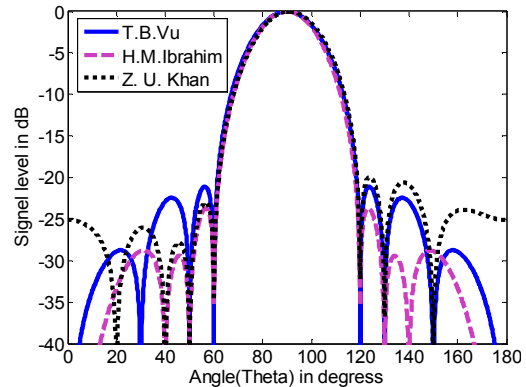

Fig. 6: Comparison of techniques for eight elements

Z_T = {1: 3.26: 4.71: 4.88: 4.71: 3.26: 1}

Z_H = {1.68: 0.10: 1.12}

Z_z = {-0.16 - 0.99i: 0.19 - 1.49i: 1.02 - 2.15i: 1.82 - 2.14i: 1.96 - 1.35i: 1.44 - 0.43i: 1}

Hence, it is clear from above calculated weights that all weights of the array factor of Vu (1984) method are altered. But Ibrahim (1991) technique and Khan *et al.* (2011) technique updated only first and second weight values and there was no need to calculate all other weights as well.

Further, it is also verified from the graph that only places of two nulls have been changed while other nulls are still pointing towards their previous locations. Main beam is not disturbed and is pointing towards $\theta_s = 90^o$.

Case-II: Linear array of eight antennas is considered in this case. NSRW method produces three randomly generated nulls towards the directions of the jammers, while using INSRW method we get independently control on three nulls. But, by implementing INSCW method, seven nulls can be steered independently in arbitrary directions and this is the maximum control on output that any array can produce. For NSRW and INSRW, this is the same situation (same number of controlled nulls) arose in previous case. Moreover, number of arbitrary nulls is same either array of seven elements is used or array of eight elements is used. But radiation pattern of the array of eight elements is much better than that of the array of seven elements. Interfering jamming signals are taken in following directions:

$$\theta_T = \{50^o, 120^o, 150^o\}$$

$$\theta_H = \{40^o, 60^o, 130^o\}$$

$$\theta_Z = \{20^o, 40^o, 50^o, 60^o, 120^o, 130^o, 150^o\}$$

While main beam is taken in the direction, $\theta_s = 90^o$. Resulted radiation pattern of the selected array with given parameters is shown in Fig. 6.
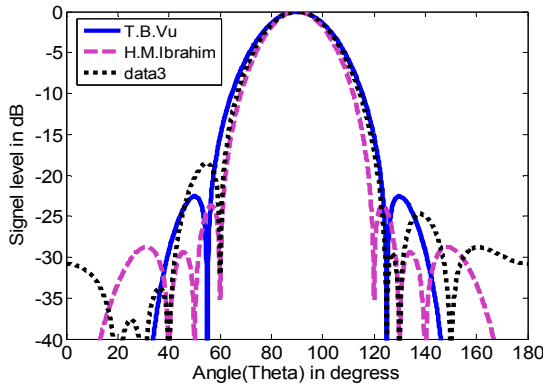
Fig. 7: Comparison of techniques for eight elements

Calculated complex weights of the array corresponding to the above set of interference directions are as follows:

Z_T = {1: 2.74: 3.76: 3.99: 3.99: 3.76: 2.74: 1}

Z_H = {0.87: -5.67e - 0.16: 0.87}

Z_z = {0.88 - 0.48i: 3.29 - 1.30i: 6.58 - 2.18i: 9.1 - 2.57i: 9.21-2.14i: 6.81-1.27i: 3.50 - 0.45i: 1}

Each null is controlled by its linked weight and in case of any change in the environment those weights which are associated to the displaced jammers will be updated. Next in order to verify the promised thing we will shift two randomly selected jammers towards new locations:

$\theta_T = \{30^o, 125^o, .\}$

$\theta_H = \{50, ., 140^o\}$

$\theta_Z = \{., ., 30^o, ., 125^o, ., ., .\}$

Weights associated to these nulls will be changed accordingly as:

Z_T = {1: 4.58: 10.44: 15.27: 15.27: 10.44: 4.58: 1}
Z_H = {1.99: 0.10: 1.14}
Z_z = {1.0 - 0.03i: 4.21 - 0.16i: 8.94 - 0.43i: 12.53 - 0.40i: 12.53 - 0.02i: 8.95 + 0.13i: 4.21 + 0.02i: 1}

Simulation of this modified version of the radiation pattern of the aforesaid techniques is shown in Fig. 7. Hence it is clear from the figure that only corresponding nulls have changed their locations while other nulls are keeping their previous locations. In this simulation, main lobe was not disturbed from its previous location, $\theta_s = 90^o$. It is still pointing towards its previous direction as shown in the Fig. 7.

CONCLUSION

Performance analysis of different null steering techniques in the field of adaptive beam forming has been presented. Three null steering techniques were considered for comparison analysis. Table 1 shows achieved steerable nulls, required hardware and side-lobe levels of each technique. Vu (1984) technique is cost efficient but is time consuming for larger arrays. Ibrahim (1991) technique is time efficient in this regard but steerable nulls are effectively halved to total number of elements in the array. However, this technique uses real weights and de-couples its weights to attain independent null steering but real weights reduces the number of steerable nulls to the half of the total number of elements in the array. In this perspective, Khan *et al.* (2011) technique is leading as it is achieving control on all possible nulls of the radiation pattern and its feature of independent null steering makes it time efficient for larger arrays. Independent null steering is an attractive feature but it increases hardware cost as well.

REFERENCES

Applebaum, S.P., 1976. Adaptive arrays. IEEE T. Antenna Propag., 24: 5.

Hejres, J.A., 2004. Null steering in phased arrays by controlling the position of selected elements. IEEE T. Antenna Propag., 52: 11.

Ibrahim, H.M., 1991. Null steering by real-weight control- a method of decoupling the weights. IEEE T. Antenna Propag., 39: 1648-1650.

Karim, H. and M. Viberg, 1996. Two decades of array signal processing research: The parametric approach. IEEE Signal Proc. Mag., 13(4): 67-94.

Khan, Z.U., A. Naveed, I.M. Qureshi and F. Zaman, 2011. Independent Null steering by decoupling complex weights. IEICE Elect. Exp., 8: 1008-1013.

Schelkunoff, S.A., 1943. A mathematical theory of linear arrays. Bell. Syst. Tech. J., 22: 80-87.

Steyskal, H., 1983. Simple method for pattern nulling by phase perturbation. IEEE T. Antenna Propag., 31(1): 163-166.

Vu, T.B., 1984. Method of null steering without using phase shifters. IEEE Proc-H, 131(4): 242-245.

Zaman, F., I.M. Qureshi, A. Naveed and Z.U. Khan, 2012a. Real time direction of arrival estimation in noisy environment using particle swarm optimization with single snapshot. Res. J. Sci. Eng. Technol., 4(13): 1949-1952.

Zaman, F., I.M. Qureshi, A. Naveed, J.A. Khan and M.A.Z. Raja, 2012b. Amplitude and directional of arrival estimation: Comparison between different techniques. Prog. Electromag. Res. B, 39: 319-335.

Zaman, F., I.M. Qureshi, A. Naveed and Z.U. Khan, 2012c. Joint estimation of amplitude, direction of arrival and range of near field sources using memetic computing. Prog. Electromag. Res. C, 31: 199-213.