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Research Article

Peak Power Reduction in OFDM Systems for Multicarrier Transmission

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Abstract: The transmit signals in an OFDM system can have high peak values in the time domain since many subcarrier components are added via an IFFT operation. Therefore, OFDM systems are known to have a high PAPR (Peak-to-Average Power Ratio), compared with single-carrier systems. In fact, the high PAPR is one of the most detrimental aspects in the OFDM system, as it decreases the SQNR (Signal-to-Quantization Noise Ratio) of ADC (Analog-to-Digital Converter) and DAC (Digital-to-Analog Converter) while degrading the efficiency of the power amplifier in the transmitter. This study describes some of the important PAPR reduction techniques for multicarrier transmission including amplitude clipping and filtering, coding and partial transmit.

Keywords: Amplitude clipping, partial transmit sequence, peak to average power reduction

INTRODUCTION

PAPR is the ratio between the maximum power and the average power of the complex pass band signal s (t) (Palicot and Louet, 2005), that is:

$$\mathrm{PAPR}\{\tilde{s}(t)\} = \frac{\max\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2}{E\left\{\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2\right\}} = \frac{\max|s(t)|^2}{E\{|s(t)|^2\}} \quad (1)$$

The above power characteristics can also be described in terms of their magnitudes by defining the Crest Factor (CF) as: Passband condition: CF = √PAPR:

In the PSK/OFDM system with N subcarriers, the maximum power occurs when all of the N subcarrier components happen to be added with identical phases. Assuming that $E\{|s(t)|^2\} = 1$, it results in PAPR = N, that is, the maximum power equivalent to N times the average power. We note that more PAPR is expected for M-QAM with M>4 than M-ary PSK. Meanwhile, the probability of the occurrence of the maximum power signal decreases as N increases. For example, suppose that there are M^2 OFDM signals with the maximum power among M^N OFDM signals in M-ary PSK/OFDM system then the probability of occurrence of the largest PAPR is $M^2/ M^N = M^{2-N}$. In other words the largest PAPR rarely occurs. Therefore we are often interested in finding the probability that the signal power is out of the linear range of the HPA. Towards this end, we first consider the distribution of output signals for IFFT in the OFDM system. While the input signals of N-point IFFT have the independent and finite magnitudes which are uniformly distributed for QPSK and QAM, we can assume that the real and imaginary parts of the time-domain complex OFDM signal s (t) (after IFFT at the transmitter) have asymptotically Gaussian distributions for a sufficiently large number of subcarriers by the central limit theorem. Then the amplitude of the OFDM signal s (t) follows a Rayleigh distribution. Let $\{Z_n\}$ be the magnitudes of complex samples $\{|s(nT_s/N)|_{n=0}^{N-1}$. Assuming that the average power of s (t) is equal to one, that is, $E\{|s(t)|^2\} = 1$, then $\{Z_n\}$ are the i.i.d. Rayleigh random variables normalized with its own average power, which has the following probability density function:

$$f_{Z_n}(z) = \frac{z}{\sigma^2} e^{\frac{z^2}{2\sigma^2}} = 2ze^{-z^2}, n = 0, 1, 2, \ldots, N-1 \quad (2)$$

where, $E\{Z_n^2\} = 2\sigma^2 = 1$. Let Z_{max} denote the crest factor (i.e., $Z_{max} = \max_{n=0, 1,\ldots, N-1} Z_n$). Now, the cumulative distribution function (CDF) of Z_{max} is given as:

$$F_{Z_{max}}(z) = P(Z_{max}<z) = P(Z_0<z).\ P(Z_1<z) \ldots P(Z_{N-1}<z) = (1-e^{-z^2})^N \quad (3)$$

where, $P(Z_n < z) = \int_0^z f_{Z_n}(x)\,dx$, n = 0, 1, 2, …, N-1. In order to find the probability that the crest factor (CF) exceeds z, we consider the following complementary CDF (CCDF):

$$\tilde{F}_{Z_{max}}(z) = P(Z_{max}>z) = 1 - P(Z_{max} \geq z) = 1 - P_{Z_{max}}(z) = 1 - (1-e^{-z^2})^N \quad (4)$$

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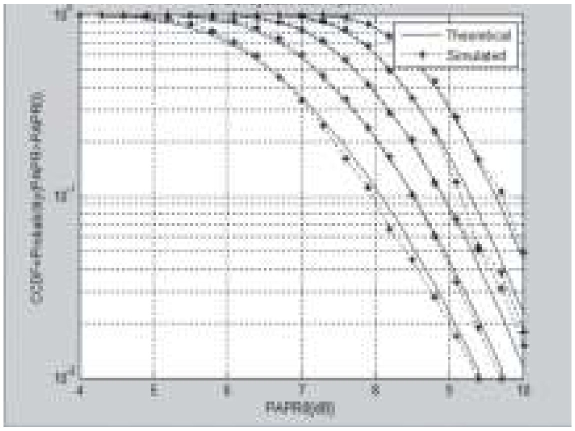


Fig. 1: CCDFs of OFDM signals with N = 64,128,256,512 and 1024

Since Eq. (3) and (4) are derived under the assumption that N samples are independent and N is sufficiently large, they do not hold for the bandlimited or oversampled signals. It is due to the fact that a sampled signal does not necessarily contain the maximum point of the original continuous-time signal. However, it is difficult to derive the exact CDF for the oversampled signals and therefore, the following simplified CDF will be used:

$$F_Z(z) \approx \left(1 - e^{-z^2}\right)^{\alpha N} \tag{5}$$

where, α has to be determined by fitting the theoretical CDF into the actual one (Van Nee and de Wild, 1998). Using simulation results, it has been shown that $\alpha = 2.8$ is appropriate for sufficiently large N. Figure 1 shows the theoretical and simulated CCDFs of OFDM signals with N = 64, 128,256, 512, 1024. Note that the simulation results deviate from the theoretical ones as N becomes small, which implies that Eq. (5) is accurate only when N is sufficiently large. It is thus of interest to find some efficient techniques that can reduce the PAPR of the signal applied at the input of the power amplifier so as to minimize the negative effects of non-linear distortions without sacrificing the power efficiency.

PAPR REDUCTION TECHNIQUES

Several techniques have been proposed in the literature to reduce the PAPR. These techniques can mainly be categorized into signal scrambling techniques and signal distortion techniques. Signal scrambling techniques are all variations on how to scramble the codes to decrease the PAPR. Coding techniques can be used for signal scrambling. Golay complementary sequences, Shapiro-Rudin sequences, M sequences, Barker codes can be used efficiently to reduce the PAPR. However with the increase in the number of carriers the overhead associated with exhaustive search of the best code would increase exponentially. More practical solutions of the signal scrambling techniques are blocking coding, Selective Level Mapping (SLM) and Partial Transmit Sequences (PTS). Signal scrambling techniques with side information reduces the effective throughput since they introduce redundancy.

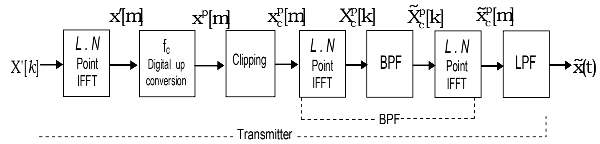


Fig. 2: Block diagram of a PAPR reduction scheme using clipping and filtering.

Table 1: Parameters used for simulation of clipping and filtering

Parameters	Value
Bandwidth, BW	1 MHz
Sampling frequency, f_s = BW/L, with oversampling factor, L = 8	8 MHz
Carrier frequency, f_c	2 MHz
FFF size, N	128
Number of guard interval samples (CP)	32
Modulation order	QPSK
Clipping Ratio (CR)	0.8,1.0, 1.2, 1.4, 1.6

Clipping and filtering: The clipping approach is the simplest PAPR reduction scheme, which limits the maximum of transmit signal to a pre-specified level. However, it has the following drawbacks. Clipping causes in-band signal distortion, resulting in BER performance degradation. Clipping also causes out-of-band radiation, which imposes out-of-band interference signals to adjacent channels (Ochiai and Imai, 2000).

Figure 2 shows a block diagram of a PAPR reduction scheme using clipping and filtering where L is the oversampling factor and N is the number of subcarriers. In this scheme, the L-times oversampled discrete-time signal x'[m] is generated from the IFFT of (X'[k]) with N/ (L-1) zero-padding in the frequency domain) and is then modulated with carrier frequency f_c to yield a pass band signal xP [m]. Let xpC [m] denote the clipped version of xP [m], which is expressed as:

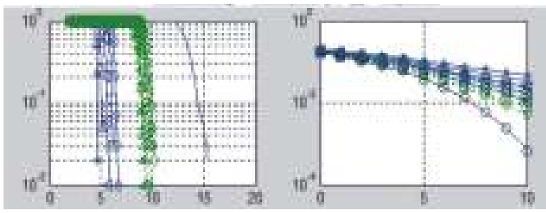
$$x_c^p[m] = \begin{cases} -A & x^p[m] \le -A \\ x^p[m] & |x^p[m] < A| \\ A & x^p[m] \ge A \end{cases} \tag{6}$$

where, A is the pre-specified clipping level.

In general, the performance of PAPR reduction schemes can be evaluated in the following three aspects (Ochiai and Imai, 1997):

- In-band ripple and out-of-band radiation that can be observed via the Power Spectral Density (PSD)
- Distribution of the Crest Factor (CF) or PAPR, which is given by the corresponding CCDF
- Coded and uncoded BER performance

Table 1 shows the values of parameters used in the QPSK/OFDM system for analyzing the performance of clipping and filtering technique.



(a) (b)

Fig. 3: (a) PAPR Distribution, (b) BER Performance

Figure 3a shows the CCDFs of Crest Factor (CF) for the clipped and filtered OFDM signals. It can be seen from this figure that the PAPR of the OFDM signal decreases significantly after clipping and increases a little after filtering. Note that the smaller the Clipping Ratio (CR) is, the greater the PAPR reduction effect is. Figure 3b shows the BER performance when clipping and filtering technique is used. It can be seen from this figure that the BER performance becomes worse as the CR decreases.

PAPR reduction code: It was shown in Wilkinson and Jones (1995) that a PAPR of the maximum 3dB for the 8-carrier OFDM system can be achieved by 3/4-code rate block coding. Here, a 3-bit data word is mapped onto a 4-bit codeword. Then, the set of permissible code words with the lowest PAPRs in the time domain is chosen. The code rate must be reduced to decrease the desired level of PAPR. It was also stated in that the block codes found through an exhaustive search are mostly based on Golay complementary sequence. Golay complementary sequence is defined as a pair of two sequences whose aperiodic autocorrelations sum to zero in all out-of-phase positions (Golay, 1961). It is stated in Popovic (1991) that Golay complementary sequences can be used for constructing OFDM signals with PAPR as low as 3dB. Van Nee (1996) showed the possibility of using complementary codes for both PAPR reduction and forward error correction. However, the usefulness of these coding techniques is limited to the multicarrier systems with a small number of subcarriers. In general, the exhaustive search of a good code for OFDM systems with a large number of subcarriers is intractable, which limits the actual benefits of coding for PAPR reduction in practical OFDM systems.

First, let us consider the basic properties of complementary sequence. Two sequences $x_1[n]$ and $x_2[n]$ consisting of -1 or +1 with equal length N are said to be complementary if they satisfy the following condition on the sum of their autocorrelations:

$$\sum_{n=0}^{N-1}(x_1[n]x[1][n+1] + x_2[n]x_2[n+i]) = \begin{cases} 2N, & i = 0 \\ 0, & i \neq 0 \end{cases} \quad (7)$$

Taking the Fourier transform yields

$$|X_1[k]|^2 + |X_2[k]|^2 = 2N \quad (8)$$

where, $X_i[k]$ is the DFT of $\{x_i[n]\}$, such that:

$$Xi[k] = \sum_{n=0}^{N-1} x_i[n]e^{-2\pi nkT} \quad (9)$$

With the sampling period of T_s: The power spectral density of $X_i[k]$ is given by DFT of the autocorrelation of $x_i[n]$. According to Eq. (7), the PSD $|X_i[k]|^2$ is upper-bounded by 2N, which means:

$$|X_i[k]|^2 \leq 2N \quad (10)$$

Since the power of $x_i[n]$ is 1, the average of $|X_i[n]|^2$ in Eq. (8) is N and thus, the PAPR of $X_i[k]$ is upper-bounded by:

$$PAPR \leq \frac{2N}{N} = 2 \text{ (or 3dB)} \quad (11)$$

Suppose that a sequence is applied as the input to IFFT. Since the IFFT is equivalent to taking the complex conjugate on the output of FFT and dividing it by N, we can replace X[k] by the IFFT of x[n] so that the PAPR can be upper-bounded by 2 (i.e., 3dB). This implies that if the complementary sequences are used as the input to IFFT for producing OFDM signals, the PAPR will not exceed 3dB.

Partial transmit sequence: The Partial Transmit Sequence (PTS) technique partitions an input data block of N symbols into V disjoint subblocks as follows:

$$X = [X^0, X^1, X^2, ..., X^{V-1}]^T \quad (12)$$

where, X^i are the subblocks that are consecutively located and also are of equal size. Then each partitioned subblock is multiplied by a corresponding complex phase factor $b^v = e^{j\Phi v}$, v = 1, 2, … , V, subsequently taking its IFFI to yield:

$$x = IFFT\left\{\sum_{v=1}^{V} b^v X^v\right\} = \sum_{v=1}^{V} b^v.IFFT\{X^v\} = \sum_{v=1}^{V} b^v x^v \quad (13)$$

where, $\{X^v\}$ is referred to as a Partial Transmit Sequence (PTS). Figure 4 shows the block diagram of this technique. The phase vector is chosen so that the PAPR can be minimized (Muller and Huber, 1996), which is shown as:

$$[\tilde{b}^1, \ldots \tilde{b}^v] = \arg\min_{[b^1, ..., b^v]}\left(\max_{n=0,1,...,N-1}\left|\sum_{v=1}^{V} b^v x^v[n]\right|\right) \quad (14)$$

Then, the corresponding time-domain signal with the lowest PAPR vector can be expressed as:

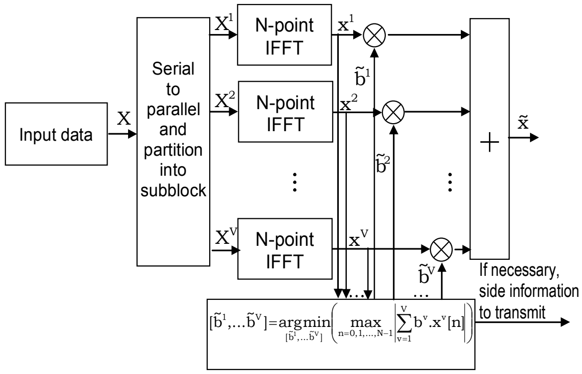


Fig. 4: Block diagram of Partial Transmit Sequence (PTS) technique for PAPR reduction

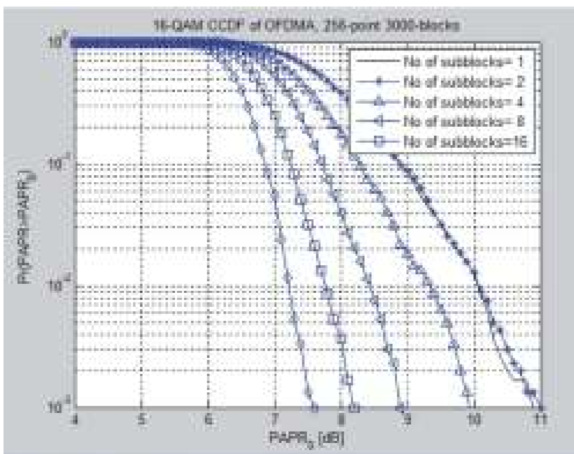


Fig. 5: PAPR performance of a 16-QAM/OFDM system with PTS technique

$$\tilde{x} = \sum_{v=1}^{V} \tilde{b}^{v} x^{v} \tag{15}$$

In general, the selection of the phase factors $\{b^v\}_{v=1}^{V}$ is limited to a set of elements to reduce the search complexity. As the set of allowed phase factors is $b = \{e^{j2\pi i/w} \mid i = 0, 1, \ldots, W-1\}$, W^{V-1} sets of phase factors should be searched to find the optimum set of phase vectors. Therefore, the search complexity increases exponentially with the number of subblocks.

Figure 5 shows the CCDF of PAPR for a 16-QAM/OFDM system using PTS technique as the number of subblock varies. It is seen that the PAPR performance improves as the number of subblocks increases with V = 1, 2, 4, 8 and 16.

CONCLUSION

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal exhibits a very high PAPR when the input sequences are highly correlated. In this study we have described several important aspects as well as provided a mathematical analysis of several techniques to reduce the PAPR, all of which have the potential to provide substantial reduction in PAPR at the cost of loss in data rate, transmit signal power increase, BER performance degradation, computational complexity increase and so on. Basic requirement of practical PAPR reduction techniques include the compatibility with the family of existing modulation schemes, high spectral efficiency and low complexity. There are many factors to be considered before a specific PAPR reduction technique is chosen. These factors include PAPR reduction capacity, Power increase in transmit signal, BER increase at the receiver, loss in data rate, computational complexity increase and so on. No specific PAPR reduction technique is the best solution for all multi carrier transmission. Rather the PAPR reduction technique should be carefully chosen according to various system requirements

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