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Research Article

Study on Fractal Characteristics of Cracks and Pore Structure of Concrete based on Digital Image Technology

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Abstract: Based on the fractal theory, this study presents a numerical analysis on the fractal characteristics of cracks and pore structure of concrete with the help of digital image technology. The results show that concrete cracks and the micro pore distribution of concrete are of fractal characteristics and the fractal dimension ranges from 1 to 2. The fractal characteristics of pores in cracked concrete and un-cracked concrete is similar and the former fractal dimension of the micro pore structure is bigger than the later.

Keywords: Concrete cracks, fractal characteristics, image technology, micro pore structure, quantitative evaluation

INTRODUCTION

Concrete is a non-homogeneous composite material with multiple phases at multiple levels. The uncertainty, ambiguity and non-linear properties of concrete exhibited on macro level reflect the complexity and irregularity of its microstructure. Many researches have proved that the microstructure has the characteristics of randomness and discreteness which is very difficult to be described using traditional tests and evaluation method. While these researches also indicate that once the disorder of the parts can be deduced, the performance of the whole, as the combination of each part, is still predictable. The key is to develop a tool to represent the complexity of microstructure and bridge the properties exhibited by parts to the whole. Currently, fractal theory is such an effective method of describing material characteristics at the micro, meso and macro levels.

The definition of fractal was put forward by Mandelbrot (Xie, 1996) as a shape made do parts similar to the whole in some way at 1986. He summarized the essential characteristic of fractal from the relationship between the whole and the part as “self-similarity”. This definition has a good suitability and has been used as criterion to judge whether an object would be of fractal features or not. Fractal theory is called one of most important scientific discoveries in the middle 1970's. It is only established for over 30 years, but the rapid progress and wide application have spread over various fields of civil engineering as it can describe complexity and nonlinearly of a phenomenon with very few parameters.

Xie (1996) was the first researcher who introduced fractal theory to rock engineering field and laid the foundation of “fractal-rock mechanics” theory. The establishment of fractal-rock mechanics marked the application of fractal theory in geotechnical engineering. In recent years, the research extended to concrete. The studies concentrated essentially on fracture damage behaviors of concrete and fractal effects of aggregate grading (Cui and Chen, 2005; Li *et al*., 2003; Liu *et al*., 2003; Xu and Xu, 2002). Beneath the fractal properties of concrete at macro level, more and more attention was put on the fractal performance of concrete at meso and micro levels. At present, measuring and evaluating the characteristic parameters of pores and cracks in concrete has become an important part of concrete material-sciences research. While whether the cracks and pores in concrete are of fractal features is still worth further research.

Due to the discreteness of concrete, the accurate extraction of cracks and pores from concrete is very important for fractal analysis. With the application of digital image processing technology in civil engineering, the researchers established several new methods to identify and extract the cracks or pores based on gray threshold principle, such as maximum entropy threshold method (Yang *et al*., 2002), segmentation based on edge intensity (Du *et al*., 2004), Sobel operator method (Chen *et al*., 2002) and maximum between-cluster variance method (Sun, 2002). But so far there is no universal algorithm that

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can isolate the complete information of concrete cracks or pores automatically and accurately in a complicated situation. Especially when concrete cracks or pores are very small, or the edges of cracks or pores have similar gray scale with the background, it is very possible to misjudge the cracks or pores from background. In addition with the denoising processing after the segment, these methods all state a large error tolerance.

This research will use fractal theory to study the fractal characteristics of cracks and pores in concrete. A digital image processing technology was adopted to segment and extract concrete cracks and pores in high accuracy with Photoshop. And the characteristic parameters of concrete cracks, such as area, fractal dimension and length, were all calculated and programmed with MATLAB. For the same reason, the characteristic parameters of concrete pores were also obtained by this method. And a comparison of fractal dimension between cracked and un-cracked concrete was conducted.

INFORMATION COLLECTION BASED ON DIGITAL IMAGE PROCESSING TECHNOLOGY

Based on digital image processing technology, the quantitative information collection has three steps, including image acquisition, preprocessing and feature extraction (Ringot and Bascoul, 2001). Firstly, the image on target object should be taken and recorded with an image acquisition device. Then, through preprocessing, the object will be extracted from digital image. Finally, the feature of the object should be collected and analyzed. The details of each procedure will be discussed as followings.

Image acquisition: Image acquisition is to capture digital pictures to collect digital information of the object. While in the captured picture, only the digital information on the object is useful. All the information exceed the target region is meaningless for characteristics analysis which is named as background. So the captured pictures should reflect the original characteristics of the object as accurately as possible. And at the same time, the characteristics of the object should be highlighted to be easily recognized from the background. Ammouche *et al*. (2000) proposed to increase the contrast between the target object and the background through pretreating the sample by dyeing. But the operation of dyeing is tedious and not easy to handle. Moreover, dyeing may cover the original appearance and conceal the useful information of the target object. So, generally, with the intense application of high solution image acquisition device, the target object is always directly captured without dyeing.

A reference scale is also very important for image acquisition. The reference scale can be a thread with known length or a plate with known area to provide a standard in describing the geometric parameters of target object. By comparing the reference scale with the target object, the information of target object can be recognized and analyzed.

Image preprocessing: The purpose of image preprocessing is to accurately recognize the object from the picture. The procedure of image preprocessing can be divided into several steps.

Firstly, with the help of commercially available software, Photoshop, the digital picture is converted into a two-dimensional gray-level image. Then the boundary of the object should be detected and captured with the magnetic lasso tool in Photoshop. The region surrounded by the boundary line contains the information of the target object, such as color and shape. In this research, the parameters, such as length, perimeter and area, are used to describe the geometric characteristics of target object. So the region in the boundary line will be snapped and filled with black color. By this way, the target object could be extracted from the picture with the Magic Wand Tool. At the same time, the background could also be extracted and filled with white color using the invert select tool of Photoshop. Generally, more pixels an image contains, more information can be collected and more accurately the object can be represented. As a result, the quality and quantity of the information collected using digital image technology is determined by the resolution of the digital acquisition device.

To numerically analyze the object, the processed two-dimensional grey-level image will be converted to a binary image using numerical analysis software MATLAB. In MATLAB, a binary image can be also taken as a binary matrix. The black pixels in binary image represent the object and their corresponding elements will be set to 0 in the binary matrix. Other elements in the binary matrix will be set to 1 which represents the background in the binary image. Then the information of the object is numerically transferred from the original digital image to the binary matrix. And the features of object can be further extracted quantitatively.

Features extraction: The binary matrix is the numerical expression of the object. The row number and column number of an element in the binary matrix represent the coordinate of the corresponding pixel in the binary image. As the pixel in one image itself has uncertain geometric characteristics, the binary matrix does not contain any specific geometric information of the object. But the binary matrix can clearly provide the structural information of the object. A numerical

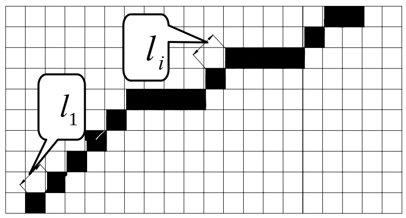


Fig. 1: Binary image of a curve

$$\begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&0&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&0&0&0&0&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1
\end{bmatrix}$$

Fig. 2: Binary matrix of a curve

representation on the length of a curve, or the area of the object, can still be established based on the binary matrix.

As shown in Fig. 1, a curve in the binary image is composed of a series of pixels that are linked to each other. Mathematically, a curve is expressed as a series of adjacent elements in which every element is the only 0-value element in the row or column of the binary matrix (Fig. 2). The distance between two adjacent pixels can be expressed by their coordinates as:

$$l_i = \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2}\, l_p \qquad (1)$$

where, l_i is the distance between two adjacent pixels; l_p is the length of one side of a pixel; (x_i, y_i) and (x_{i+1}, y_{i+1}) correspond to the coordinates of two adjacent pixels.

If there are m pixels on a curve in binary image, the length of the curve l_c can be deduced from Eq. (1) as:

$$l_c = \sum_{i=1}^{m-1} \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2} \cdot l_p \qquad (x_i, y_i) \in C \qquad (2)$$

where, C is the set of pixels on the curve.

If an object overs m pixels in the binary image, the area of the object, S_o, can be mathematically expressed as the sum of all the areas of the pixels, as:

$$S_o = m \cdot l_p^2 \qquad (3)$$

Equation (2) and (3) indicate the geometric feathers of an object are determined by the geometric size of the pixel in binary image. Although the size of the pixel varies in different images, while in this research, its specific geometric properties should be assigned by the reference scale.

Fig. 3: Cracks in concrete slab

If the reference scale is a thread with known length, it will be processed as a curve in binary image. Based on Eq. (2), the side length of one side of pixel can be represented by the length of the reference scale, L_r, as:

$$l_p = \frac{L_r}{\sum_{i=1}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}} \qquad (x_i, y_i) \in R \qquad (4)$$

where,

R = The set of pixels on the reference scale

N = The number of pixels in the reference scale

By the similar way, if the reference scale is a plate and it occupies n pixels in the binary image, the length of one side of a pixel can be represented by the area S_r of the reference scale.

$$l_p = \sqrt{\frac{S_r}{n}} \qquad (5)$$

EXPERIMENTAL RESEARCH

To investigate the fractal properties of concrete, the fractal characteristics of cracks and pores generated form a concrete slab cracking test were analyzed in this research.

The concrete with the strength grade of C30 and w/c (water/cement) ratio of 0.53 was casted into a steel mold with the size of 600 mm×600 mm×63 mm. To induce cracking in concrete, all the four sides of the steel mold were reinforced with two rows of steel bolts. The reinforced steel mold will provide a strained boundary condition on concrete slab. Due to the shrinkage of concrete, the tensile stress will grow gradually. Once the tensile stress exceeds the tensile strength, the concrete slab will crack. After casting, concrete slab was put into a curing room with the temperature of 30°C±3°C and relative humidity of

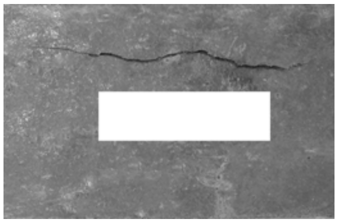


Fig. 4: Original image of concrete crack

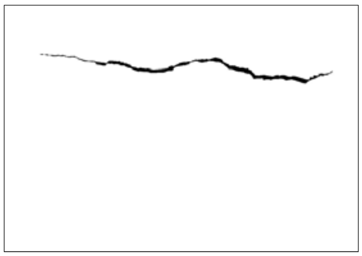


Fig. 5: Binary image of concrete crack

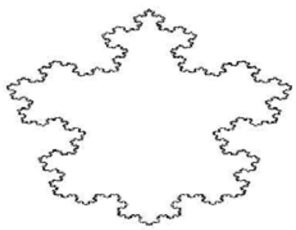


Fig. 6: Koch curve

65%±5%. To accelerate concrete cracking, the wind with the speed of 3m/s was conducted on the concrete slab. At 24 h after casting, as shown in Fig. 3 and 6 cracks were observed using a reading microscope and recorded with a digital camera. The camera used in this research was Canon Power shot A2000IS with the image resolution of 3648×2736 pixels.

FRACTAL FEATURES OF CONCRETE CRACKS

Fractal dimension of concrete cracks: As show in Fig. 4, an image of concrete crack was captured with the digital camera. The reference scale was a piece of paper (white part in Fig. 4) with the size of 70 mm×23 mm. Based on the digital image processing technology; the original image was transferred into a binary image (Fig. 5).

Crack is a narrow opening on the surface of concrete. Generally, the length of crack is much bigger than its width. In a binary image, as shown in Fig. 5, a crack is composed of two curves sharing the same beginning node and ending node and the area between the two curves. In this research, the fractal dimension of concrete crack is calculated and programmed according to the box counting algorithm (Xie, 1996).

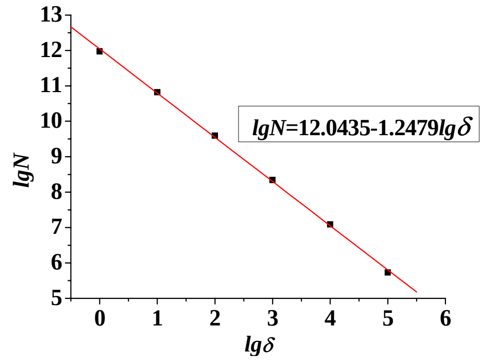


Fig. 7: Fractal dimension of Koch curve

Box counting is to calculate the box dimension by covering the pixels in binary image continuously with newly established sub pixel-matrixes which is named as box (Ding *et al*., 1999). With the given size of the box, δ, the initial number of the boxes, N, is set as 0 in each mesh generation. If the binary value of every pixel in the box is 1, the box represents the background of the image and N will remain unchanged. If the binary value of any pixel in the box is 0, the box, at least partially, covers the crack in the image and N will be reset as $N+1$. Finally, N is determined by traversal of all possible sub pixel-matrixes with the given size of δ. It should be noticed that the value of N corresponds to the size δ. According to fractal theory, the logarithm of N is linearly related the logarithm of δ. And the slope coefficient is the fractal dimension D. So based on the box counting method, the fractal dimension D of concrete crack can be derived from the analysis of binary image.

In the research, the box counting algorithm is programmed with MATLAB. To verify the algorithm, the fractal dimension of Koch curve, as shown in Fig. 6, is calculated. As illustrated in Fig. 7, the calculated fractal dimension of Koch curve is 1.2479. The theoretical fractal dimension of Koch curve is 1.2618 according to its generation principle (Xie, 1996). It can be concluded that the calculated result has a good agreement with the theoretical value and the program can be used to calculate fractal dimension of concrete cracks.

The fractal dimension D_c and the correlation coefficient r_D of concrete cracks are listed in Table 1. All the correlation coefficients of cracks calculated based on fractal theory are close to 1 which indicates that concrete cracks are of fractal natures.

Relationship between length and fractal dimension of concrete cracks: To simplify the calculation, the concrete crack was displayed horizontally during image acquisition. A scanning will be conducted on the corresponding binary matrix from the first column to the last. The coordinate of 0-value elements at the top and bottom in each column will be stored into array P

Table 1: Characteristic parameters evaluation of concrete cracks

Crack No.	D_c	r_D	L_I (mm)	L_{II} (mm)	L_c (mm)
1	1.4035	0.9948	103.821	98.589	100
2	1.3700	0.9946	76.440	73.667	75
3	1.2745	0.9942	111.003	113.786	108
4	1.4193	0.9953	133.152	122.649	130
5	1.3063	0.9952	51.823	53.960	50
6	1.3269	0.9945	42.609	42.108	44

and Q respectively. Based on the image processing technology, array P represented the upper boundary of concrete crack. And Q stated the lower boundary coordinate.

To simplify the calculation, the concrete crack was displayed horizontally during image acquisition. A scanning will be conducted on the corresponding binary matrix from the first column to the last. The coordinate of 0-value elements at the top and bottom in each column will be stored into array P and Q respectively. Based on the image processing technology, array P represented the upper boundary of concrete crack. And Q stated the lower boundary coordinate.

This research provides two methods to calculate the length of concrete cracks. The first method is to calculate the length of upper boundary and lower boundary of concrete crack based on Eq.2. The length of concrete crack is represented as the larger one of these two values as L_I:

$$L_I = \sum_{i=1}^{t-1} \sqrt{(x_{i+1}-x_i)^2 + (y_{i+1}-y_i)^2} \cdot l_p \qquad (x_i, y_i) \in P \; or \; (x_i, y_i) \in Q \tag{6}$$

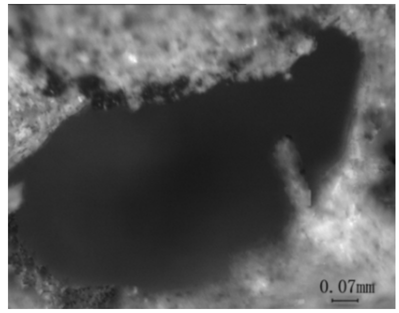
The second method is established based on the fractal theory. The crack can be seen as a tortuous curve joining the beginning point and ending point. And the length of concrete crack is theoretically related with its tortuosity which, according to fractal theory, can be quantitatively represented by its fractal dimension. Then the length of concrete crack L_{II} can be demonstrated by its beginning point (x_a, y_a) and the ending point (x_b, y_b) as:

$$L_{II} = D_c \cdot \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \cdot l_p \tag{7}$$

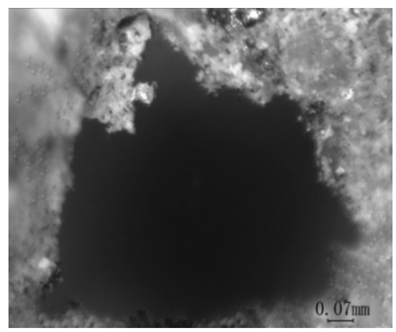
The length of concrete cracks calculated with two methods, L_I and L_{II}, are listed in Table 1. A comparison between the calculated results and measured results L_c indicates that both methods can be used to analyze the length of concrete crack.

FRACTAL FEATURES OF CONCRETE PORES

The pore structure is very important for service performance evaluation of concrete. The durability, shrinkage and mechanical properties of concrete are all influenced by its pore structure. While the pores



(a) Pore in cracked concrete



(b) Pore in un-cracked concrete

Fig. 8 Original images of pores in concrete

distribute disorderly in concrete and its geometric feathers all exhibit, to some extent, irregularity. So the fractal dimension is a very important parameter to represent the characteristic of concrete pores. Although some investigations were conducted on the fractal properties of the pore structures in rocks, clayey soil and sandy soil etc (Moore and Donaldson, 1995; Ma *et al*., 2007; Wang *et al*., 2004; Yuan, 2005), there is no much intensive research on the fractal features of concrete pores.

In this research, the fractal dimensions of pores in cracked concrete and un-cracked concrete collected from the concrete slab were analyzed. The images of pores were taken by an optical microscope, CMS-200, with the maximum magnification of 200 time. The measurement of the optical microscope ranges from 10^{-6} to 10^{-3}m. The original and binary images of pores in cracked concrete and un-cracked concrete were shown in Fig. 8 and 9. It is also shown in Fig. 8 and 9; a scale of 0.07mm is denoted, which can be used as the reference scale.

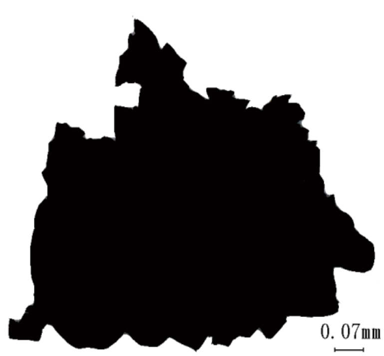
Table 2: Areas and perimeters of pores in un-cracked concrete

Pore No.	Area (mm^2)	Perimeter (mm)	Pore No	Area (mm^2)	Perimeter (mm)	Pore No	Area (mm^2)	Perimeter (mm)
1	0.072	1.043	13	0.372	2.676	25	1.313	4.488
2	0.136	1.430	14	0.424	2.846	26	1.888	4.989
3	0.136	1.483	15	0.409	2.980	27	1.963	5.250
4	0.183	1.558	16	0.700	3.066	28	1.781	5.375
5	0.209	1.712	17	0.635	3.081	29	2.105	5.952
6	0.188	1.913	18	0.638	3.096	30	3.565	8.410
7	0.307	2.092	19	0.815	3.298	31	3.524	9.607
8	0.490	2.287	20	0.604	3.323	32	3.559	9.906
9	0.372	2.331	21	0.841	3.334	33	9.459	16.716
10	0.482	2.347	22	0.891	3.526	34	10.331	19.878
11	0.363	2.365	23	1.061	3.827	35	19.133	26.499
12	0.457	2.453	24	1.449	4.406	36	20.638	33.453

Table 3: Areas and perimeters of pores in cracked concrete

Pore No.	Area (mm^2)	Perimeter (mm)	Pore No	Area (mm^2)	Perimeter (mm)	Pore No	Area (mm^2)	Perimeter (mm)
1	0.001	0.149	13	0.004	0.300	25	0.027	1.495
2	0.002	0.172	14	0.005	0.300	26	0.096	1.812
3	0.002	0.183	15	0.005	0.310	27	0.467	3.207
4	0.002	0.184	16	0.010	0.436	28	0.458	3.612
5	0.002	0.194	17	0.010	0.462	29	0.507	4.021
6	0.002	0.207	18	0.009	0.494	30	0.524	4.857
7	0.002	0.215	19	0.012	0.532	31	1.338	6.141
8	0.003	0.231	20	0.012	0.611	32	1.712	15.565
9	0.002	0.248	21	0.008	0.665	33	1.887	17.557
10	0.004	0.252	22	0.026	0.782	34	2.438	19.576
11	0.005	0.258	23	0.021	0.964	35	4.150	26.376
12	0.004	0.299	24	0.052	1.355	36	7.560	35.192

(a) Pore in cracked concrete



(b) Pore in un-cracked concrete

Fig. 9: Binary images of pores in concrete

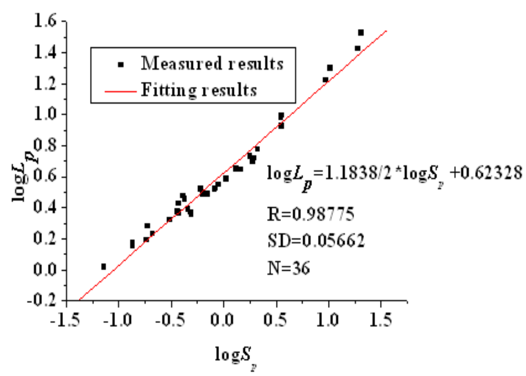


Fig. 10: Relationship between L_p and S_p of pores in un-cracked concrete

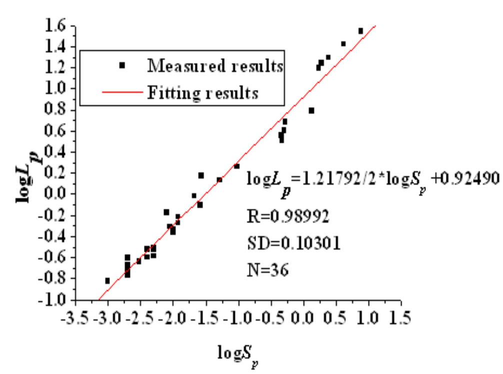


Fig. 11: Relationship between L_p and S_p of pores in cracked concrete

A total of 36 pores were captured from the cracked and un-cracked concrete by the optical microscope respectively. The areas and perimeters of all the pores were calculated with Eq. (2) and (3) based on image processing technology. The calculated results are listed in Table 2, 3.

As reported by Voss *et al*. (1985), if a pore is of fractal properties, the logarithm of perimeter L_p is determined by the logarithm of area S_p and fractal dimension of the pore. In this research, the fraction dimension D_p of the pores in cracked concrete and un-cracked concrete is calculated with Eq. (8):

$$\log L_p = D_p / 2 \times \log S_p + c \quad (8)$$

where, c is the material parameter.

In this research, the data analysis was performed with software ORIGIN 7.0. As demonstrated in Fig. 10 and 11, the logarithm of S_p was plotted against the logarithm of L_p of pores. A linear relationship between the logarithms of L_p and S_p can be observed. Based on Eq. (8), the fractal dimension of pores in cracked concrete, or un-cracked concrete, is twice the slop of logarithms of S_p and L_p. A linear fitting indicates that, in this research, the fractal dimension is 1.1838 for un-cracked concrete and 1.21792 for cracked concrete. Both correlation coefficients are close to 1 which means that the pores in concrete are of fractal properties. The fractal dimension of pores in cracked concrete is a little higher than that in un-cracked concrete. This phenomenon states that cracking may influence complexity of the microstructure of concrete.

CONCLUSION

- The cracks and micro pore structure of concrete state very clear fractal properties. It is a good way to represent the irregularity of concrete cracks and homogeneous degree of micro pore distribution with fractal principle. And the fractal dimensions can be taken as a characteristic parameter to describe the tortuosity of cracks.
- In comparison with the traditional measuring method, this research put forward a new algorithm in calculating the characteristic parameters of concrete cracks or pores, such as area, length and perimeter, *et al*. with digital image technology. The new method will improve the quantitative identification of early age cracking or pores of concrete in accuracy, convenience and efficiency.

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