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Research Article

Study on Traditional Pricing Models Based on Terms of Convertible Bonds in China

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Abstract: The study has gotten the conclusion that the binary tree model is more suitable for convertible bonds pricing in China by analyzing terms of convertible bonds in China and deducing traditional pricing models of convertible bonds. The characteristics of terms embedded in convertible bonds decide which pricing models will be elected and which model parameters to be estimated. By analyzing terms of convertible bonds in China, get the expressions of terms value, which are called as terminal conditions and boundary conditions and then deduce the traditional pricing models. Based on the above processes, it can be found that the binary tree model use terminal conditions and boundary conditions better.

Keywords: Convertible bonds, pricing efficiency, traditional pricing models, terms of convertible bonds

INTRODUCTION

The convertible bond is a financial derivative with good investment and financing functions and the pricing efficiency of the existed models is generally not high. Overall, aspects which mainly impact the accuracy of convertible bond pricing are:

- The selection or development of pricing models
- The estimation of model parameters
- The mathematical expression of the value of the terms embedded in convertible bonds

The analysis of terms embedded in convertible bonds decides which pricing models will be elected and which model parameters to be estimated and the pricing efficiency of pricing models. In the 1st stage of research about the pricing of convertible bonds: 1960s to 1970s, people did not take the term of redemption into account, which makes the convertible bond price was overvalued. In the next stage of research about the pricing of convertible bonds: 1970s to 1990s, people thought the value of convertible bonds equaled the value of ordinary bonds and the value of the call options which was the sum of value of embedded terms. Such as Ingersoll (1977), he assumed there were no cash dividends and the convertible bond was a discount bond which did not pay coupon interest. He developed and used the arbitrage theory to make sure the optimal redemption strategy and the optimal conversion strategy for investors and the issuers. He thought the value of convertible bonds = the value of common bond+the value of the conversion term-the value of the redemption term. Ho and Lee (1986) proposed a two-factor model based on the stock price and stochastic interest rates and got: The value of convertible bonds = the value of common bond+the value of sale back term+the value of the conversion term-the value of the redemption term. Lin (1988) discuss the pricing problem and the value of the redemption term of the convertible bonds with Black-Scholes model and tried to calculate the total value of the convertible bonds by sum the value of embedded terms . Wang and Wu (2001) using Monte Carlo simulation, got the probability of term triggered for redemption and return sale. Using the finite difference method for solving, they pointed out that the contribution of the return sale term on investment was small. This pricing model does not consider the value of interdependence between the terms, making the model pricing is often inefficient. The majority of scholars preferred a comprehensive analysis of the value of all terms and got a higher pricing model efficiency, such as Tsiveriotis and Fernandes (1998) and Lai *et al*. (2008), both chose binary tree model according to the characteristics of terms embedded in convertible bonds and got a higher pricing model efficiency. Above are illustrated that it is very important to choose pricing models according to the characteristics of terms embedded in convertible bonds. Next, by analyzing terms of convertible bonds in China and deducing traditional pricing models of convertible bonds, the study will choose pricing models which are more suitable for Chinese convertible bonds.

TERMS OF CONVERTIBLE BONDS IN CHIAN

A issue announcement of convertible bonds with Chinese characteristics usually includes the term of bond, the term of conversion , the term of redemption

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and the term of sale back and the term downwardly revised.

- **The term of conversion:** This is the most essential term of the convertible bond and other terms are designed to this term, which is also the main difference between convertible bonds and ordinary bonds. It makes the convertible bond can be converted into a certain number of underlying securities by a certain percentage or price within a certain period .This period is called as the conversion period ,which is about 6 months since the issue of the convertible bond.
- **The term of sale back:** This is an embedded terms which protects investors, makes the investors can sell convertible bonds back to the issuer at a certain price under certain conditions. In China, generally when the stock price fall to 60-80%, respectively of the conversion price (usually called the stock price as the trigger price of back sale), investors can sell back at certain price, such as 103-105%, respectively of the convertible bond denominations (generally, this price is called as the sale back price which is abbreviated as P) .The period during which the investors can sell convertible bonds back to the issuer is usually called as the sale back period, which is usually same with the conversion period in China. The term of the sale back gives investors an American put option, which helps to protect the interests of investors and therefore there is a certain degree of drop-down trend for the convertible bond coupon rate.
- **The term of redemption:** Actually, most of the corporate bonds have redeemable characteristics. This term embedded in the convertible bond is valuable, whose value is bound to have a significant impact on the pricing of convertible bonds. It makes the issuer can redemptive convertible bonds at a certain price under certain conditions. In China, generally when the stock price increase to 130% of the conversion price (usually called the stock price as the trigger price of redemption), the issuer can redemptive convertible bonds at certain price, such as 105-107%, respectively of the convertible bond denominations (generally, this price is called as the redemption price which is abbreviated as C) .The period during which the issuer can redemptive convertible bonds under certain conditions is usually called as the redemption period, which is usually same with the conversion period in China. There is no doubt that this term is conducive to protect the interests of the issuer and as compensation, the coupon rate of convertible bonds with this term is generally higher than the one without this term.
- **The term downwardly revised:** The term downwardly revised makes when the company's stock continue to fall to a certain extent which has a large gap with the initial conversion price, in order to avoid the bondholders convertible bond sale back, the issuer can adjust the initial conversion price before the stock price falls back to the trigger price of back sale (generally the 60-80%, respectively of initial conversion price). Usually, the previous trigger price of back sale will set as the new conversion price, which is the 60-80%, respectively of initial conversion price. To some extent, the revised downward term is benefit for both investors and issuers, but its original intent of design is to consider the interests of the issuer more.

The expressions of terms value: The term of the bond and the term of conversion term make convertible bonds with features of ordinary bond and option. Other terms embedded have a certain trigger conditions, in fact, are the dependent variables which depend on the underlying stock price path. They can be defined as a variety of boundary conditions and terminal conditions and should be taken into account when pricing in each node. Value of terms embedded in convertible bonds are not independent from each other, so we cannot separate them when pricing, or there will be a great difference between the theoretical price and the actual price.

TERMINAL CONDITIONS

The terminal value is the maturity value of the bonds. At terminal node, only the term of the bond and the term of conversion affect the value of the convertible bond .Other terms cannot be triggered. Based on the foregoing analysis, the terminal conditions expression can be gotten:

$$V(S_t, T) = Max(nS_T, B);$$

$$B(S_t, T) = \begin{cases} B_0 e^{rT}; & B_0 e^{RT} > nS_t \\ 0; & B_0 e^{RT} \le nS_t \end{cases};$$

$$C(S_t, T) = \begin{cases} 0; & B_0 e^{RT} > nS_t \\ nS_t; & B_0 e^{RT} \le nS_t \end{cases}$$

$V(S_t, T)$ = The maturity value of convertible bonds
$B(S_t, T)$ = The value of the cash portion of convertible bonds at the end of the period
B_0 = The value of pure bond value of convertible bonds at initial node
$C(S_t, T)$ = The value of the equity portion of convertible bonds at the end of the period
R = The discount rate of risk premium
n = Ratio of conversion;
S_t = The underlying stock price at time t

In addition, the issue of the company's cash dividend payment will naturally impact the value of the convertible bonds. If issuers do not need adjust the conversion price of convertible bonds according to the issue of cash dividends , the issuance of cash dividends will naturally affect the price of the underlying stock, which may possibly trigger the above mentioned various embedded terms and makes the convertible bond value change. At this time the number of the cash dividend and the remaining period of convertible bonds and other factors should be taken into account. A relatively simple and quantitative approach is to adjust the Brownian motion which the price movement of the underlying stock meets as:

$$dS = (r-r_d)\, Sdt+S\sigma dz$$

r_d is stock bonus rate; If issuers have to adjust the conversion price of convertible bonds according to the issue of cash dividends , the issuance of cash dividends is equivalent to add a more piece of protective overcoat for the terms of the sale back not to be triggered. The terms of the sale back will not be triggered and the probability of the downward revision of the terms been triggered is small, so term of redemption embedded is most likely to be triggered. For the issuer of China convertible bonds rarely pay cash dividends, this point will not be considered in this study.

Boundary conditions: Consider the terms embedded, the value of convertible bonds at intermediate nodes can be summarized as boundary conditions:

$$V_{i,j} = \max\{nS_t, P, \min[C, H_{i,j}]\}$$
$$H_{i,j} = C_{i,j} + B_{i,j}$$
$$C_{i,j} = e^{-r\Delta t}(p_u C_{i+1,j+1} + p_m C_{i,j+1} + p_d C_{i-1,j+1})$$
$$B_{i,j} = e^{-R\Delta t}(p_u B_{i+1,j+1} + p_m B_{i,j+1} + p_d B_{i-1,j+1})$$

where,

C = The redemption price
P = The sale back price
P_u = The probability of the logarithm price or price increasing at each node
P_m = The probability of the logarithm price or price not changing at each node
P_d = The probability of the logarithm price or price declining at each node
r = Risk-free discount rate
j = The number of time periods
i = The number of increase in these periods

In summary, the value of terms embedded in convertible bonds can be summarized as boundary conditions and terminal conditions, which can solve the problem that for there is interdependence between value of terms embedded in convertible bonds, we cannot separate the value of terms embedded when pricing. It means the pricing model is the better pricing model, which can take boundary conditions and terminal conditions into account better. Based on this point and the following model derivation, the pricing model which is more suitable for convertible bonds in China will be gotten.

TRADITIONAL PRICING MODELS OF CONVERTIBLE BONDS

Introduction of traditional pricing models: Traditional pricing models of convertible bonds include: the analytical method: BS model; the numerical method, which are mainly the Monte Carlo simulation, the binary tree model and the finite difference method.

Derivation of traditional pricing models:

- **The analytical method:** The analytical method is the Black-Scholes pricing model referred to as the BS model, which has seven harsh assumptions and no longer repeat it here. Under these assumptions, the movement of stock prices follows the law of geometric Brownian motion which in mathematics was manifested as Ito process:

$$dS = \mu^* S d_t + \sigma S d_z = (\mu + \frac{1}{2}\sigma^2) S d_t + \sigma S d_z \qquad (1)$$

S = The price of the underlying stock
μ^* = The expected return rate of the stock in unit time
μ = The mathematical expectation of the natural logarithm of the stock return in the unit time
σ = The volatility which is the standard deviation of the natural logarithm of the stock return in unit time
$d_z = \varepsilon\sqrt{d_t}$ = A random process called Wiener process, which meets the standard normal distribution

Based on (1), the further study is available to another important mathematical result the Ito lemma:

$$df = (\frac{\partial f}{\partial S}\mu^* S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial f}{\partial S}\sigma S dz \quad (2)$$

$f = (S, t)$ is the price of derivative securities. Take the typical dynamic non-arbitrage equilibrium analysis technology and take a dynamic trading strategy under above assumptions to copy the end cash flow of European call variable. Using Ito process and Ito lemma to eliminate the random, we can get the famous Black-Scholes stochastic differential equation:

$$\frac{\partial f}{\partial t} + r_f S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_f f$$

combined with the terminal conditions of the European underlying assets: C = max [S(T)-X, 0] or P = max [X-S(T), 0], we can get the Black-Scholes pricing formula:

$$c = S(t)N(d_1) - X e^{-r_f(T-t)}N(d_2);$$
$$p = X e^{-r_f(T-t)}N(-d_2) - S(t)N(-d_1)$$

N(.) is the cumulative normal distribution function:

$$d_1 = \frac{\ln(S(t)/X) + (r_f + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

C = The price of the European call variable
p = The price of the European put variable
X = The exercise price of variable
T = The expiration date of variable
t = On behalf of the present

As described above, BS model can't consider the value of terms embedded complexly and therefore can't be used for pricing convertible bonds accurately. However, as we know, BS model is the basis of many pricing models such as tree graph model including the binary tree model and the trinomial tree model and so on.

The numerical method:

- **The binomial method:** In the binomial method, assume that the underlying asset price may rise and reduce at each stage:

n = The number of time stage
$\Delta t = T/n$ = The length of each stage
u = The proportion of the underlying asset price increasing in each stage
d = The proportion of the underlying asset price declining in each stage
p = The probability of the underlying asset price increasing in each stage
1-p = The probability of decreasing
v = The degree of the natural logarithm of the underlying asset prices rising in each stage
w = The degree of the natural logarithm of the underlying asset prices dropping in each stage.

We can get one-stage binary tree model diagram:

$$\begin{tikzcd}[row sep=small] & uS_0 = S_u \ (f_u) \\ S_0 \arrow[ur] \arrow[dr] & \\ & dS_0 = S_d \ (f_d) \end{tikzcd}$$

$$\ln S_u = \ln u + \ln S_0 = v + \ln S_0; \ln S_d = \ln d + \ln S_0 = w + \ln S_0$$

It is easy to get $u = e^v$; $d = e^w$ from the above diagram. The eigenvalues of the binary tree model include size and probability of price changing. Focus on the logarithm price of the underlying asset at the first phase, which is In $S_{\Delta t}$. In $S_{\Delta t}$ fits the normal distribution whose mean is In $S_0+\mu\Delta t$ and variance is $\sigma^2\Delta t$. Parameters should ensure that the mean and variance of the discrete binomial distribution are same with the mean and variance of BS model of the consistent normal distribution, in fact when $\Delta t \to 0$, the binary tree model is the BS model. With the above tree diagram and mean constraint and variance constraint, if add the parameter imposed by Cox, Ross, Rubinstein: $w = -v$, we can get the solution to the model:

$$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}; p = \frac{1}{2} + \frac{1}{2}(\frac{b - 0.5\sigma^2}{\sigma})\sqrt{\Delta t}$$

If add the parameter imposed by Jarrow, Ruddy: P = 1/2, we can get the solution to the model:

$$u = e^{(b-\sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}}; d = e^{(b-\sigma^2/2) - \sigma\sqrt{\Delta t}}; p = \frac{1}{2}$$

with the risk neutral assumption, we can get the solutions to the model:

$$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}; p = \frac{e^{r\Delta t} - d}{u - d}$$

As described above, the binary tree model calculates value of convertible bonds by sub-node and therefore the value of terms embedded can be considered complexly, which means the binary tree model can use terminal conditions and boundary conditions better.

- **Monte carlo simulation method:** The same point between Monte Carlo simulation and the tree diagram method is both required to simulate the path of asset price changing within the validity period. The difference is that the tree diagram method calculates the characteristic values of model and the variable path has been designed firstly, while Monte Carlo simulation is to examine a random sample of price changing in sequence and the variable path is a random. The time Δt corresponds to a value randomly selected and during T-t = $n\Delta t$, n random values are taken to produce a simulated path with which we can calculate the price of underlying asset and it is called a simulation or an experiment. After many simulations, such as 10,000 simulations, we can get 10 000 the final price of the underlying asset. The final value of the derivative variable can be calculated with the final price of the underlying asset and the arithmetic mean of the derivative variable can be gotten, then using the risk-free rate to discount the arithmetic mean $E(\overline{P}_t)$. Finally get the present value of the derivative variable $f = e^{-rt}E(\overline{P}_t)$. As described above, Monte Carlo simulation also can't consider the value of terms embedded complexly alone. But it can combine with other pricing model to improve the pricing efficiency such as with genetic operator method.

- **The finite difference method:** Object handled by the finite difference method is the partial differential equation of the BS model and the numerical method is used to solve the partial differential equation of the derivatives and then price the derivative. The finite difference method first divides the partial differential equation into a series of differential equations which are simplified by the iteration method and then the price of the derivative is calculated. In general, a Partial Differential Equation (PDE) has infinitely many solutions and we are only interested in solutions that meet specific criteria and initial value set. For the finite difference method, it is necessary to know the provisions of the boundary and initial conditions. The basic idea of the finite difference method is basically same with the tree diagram model. But it doesn't calculate the value of convertible bonds by sub-node and therefore the value of terms embedded can be considered complexly.

CONCLUSION

By analyzing terms of convertible bonds in China, get the expressions of terms value, which are called as terminal conditions and boundary conditions and then deduce the traditional pricing models, we can find the binary tree model can use terminal conditions and boundary conditions better. It means that the binary tree model is more suitable for China convertible bond pricing than other traditional pricing models.

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