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Research Article

Technical Analysis on Mechanical Model Based Football Curveball

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Abstract: In this study, from the angles the physics and biomechanics, in the case of the curveball generated by the rotation problem analysis and exposition, considering the speed, rotation, the wall, the goalkeeper, goaltender factors and football running track and the theory trajectory deviation factor, making the model as much as possible to simulate the actual effect and using the MATLAB software to draw the flight of the ball trajectory simulation. Reference designed for teaching, training and competition as well as to further deepen the awareness and understanding of football curveball. It can improve the free kick guidance and not only help the shooter to select the best shooting methods, but also for the goalkeeper has targeted to fighting with reference.

Keywords: Mechanical model, simulation trajectory, the curveball

INTRODUCTION

In 2005, China and Panama's competition Lulin free kick that directly remain fresh, the ball and the Holland team goal and was rated as the best goal of the FIFA World Youth championship. Well, how to create such a wonderful world "wave"? Which not only has a football player superb footwork as well as a certain luck factor, do you think there is also contains a wealth of physics and mathematical principles.

The modern football sport is developing the comprehensive and rapid, national team treat the ball as one of the important means of scoring. In a variety of positioning the ball, the positioning of the arc trajectory ball is more powerful, its flight trajectory is banana, commonly known as the "banana ball". It is due to the different contact with the ball of the foot lateral, medial and a front, forming a different direction of rotation of the ball in flight. Playing well "banana ball" should be fast, strong spins, the penalty can be around the defensive wall and to the harsh angle to send the ball. In the 12th World Cup, the French team in the game with the Austrian team, a player kicked out of the "banana ball " around the wall from the far corner of the net nets, laid the foundation for the French team's victory, banana ball " might thus can peep one spot. Therefore, from the 12th World Cup match, the curve ball technique is popular zed in the soccer game, so that the flight path of the ball change constantly, the ball can be around the defensive wall, with harsh angle to make impossible to guard against network. Because the curveball has a strong offensive and cleverly concealed, national teams treat it as the important means of scoring. Principles of mechanics and technical essentials of the curveball, China has conducted a lot of research. Among

them, Li Shuping at the university "Sports Biomechanics" tutorial book in sports fluid mechanics problems expounds the football ball rotation generation and motion trajectory. He pointed out that if rotation in a fluid motion ball, it will do a curvilinear motion (Gelong, 1993); Harbin Institute of Technology Liu Dawei sphere flight path "College Physics" abnormal explore pointed sphere flying in the air campaign to generate curvilinear motion in the horizontal direction of rotation occur parabolic movement does not involve the sphere rotating with the ball through the air, the air is fluid and football movement is bound to be some changes in the fluid (Gelong, 1991). Research on the curveball the most noteworthy Jiangxi Normal University Graduate Xiong Zhifeng, the world famous football player, former England captain Beckham Bezier curve ball technique, biomechanics characteristic and competitive ability analysis of Beckham curve ball technique and technical characteristics of Liu (1987), for Beckham type training, competition and study the world's advanced curve ball technique to provide a reference for the development. As a result, it can be seen that involves multiple aspects curveball, curveball in the importance of the various competitions are obvious. The following analysis and calculation by the principles of physics and mathematics, scientific analysis of the sports issues of the kick (Liu and Zoubao, 1998).

PARAMETER CHOICES AND ASSUMPTIONS OF THE STUDY

The basis of the selection of the parameters: After a lot of kick score directly observed, discovered several law as the basis of the simplified model:

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The goal of the multi-location close to both the right and left of the uprights.

Goalkeeper standing goal at the beginning of the central, the center of the wall roughly connection goalkeeper and striker.

The wall roughly perpendicular to the shooter and the goal line connecting the center.

If the goalkeeper to make saves reaction, they may take the "diving" type of scrambling.

Prior goalkeeper will be directing the wall, so that they can see the ball.

Environmental assumptions:

- In the process of the football environmental factors did not change (such as: air viscosity η).
- The rotational speed ω of the ball in the football process unchanged.
- Football in the course of the campaign does not come into contact with the ground.
- Does not consider the role of air buoyancy and wind.
- The wall is equivalent to the "wall" that does not consider the gap between the players head and head, squeezed between the players is very tight, there is no gap, but does not consider the deformation caused by squeezing and that between the player's actions consistent.
- Wall and perpendicular to the shooter and the goal line connecting the center.
- Goalie to start the reaction time and the time of take-off of a wall with the striker foot time consistent goalkeeper sight is not taken into account the wall, so that the goalkeeper after a reaction time begins to move.
- Goalie movement in one plane.
- Goalkeeper football contact with at least one football radius thinks he can get the ball or dragged to the bottom line, or that the fighting failed.
- That the goalkeeper's reaction time to meet the normal distribution.
- Taking into account the goalie reaction time less description of his state, since the state is a good jump must be high, so the approximation that the reaction time and the take-off of the product of the maximum height of a certain value.
- The shooter shot normal logic (rely on which side of the radio, close to which side of the goal) and goalie reaction time for normal; "surprise" (depend on which side of the shot, close to the other side of the goal) that goalie reaction time is doubled.
- When you the shooter choose to play strategy, choose from a wall beneath drilled, nor consider the factors of the second attack.

Data description:

• Some data which varies from person to person (for example, after the goalkeeper outstretched arm length), for the sake of consistency, certain values in accordance with the general situation.

 In addition, the football rotation angular velocity ω in peacetime no one cares, I just repeated viewing some video later combined with their own experience to make the estimates, there may be some discrepancies with the actual situation.

FOOTBALL FLIGHT TRAJECTORY MECHANICS MODEL

Football trajectory in the air: Access to information that, when the movement of the objects in the fluid, the fluid adhering to the surface of the object to move together with the object, so that the surface of the object to produce relative movement between the fluid layer and the fluid near the viscous force generated by such relative movement will impede the motion of the object. Laterally is proportional to the rate of change of the viscous forces and the relative velocity of the fluid layer and the relative motion is proportional to the contact area, so the relative velocity of the fluid and the object of the viscous force and the object is directly proportional to the surface area of both. For the movement of the ball in the fluid, viscous forces:

$$F_f = 4\pi r^2 \eta \left(\frac{dv}{dr}\right) \approx 4\pi r v \eta$$

Stokes formula: v small (football speed in our view, is very fast, but also within the context of physics is a smaller speed), pressure drag proportional to v and viscous half of the force, then:

$$F_f = F_{pv} + F_{fv} = 6\pi\eta \, rv$$

Take v = 27.8 (100 km/h), be regarded as resistance is approximately 0.01422 cattle, shows that within a limited distance, resistance to impact on the speed of a very small, as well as to facilitate the calculation, in the study, this study ignores the role of the air resistance. When the movement of the spherical object around its own rotation in the fluid, due to the viscous force of the objects surrounding the formation of circulation, resulting in the object is formed around asymmetry under steady flow, the formation of a moving direction of the object and self are perpendicular to the lateral force in the rotating axis direction - referred to herein as "swirl force", "Magnus effect". For the sphere in terms of the Magnus effect roundabout force size:

 $F_l = \pi \rho v r^3 \omega$

In the vertical plane, soccer, by the swirl force and the influence of gravity, due to the swirling direction of the force is always perpendicular to the direction of the velocity, it is directly calculated trajectory is very difficult to speed decomposition of the original, wherein the movement of a sub swirl force and gravity phase offset. Based on the force balance: $v = \frac{mg}{\pi\rho\omega r^3}$.

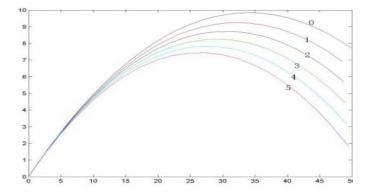


Fig. 1: Matlab rotating sphere flight path

Speed vertical decomposition easily obtained by the following formula:

$$v' = \sqrt{(v_0 \cos \theta + v)^2 + (v_0 \sin \theta)^2}$$
$$tg \alpha = \frac{v_0 \sin \theta}{v_0 \cos \theta + v}$$

In a vertical direction and the z-axis, d that football starts moving after the projective length in the horizontal plane, the initial position as the origin, to establish a Cartesian coordinate system and then synthesis principle according to the displacement and the two decomposition velocity generating uniform linear motion and uniform circular motion together, we can obtain the following formula:

$$d = R \left[\sin \alpha - \sin(\alpha - \omega' t) \right] - vt$$

$$z = R \left[\cos(\alpha - \omega' t) - \cos \alpha \right]$$

In addition, when the football is not rotated, it is easy to obtain the equation:

$$z = xtg \ \theta - \frac{g}{2v_0^2 \cos^2 \theta} x^2$$

Order to analyze the difference between rotating and non-rotating, I take a special set of data: $\theta = 60$ degrees early speed from 50 km / ω 0 has been by way of 5. From top to bottom painted with Matlab Fig. 1:

Figure 1, we can find that the difference is not large, with or without a rotating just pilling, close to the highest point, a few track separation, the rotation of the ball quicker than non-rotating whereabouts and the faster rotation, the whereabouts of the faster, if the goalkeeper is estimated in accordance with the general parabolic ball, it will lead to promote off guard.

Because air resistance is ignored after the football in the horizontal direction only under the action of the "swirl force" uniform circular motion, this case is much simpler than the case in a vertical plane.

We can easily obtain the following parametric equations: gyro radius cyclotron $R = \frac{mg}{\pi\rho\omega r^3}$ angular velocity $\omega^{\sim} = \frac{v^{\sim}}{r}$:

$$x = R \left[\sin \phi - \sin(\phi + \omega' t) \right]$$

$$y = R \left[\cos \phi - \cos(\phi + \omega' t) \right]$$

Plan circle equation:

$$(x - R\sin\phi)^2 + (y - R\cos\phi)^2 = R^2$$

Taking the above points, we can draw on the time t in the unified coordinate parameter equation: In the midpoint of the goal line as the origin, the direction of the goal line to the y-axis, perpendicular to the direction of the goal line for the x-axis, the vertical direction the zaxis: Globe initial position is disposed football (x_0, y_0, r), In the transformation of the reference system, the new parameter equation of time t as follows:

$$x = x_0 - R_{xoy} [\cos \phi - \cos(\phi + \omega'_{xoy} t)]$$

$$y = y_0 + R_{xoy} [\sin \phi - \sin(\phi + \omega'_{xoy} t)]$$

$$z = r + R_c [\cos(\alpha - \omega'_c t) - \cos \alpha]$$

Plane circle equation becomes:

$$(x - x_0 + R_{xoy} \cos \phi)^2 + (y - y_0 - R_{xoy} \cos \phi)^2 = (R_{xoy})^2$$

 x_0, y_0, θ, ϕ Known quantity, the rest of the amount is determined in accordance with the following equation:

$$R_{xoy} = \frac{mv_0 \cos \theta}{\pi \rho \omega_{xoy} r^3}$$

$$\omega'_{xoy} = \frac{v_0 \cos \theta}{R_{xoy}}$$

$$R_z = \frac{mv'}{\pi \rho \omega_z r^3}$$

$$\omega'_z = \frac{v'}{R_z} v' = \sqrt{(v_0 \cos \theta + v)^2 + (v_0 \sin \theta)^2}$$

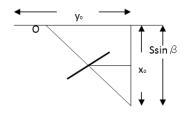


Fig. 2: Wall, goalkeeper, striker diagram

$$tg \alpha = \frac{v_0 \sin \theta}{v_0 \cos \theta + v} \quad v = \frac{mg}{\pi \rho \omega_z r^3}$$

We can be found in the horizontal plane, rotating the faster, more curved trajectory football around the wall, this is very helpful.

Field goal constraints: Make the issue of a direct free kick score, the main resistance came from the wall and goalkeeper, will be analyzed below:

Formula based on the acceleration, it is easily obtained, that the wall at the top equation of motion in the vertical direction, the Master:

$$z = h + v_2 t - \frac{1}{2} g t^2$$

According to assumptions wall and center in the connection of the goalkeeper and striker and the rules of football: the wall must be at least leave the shooter 9.15 m. Although the "at least", but the wall away from the shooter nearer the blockade angle, so in the actual game, the wall is generally as close as possible to or even less than this value, so later in this study do not consider "at least" that 9.15 value. Then $x = x_0 - s \sin \beta$, β is the angle between the shooter and the goal line connecting the center with the goal line, Fig. 2.

Based on the principle of similar triangles and Fig. 2 is easy to come to the coordinates of the center of the wall in the y-axis:

$$y_0 - \frac{y_0}{x_0} s \sin \beta$$

The position of the wall in the horizontal plane is two points of connection, these two points are:

$$(x_{0} - s\sin\beta - \frac{n}{2}e\cos\beta , y_{0} - \frac{y_{0}}{x_{0}}s + \frac{n}{2}e\sin\beta)$$
$$(x_{0} - s\sin\beta + \frac{n}{2}e\cos\beta , y_{0} - \frac{y_{0}}{x_{0}}s - \frac{n}{2}e\sin\beta)$$

Plane linear equation:

$$\left(y - y_0 + \frac{y_0}{x_0}s\sin\beta\right) = -tg\beta(x - x_0 + s\sin\beta)$$

The points obtained by plugging a wall, but also hit the goal within the constraint equations: $f(t_0) = 0$, $f(t_1) = x$

where in the value of x 'is a linear equation of the intersection of the abscissa of the plane equation of a circle with the wall to enter the goal range must be met:

$$|g(t_0)| \le \frac{b}{2} - r \quad r \le h(t_0) \le a - r$$

From the top of the wall, must satisfy the following requirements: $h(t_1) \ge r + h + v_2t_1 - \frac{1}{2} gt^2_1$, On the y-direction is not required.

From around the wall next to be met: $g(t_1) \le y_0 - y_0/x_0 \ s - (n/2e + r) \sin \beta$ or $g(t_1) \le y_0 - y_0/x_0 \ s - (n/2e + r) \sin \beta$, Z direction is not required.

Satisfy the above formula, there exists the possibility to score, but the above does not take into account the goalkeeper factors. Hereinafter discussed in this analysis due to the more complex factors goalkeeper.

Based on the trajectory of the football in the vertical plane (Fig. 1) can be known: the football in the rising phase rotation and no rotation difference is not large, football through the wall, must be in the rising phase, in this case, reaching the wall, you can approximate the trajectory parabola to simplify the model.

According to the previously derived equation:

$$R_{xoy} = \frac{mv_0 \cos\theta}{\pi \rho \omega_{xoy} r^3} \omega'_{xoy} = \frac{v_0 \cos\theta}{R_{xoy}}$$

Horizontal direction cyclotron angular velocity ω 'and θ , determined, goal time $t_0 = \gamma/\omega$ '. Where γ is the central angle, obviously, to make requirements as soon as possible to reach the goal, the most critical is to make γ small.

The observation level diagram (Fig. 2), if the shooter at point A wanted kicked the ball into (0, y (denoted as point B), so the length of the chord AB settle down, but in the chord length of certain circumstances, for the central angle is small, it is necessary to make the cyclotron radius as large as possible, however $R_{xoy} = \frac{mv_0 \cos\theta}{\pi \rho \omega_{xoy} r^3}$. We can find: the radius of gyration is proportional to the cosine of the angle between the initial velocity and the horizontal, depending on the nature of the cosine, let θ be as small as possible.

If the shooter to select from above through the wall: The smallest θ, apparently is the football just from the people on the wall side grazed corresponding value of θ. Also know football passes over the wall, when the horizontal position of the

goal point settle down after the goal time will settle down, not as we imagine, scoring points higher the more time short (in fact, the level of scoring points is determined by the angular velocity).

To calculate the value of θ , based on the above analysis, we can use "Football is just from the people on the wall side" as an equation and the goal position into equation to solution.

Consider using the parabolic equation $(t_1 \text{ is the time to reach the wall after the wall):}$

$$z = r + v_0 \sin \theta t_1 - \frac{1}{2}gt_1^2 = h + v_2 t_1 - \frac{1}{2}gt_1^2 + r$$

Goal position is (0, y, z), then there is:

Listed in the y direction to obtain the equation: $g(t_0) = y$, t_0 can be calculated by $f(t_0) = 0$. Simplification can get two the equation θ , ϕ , according to the principles of the equations, possible solutions. This allows a view of the goal time t_0 .

If the shooter choose from around the wall next to: Set the goal position in the (0, y and z) (Note, due Football whereabouts stage, so the vertical direction can not be used parabola simplified model) has now been considered with the same reason as above, to make the ball to reach the goal line as soon as possible, it is necessary the radius of gyration of the football in the horizontal plane as large as possible, But now the problem is : from the top of the bypassing of the radius of gyration of the decisive factors is the initial velocity and the angle between the horizontal plane θ corresponding to the minimum, is fired at the bottom, in fact, the bottom is not necessarily a goalkeeper hardest fighting, so we can not think that this is the best strategy. Then, proceed from the other aspects, taking into account the three points of the horizontal plane can be determined a circle, when football clockwise path of movement, a bit of the A, B and another point is $(x_0 - ssin \beta + (\frac{n}{2}e + r) \cos \beta, y_0 = \frac{y_0}{x_0}s - (\frac{n}{2}r + r \sin \beta)$. Easy to know, to make three points determine the radius of maximum, B is (0, b/2+r). The football counterclockwise movement, B is (0, b/2 =-r), the third is $(x_0 - s \sin \beta + (\frac{n}{2}e +$ $r\cos\beta$, $y0=y0x0s-n2e+r sin\beta$ Find three, you can draw the radius of gyration Rxoy of the horizontal plane and then according to the equations:

$$\begin{cases} f(t_0) = 0\\ g(t_0) = \frac{b}{2} - r \text{ (Conterlockwise rotation)}\\ \text{or } \left(-\frac{b}{2} + r\right) \text{ (Clockwise rotation)} \end{cases}$$

This can be solved to reach the time of the goal line t_0 .

• Introducing goalkeeper factors: We first analyze the analysis goalkeeper may arrive within a certain time range: Do not consider the goalkeeper's body length and starts moving is "diving", i.e. with the ground with no contact, so only by the action of gravity, may reach the range is calculated as follows:

For any time t, exzists $\overrightarrow{v_{3t}} = \overrightarrow{v_3} + \overrightarrow{gt}$ On dt integral:

$$\int_{0}^{t} \overrightarrow{v_{3t}} dt = \int_{0}^{t} \overrightarrow{v_{3}} dt + \int_{0}^{t} \overrightarrow{g} t dt$$

It can be reached at t position vector:

$$\vec{s} = \vec{v}_3 t + \frac{1}{2} \vec{g} t^2$$

Considering the goalkeeper length, due to the fighting process position and there is no big change, consider only the goalkeeper hand reaches the range, then, can be approximated those concentric circles pan up a goalie stretching the length of the arm. Of course, we only take that part of the xoy above the plane. This gives the range equation:

$$y^{2} + \left(z + \frac{1}{2}gt^{2} - l\right)^{2} = (v_{3}t)^{2}$$

The formula that, with the passage of time, the scope of the goalkeeper can Saves growing, so the shooter must as soon as possible so that the ball to reach the goal line.

• Calculate the probability of scoring: Disposed football reach the goal line time t_0 , The goalkeeper in the air movement time $t_0 - \Delta t$, Another Δt is changed with the goalie play, easy to know, goalie play better, not only the decrease in reaction time and jump the higher approximation that the maximum height of the jump with the product of the reaction time for a given value, then exist $(v2^3_0/2_g)\Delta t_0 = (v^2_3/2_g)\Delta t_3$, Thus the goalkeeper may reach a range of equation is:

$$y^{2} + \left(z + \frac{1}{2}g(t_{0} - \Delta t)^{2} - l\right)^{2} = \frac{\Delta t_{0}}{\Delta t}[v_{30}(t_{0} - \Delta t)]^{2}$$

When t_0 the coordinates of the center of the sphere of football(0, y, z), the goalkeeper just threw herself into a ball.

Since only one unknown, the equation solution, The solution is $\Delta t(k)$, Obviously, when $\Delta t \leq \Delta t(k)$, It saves the success, or other score.

Then calculate the probability of the scoring play index, based on the assumption that, to meet the normal distribution:

$$y = \frac{1}{\sqrt{2 \pi \sigma}} e^{-\frac{\left(\Delta t - \Delta t_0\right)^2}{2 \sigma^2}}$$

where, σ is the standard deviation goalie reaction time.

Through access to information, we can found, σ can also be used the " 3σ principles to determine, Function with 99.7% of the area surrounded by the x-axis between [Δt_0 - 3σ , Δt_0 + 3σ], Δt_0 - 3σ and Δt_0 + 3σ Respectively in normal circumstances the minimum and maximum, consider minimum is 0, then $\sigma = 0.05$.

Note that the reaction time is of no practical significance, in fact, this probability is very low. When the location used to reach the goal line time and football is known, according to the equation:

$$y^{2} + \left(z + \frac{1}{2}g(t_{0} - \Delta t)^{2} - l\right)^{2} = \frac{\Delta t_{0}}{\Delta t}[v_{30}(t_{0} - \Delta t)]^{2}$$

Can solve Δt , the probability of scoring $p = \delta(\Delta t)$, $\delta(\Delta t)$ value $\tau(\Delta t - \Delta t_0/\sigma)$, $\tau(u)$ can by inspection standard normal distribution table.

The best strategy of the soccer game: Through the analysis, we can know that the optimal incident point only 4 possible, which is the 4 " dead", our main research is between them 4 merits. Following consider only the vertical or horizontal direction of the angular velocity that passes over the wall of the ball only in the vertical plane within the angular velocity, then ball around the wall from the next only in the horizontal plane within the angular velocity. Further, since the kicked out of the vertical plane of the angular velocity and kicked out of the horizontal plane of the difficulty of the angular velocity of the relationship can not be learned, so both cases are calculated separately.

Excellent index are defined: $\psi_0 = \Omega \psi_1 + (1 - \Omega)\psi_2 \psi_1$ is Torture goalkeeper Index, equal 4 probability values do standardized xpressions:

$$\psi_{1i} = \frac{p_i - \sum_{i=1}^{4} p_i}{\max\{p_i\} - \min\{p_j\}}$$

 ψ_2 is Striker easily index, Defined as four angular velocity values do the opposite number expressions standardized(because the smaller angular velocity the easier to kick):

$$\psi_{2i} = \frac{\sum_{i=1}^{4} \omega_i}{\max\{-\omega_j\} - \min\{-\omega_j\}}$$

Finally, the program is which ψ_0 is maximum.

CONCLUSION

Good results and efficiency are closely linked and science is the best method to improve the efficiency, sports is also like this, national annually invest a lot of material resources, manpower engaged in sports scientific research purpose is also obviously. In this study, it has no direct role to improve the level of football players, but it obvious that has a significant guiding significance, which help the shooter choose the best shot, also provide a reference for the goalkeeper to fight the target. If a team kicks can handle good and enhance the performance of an inevitable thing.

Although this model is only for football, but has very good popularization. First of all, almost all with rotation of the ball games can use this model to apply, Such as: golf, baseball, etc. There are a lot of similarities with the model. In baseball, the pitcher corresponds to Sagittarius, the batter corresponding wall and the keeper, the strike zone corresponding to the doorframe, only need to modify the corresponding data, then according to the actual situation of changing the constraint conditions and does some useful details; even the strategy selection can also use this model to formulate the scheme. In fact, the model can also be extended to a wider range, satisfied Departure-Around obstacles-Reach the designated regional movement process. Just more constraint conditions are to be modified

EVALUATION OF THE MODEL

- Strong universality, as long as they are not in strong winds heavy rain and extreme weather in competition, the models are suitable.
- Add a lot of humanized variable (Such as: goalkeeper to play, strength index).
- Strong practicality, easy programming.

Main innovations:

- The mathematical model to fit the person's activities have always been a difficult mathematical modeling, In this study, considering the ball speed, rotation, obstruction of a wall, the state of the goalkeeper, the goalkeeper's physique and experience the power of the shooter and the accuracy of such factors, make the model as much as possible to achieve the simulation of the actual effect.
- Strength index is introduced, not only to make the model more value in use, but also greatly reduce the dependence of the model for the original data.

Dealing with football in the angular velocity of the almost no real data quantity, the angular velocity as a strategy selection basis rather than a known quantity, by solving equations to make the football at a certain point of the angular velocity and use the standard method to avoid the quantitative analysis, only to compare size qualitative.

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