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Research Article

Analysis of Signal Attenuation of Continuous Wave in Drill String

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Abstract: In this study, the friction model in non-Newtonian drilling fluid is developed to evaluate the signal attenuation in the information transmission with the continuous wave. A model of transient non-Newtonian power-law pipe flow is developed by assuming a steady viscosity varied only with the radius and the solution is derived analytically in complex domain and time domain. The frequency-dependent friction is developed based on the solution in the time domain and is used in the pressure wave transmission. And the analysis results show that the highest pressure amplification with resonant frequency increases with the power-law index *n* increase and the resonant frequency increases with the *n* decrease.

Keywords: Continuous wave, frequency-dependent friction, non-Newtonian, propagation constant

INTRODUCTION

Mud pulse telemetry has been the global standard for real time data transmission in the Measurement While Drilling/Logging While Drilling (MWD/LWD) technology for the past thirty years (Klotz *et al*., 2008a). The data rate of the mud pulse telemetry had reached to 6bits per second (bps) in the 1990s (Martin *et al*., 1994), then this technology reached mature and now the data rate is up to 20 bps (Klotz *et al*., 2008a, b). The practical limit of 50 bps with a 35 Hz bandwidth fairly clear of noise is estimated by Montaron *et al*. (1993). However, the theoretical analysis of data rate limit for the mud pulse telemetry has not been found in the relevant literature. For the same bandwidth, with the new measurement technique, signal generator type and data compression technique (Bernasconi *et al*., 1999), the maximum data rate can be improved. Therefore, the analysis of the maximum limit for the data rate makes no sense and the analysis of bandwidth limit is theoretical analysis in this study.

In the earlier studies about the pressure pulse transmission, the signal attenuation is based on the friction coefficient of the Newtonian fluid (Chen and Aumann, 1985; Desbrandes *et al*., 1987; Kytomaa and Crosso, 1994). Wang *et al*. (2009) developed a frequency-dependent friction model for the non-Newtonian drilling fluid, in this model; the shear rate of the Newtonian fluid was used in the constitutive equation of non-Newtonian fluid to calculate the frequency-dependent friction.

The attenuation models of the pressure wave cited above have been limited to the Newtonian fluid which cannot fully reflect the transmission characteristics of a pressure wave transmitted in the non-Newtonian fluid flowing. In this study, the frequency-dependent attenuation model for the non-Newtonian power-law fluid is developed based on the approach developed by Zielke (1968) for Newtonian laminar flows.

MATHEMATICAL MODEL

Governing equation: In order to simplify the problem, the geometry is defined as a long horizontal pipeline in which the axial coordinate is *x* and radial coordinate is *r*. By assuming the fluid motion of the power-law fluid inside the drill string as incompressible and laminar, the motion equation for the unsteady-state flow can be written as:

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t} + \left(\frac{\partial \tau}{\partial r} + \frac{\tau}{r} \right) \tag{1}$$

where, u is the axial velocity, ρ, the drilling fluid density, p, the pressure, τ, the shear stress, t , the time, x, the axial coordinate, r, the radial coordinate.

The rheological equation used to represent power-law fluids in cylindrical coordinates can be written as:

$$\tau = -K \left(\frac{\partial u}{\partial r} \right)^n = -\mu_a \frac{\partial u}{\partial r} \tag{2}$$

where,
K = The consistency coefficient
n = The power-law index

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μ_a = The apparent viscosity which can be expressed as

$$\mu_a = K\left(\frac{\partial u}{\partial r}\right)^{n-1} \tag{3}$$

Substituting the Eq. (2) into the Eq. (1), we can get a nonlinear partial differential equation which is difficult to be solved analytically. Before the pressure wave traverses the drilling fluid in the drill string, the flow of the drilling fluid has been steady and the pressure wave will disturb the steady flow of the drilling fluid. In fact, for the information transmission, the amplitude of the pressure wave is small, so that the velocity disturbance and shear rates caused by the pressure wave are smaller than that of the steady-state flow. Therefore, in this study the shear rate in steady power-law flow is used in the apparent viscosity μ_a.

Substituting Eq. (2) into the Eq. (1), the motion equation is written as:

$$\frac{1}{\mu_{a0}}\frac{\partial p}{\partial x} = r^{\frac{n-1}{n}}\frac{\partial^2 u}{\partial r^2} + \frac{2n-1}{n} r^{\frac{-1}{n}}\frac{\partial u}{\partial r} - \frac{\rho}{\mu_{a0}}\frac{\partial u}{\partial t} \tag{4}$$

where, the μ_{a0} is similar to the dynamic viscosity of the Newtonian fluids and is defined by:

$$\mu_{a0} = K^{\frac{1}{n}}\left(\frac{f}{2}\right)^{\frac{n-1}{n}} \tag{5}$$

In which, f is the pressure gradient of the steady flow.

Boundary conditions: The following equations represent the boundary conditions for all time which must be applied to solving the governing equation:

$$u\big|_{r=R} = 0 \tag{6}$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = 0,\quad u\big|_{r=0} = u_m \tag{7}$$

Solution: Equation (4) can be converted into ordinary differential equations by means of the Laplace transform and can then be integrated subject to appropriate boundary conditions. The Laplace transform of Eq. (4) can be expressed as:

$$\frac{1}{\mu_{a0}}F(s) = r^{\frac{n-1}{n}}\frac{d^2U(s,r)}{dr^2} + \frac{2n-1}{n} r^{\frac{-1}{n}}\frac{dU(s,r)}{dr} - \frac{\rho}{\mu_{a0}} sU(s,r) \tag{8}$$

where, $F(s) = \frac{\partial P(s,x)}{\partial x}$ is the Laplace transform of $\frac{\partial p(t,x)}{\partial x}$, $U(s, r)$ is the Laplace transform of u and s is the Laplace transform variable.

The general solution to the Eq. (8) can be written as:

$$U(s,y) = C_1 I_\sigma\left(\frac{1}{\sqrt{m}}y\right) y^{\frac{m-1}{2m}} + C_2 K_\sigma\left(\frac{1}{\sqrt{m}}y\right) y^{\frac{m-1}{2m}} - \frac{F(s)}{\rho s} \tag{9}$$

where, C_1 and C_2 are constants of integration to be determined from the boundary conditions, I_σ and K_σ are modified Bessel functions of first and second kinds and σ order and

$$\sigma = \left|\frac{1-n}{1+n}\right|,\quad m = \frac{n+1}{3n-1}$$

$$y = \sqrt{\frac{\rho s}{\mu_{a0}}}\frac{2n}{3n-1}\frac{1}{\sqrt{m}} r^{\frac{n+1}{2n}} \tag{10}$$

With the boundary conditions in Eq. (6) and (7), the constants of integration in Eq.(9) are:

$$C_2 = \frac{2\sin(\sigma\pi)F(s)y_R^{\frac{1-m}{2m}}}{\pi\rho s I_{-\sigma}\left(\frac{y_R}{\sqrt{m}}\right)},\quad C_1 = \frac{F(s)y_R^{\frac{1-m}{2m}}}{\rho s I_{-\sigma}\left(\frac{y_R}{\sqrt{m}}\right)} \tag{11}$$

And finally, the Eq. (9) is transformed as:

$$U(s,y) = \left(\frac{I_{-\sigma}\left(\frac{y}{\sqrt{m}}\right)y^{\frac{m-1}{2m}}}{I_{-\sigma}\left(\frac{y_R}{\sqrt{m}}\right)y_R^{\frac{m-1}{2m}}} - 1\right)\frac{F(s)}{\rho s} \tag{12}$$

In which:

$$y_R = \sqrt{\frac{\rho s}{\mu_{a0}}}\frac{2n}{3n-1}\frac{1}{\sqrt{m}} R^{\frac{n+1}{2n}} \tag{13}$$

FREQUENCY-DEPENDENT FRICTION

Pressure wave transmitted inside the drill string: In the continuous wave information transmission, the pressure wave inside the drill string can be seen as the periodic flow in the steady pipe flow and only the periodic flow plays the leading role. The amplitude of velocity oscillation for the periodic flow is smaller than the velocity in the steady pipe flow, therefore, in the periodic flow, a viscosity varied with radius but not varied with time is used.

The time domain solution for the periodic flow is obtained using the inverse Laplace transform of the Eq. (12) by partial fraction expansion method. the pressure gradient varies sinusoidally with time:

$$\frac{\partial p(t,x)}{\partial x} = \rho P_p \exp(i\omega t) \rightarrow F(s) = \frac{\rho P_p}{s - i\omega} \tag{14}$$

For the periodic flow, only the real part of the solution gives the physical value and for a long time steady oscillation, the transient term of the solution can be neglected. Therefore, for the long time steady periodic flow of non-Newtonian fluid, the result can be simplified as:

$$u(r,t)=\frac{P_p}{i\omega}\left(1-\frac{J_{-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}r^h\right)r^{h-1}}{J_{-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}R^h\right)R^{h-1}}\right)\exp(i\omega t) \quad (15)$$

where, P_p is the amplitude of the sinusoidal pressure gradient, ω is the angular frequency. t is time, j is Bessel function of first kind and $h=\frac{n+1}{2n}$, $v=\frac{\mu_{a0}}{\rho}$. The h is a parameter relating to the power-law index n, the larger the n deviates from 1, the more obvious the non-Newtonian characteristic will be.

Frequency-dependent friction: Base on the Eq. (15), the mean velocity and the wall shear stress are

$$\overline{U}(t)=\left(1-\frac{1}{\Pi_{1-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}R^h\right)}\frac{2}{h}\right)\frac{P_p}{i\omega}\exp(i\omega t) \quad (16)$$

$$\tau_w(t)=K\left(\frac{R^{2h-1}P_p}{vh}\frac{\exp(i\omega t)}{\Pi_{1-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}R^h\right)}\right)^n \quad (17)$$

In which the function $\Pi_{1-v}(z)=z\frac{J_{-\sigma}(z)}{J_{1-\sigma}(z)}$ is defined as a modified quotient of Bessel functions of $1-\sigma$ order.

The frequency of τ_w is physically same as the $\overline{U}$(t), however, the frequency of the real part of Eq. (17) is n times of the frequency of $\overline{U}$(t). If the real part in the bracket is only used, for some values of n, such as n = 2, the τ_w is non-negative, which are contrary to the fact. Therefore, the steady viscosity is used here and according to the Eq. (2),τ_w can be derived as:

$$\tau_w(t)=\frac{\omega\rho R}{2}\frac{1}{\eta}\mathrm{Re}\overline{U}(t)+\frac{\rho R}{2}\xi\frac{\partial\,\mathrm{Re}\overline{U}(t)}{\partial t} \quad (18)$$

$$\frac{1}{\eta}=\mathrm{Re}\left(\frac{\frac{2}{h}i}{\Pi_{1-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}R^h\right)-\frac{2}{h}}\right) \quad (19)$$

$$\xi=\mathrm{Im}\left(\frac{\frac{2}{h}i}{\Pi_{1-\sigma}\left(\sqrt{\frac{-i\omega}{v}}\frac{1}{h}R^h\right)-\frac{2}{h}}\right) \quad (20)$$

Equation (18) is the frequency-dependent friction of the non-Newtonian power-law pipe flow, which varied with time, frequency, power-law index, pipe radius, consistency coefficient and acceleration of the fluid.

FREQUENCY-DEPENDENT FRICTION USED IN THE PRESSURE WAVE ATTENUATION

The equations of the pressure wave propagation are used in a simplified form by neglecting the nonlinear terms and then the motion equation and continuity equation become:

$$\frac{\partial p}{\partial x}+\frac{\rho}{\pi R^2}\frac{\partial Q}{\partial t}+\frac{2\tau_w}{R}=0 \quad (21)$$

$$\frac{\partial p}{\partial t}+\frac{\rho a^2}{\pi R^2}\frac{\partial Q}{\partial x}=0 \quad (22)$$

where, Q is the flow rate, a is the wave speed in the drilling fluid, its calculation can be found in the study by Liu *et al*. (1999).

Substituting Eq. (18) into Eq. (21), the motion equation becomes:

$$\frac{\partial p}{\partial x}+\frac{\rho}{\pi R^2}(1+\xi)\frac{\partial Q}{\partial t}+\frac{\rho\omega}{\pi R^2\eta}Q=0 \quad (23)$$

Let:

$$R_f=\frac{\rho\omega}{\pi R^2\eta},\quad L_f=\frac{\rho}{\pi R^2}(1+\xi),\quad C_f=\frac{\pi R^2}{\rho a^2} \quad (24)$$

The propagation constant $\gamma=\sqrt{C_f\omega(-L_f\omega+iR_f)}$ becomes:

$$\gamma=\frac{\omega}{a}\sqrt{-(1+\xi)+\frac{i}{\eta}} \quad (25)$$

where, R_f, L_f, C_f are fluid resistance, fluid capacitance and fluid inductance respectively they are equivalent to the ones used in the analysis of electric transmission lines.

The characteristic impendence is:

$$Z_C=\sqrt{\frac{R_f+iL_f\omega}{iC_f\omega}} \quad (26)$$

Z_c Is the function of the physical properties of the pipe radius, oscillation frequency and consistency coefficient and power-law index.

As shown in Fig. 1, if the values for the p and Q at one cross section x_1, the values at another cross section x_2 can be calculated using the following transfer functions:

$$\begin{bmatrix}p(x_2)\\Q(x_2)\end{bmatrix}=\begin{bmatrix}\cosh(\gamma l) & -Z_C\sinh(\gamma l)\\ -\frac{\sinh(\gamma l)}{Z_C} & \cosh(\gamma l)\end{bmatrix}\begin{bmatrix}p(x_1)\\Q(x_1)\end{bmatrix} \quad (27)$$

where, l is the distance between cross section x_1 and x_2.

Fig. 1: Cylindrical pipe with flow rate and pressure at two ends

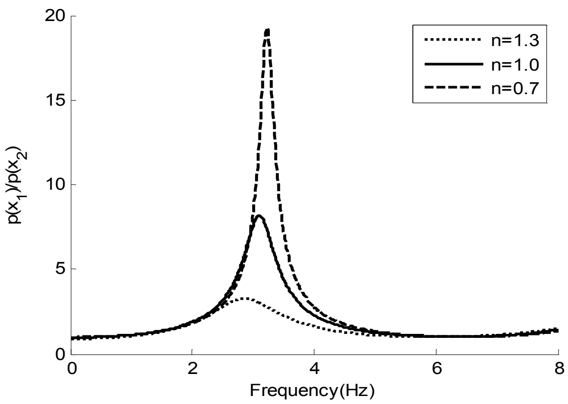

Fig. 2: Ratio of the pressure amplitude between upstream and downstream versus frequency for different n

For the pressure wave transmission, the flow rate of the system in the upstream and the pressure amplitude generated by the mud pulser in the downstream are known and the pressure amplitude at the standpipe in the upstream is that we are interesting, which can be written as:

$$p(x_1) = \frac{p(x_2) - Q(x_1) Z_C \sinh(\gamma l)}{\cosh(\gamma l)} \tag{28}$$

The Eq. (28) is derived by assuming the drilling fluid channel is a long horizontal pipeline without sudden expansions, contractions, interfaces and other fluid component. In this study, we just want to get the influence of non-Newtonian fluid on the attenuation and frequency characteristic of the drilling fluid channel.

Figure 2 shows the numerical results obtained from Eq. (28) with the frequency dependent terms for L_f and R_f. It can be seen that the highest pressure amplification with resonant frequency increases with the power-law index *n* increase and the resonant frequency increases with the *n* decrease.

CONCLUSION

- A model of transient non-Newtonian power-law pipe flow has been developed by assuming a steady viscosity only varied with the radius and the solution is derived analytically in complex domain
- A model of the frequency-dependent friction has been developed based on the solution of the transient non-Newtonian power-law pipe flow in the time domain, which is in good agreement with the Newtonian results arrived by earlier studies for $n = 1$
- The frequency-dependent friction has been used in the pressure wave transmission. And the results show that the highest pressure amplification with resonant frequency increases with the power-law index *n* increase and the resonant frequency increases with the n decrease.

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