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Research Article

Floating Shaft Load Sharing Method for Face Gear Split Torque Transmission System

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Abstract: In this study, the equivalent gear meshing error principle was used to analysis face gear split torque dynamics modeling, the mathematical model includes stiffness of shaft supporting, position of gears, backlashes, time-varying stiffness, damping, manufacturing error, assembly error. The result shows the floating shaft can achieved load sharing, the support stiffness is an important factor, decreasing the input shaft support stiffness, load sharing performance becomes better and vibration decreases by increasing the displacement of shaft. To study effect of errors, every errors change separately while others' ideal. Every errors affect load sharing in different way, gear 4 manufacturing error and shaft 2 assembly error affect load sharing most, load sharing cannot achieve by decreasing one error.

Keywords: Face gear, floating shaft, split torque, transmission system

INTRODUCTION

Main rotor transmission system has a significant influence on overall performance of the helicopter. A high performance main rotor transmission is especially demanding to meet the development of helicopter. Split torque transmission has larger reduction ratio, small size and lower overall drive system weight, high efficiency and reliability, which was promising, attractive alternatives to planetary transmission. In addition, the safety, reliability, lightweight and energy efficient are of crucial importance for drive system as well as the ability of producing little vibration and noise.

Split torque transmission was studied for a long time (White, 1974, 1981, 1983, 1989, 1998). A split torque design for a helicopter main rotor gearbox was developed by Westland Helicopters (Cocking, 1986). While comparing split torque arrangement with planetary arrangement transmission system, split torque arrangement has larger reduction ratio (split torque maximum reduction ratio of 14:1, while planetary arrangement maximum reduction ratio of about 7:1), planetary has more power paths (planetary arrangement usually has 3 to 18 parallel power paths, split torque arrangement has two parallel power paths). A large reduction ratio at the final gear stage tends to reduce the overall transmission weight. Split torque can offer the following advantages over the traditional planetary design (Krantz and Delgado, 1996):

- A high speed reduction ratio at the final stage
- A reduced number of gear stages
- Lower energy losses
- Increased reliability because of separate drive paths
- Fewer gears and bearings
- Lower noise levels from gear meshes
- Lower overall drive system weight

Though split torque transmission is sensitive to manufacture tolerances, which can carry unequal loads in the two paths, attracted by the performances of split torque, more and more split torque designs were studied. Many different split torque gearbox conceptual designs have been proposed. All design concentrates the equality of the torque split.

Kish (1993) made a development of a torsion ally compliant load sharing device, which shows excellent performance under nominal laboratory but it was limited for field operation because of operation temperature.

Heath (1993) also ever developed face gear split torque transmission system and it shows excellent weight saving, the total calculated weight of the transmission assembly is 40% weight saving.

Krantz (1994) used a balance beam mechanism for load sharing. The result shows:

- The loads of the two power paths differ, although the gearbox has symmetric geometry.
- Friction must be considered to properly evaluate, the balance beam is not an effective device for load sharing unless the coefficient of friction is less than 0.003.

Krantz and Delgado (1996) developed a method to analyze the load sharing of split torque gearboxes, which achieved load sharing by control clocking angles. The study indicates that split-path gearboxes can be used

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successfully in rotorcraft and can be manufactured with existing technology.

Gu *et al*. (2009a, b) further developed spiral bevel gear split torque transmission, which reduction ratio is 1, which also showed good performance.

Although different designs can balance the unequal loads of the two paths, risk inherent to this design is the manufacture precise. To analysis how transmission system manufacturing error affect load sharing, the equivalent gear meshing error principle (Lu *et al*., 2009) was used to analysis face gear split torque dynamics modeling, the model includes stiffness of shaft supporting, position of gears, backlashes, time-varying stiffness, damping, manufacturing error, assembly error. The mathematical model is used to analysis how manufacturing error, assembly error affect load sharing, it can provide information to analysis and design face gear split torque transmission system. The research reported herein was done to help enable the use of split-path transmissions with the load-sharing properties, optimal clocking angle and effect of manufacturing tolerances are calculated for future rotorcraft.

In this study, the equivalent gear meshing error principle was used to analysis face gear split torque dynamics modeling, the mathematical model includes stiffness of shaft supporting, position of gears, backlashes, time-varying stiffness, damping, manufacturing error, assembly error. The result shows the floating shaft can achieved load sharing, the support stiffness is an important factor, decreasing the input shaft support stiffness, load sharing performance becomes better and vibration decreases by increasing the displacement of shaft. To study effect of errors, every errors change separately while others' ideal. Every errors affect load sharing in different way, gear 4 manufacturing error and shaft 2 assembly error affect load sharing most, load sharing cannot achieve by decreasing one error.

SPLIP TORQUE MODEL ANALYSIS

Face gear split torque model: Face gear split torque transmission system was presented in Fig. 1, which includes two stages, first stage face gear divides power to two branches and the second stage gathers the two branch power and output.

System support, errors in the transmission system cannot be ignored, to simplify the calculation, the axis of bending was ignored and the establishment of the coordinate system shown in Fig. 2, global coordinate system is: straight up to the Z axis, the input pinion axially parallel direction of the Y-axis right-hand Cartesian coordinate system.

Manufacturing error analysis: In short, the basic model of robotic system is a.

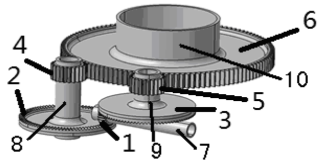

Fig. 1: Face gear split torque transmission
1: Input gear; 2: Gace gear 1; 3: Face gear 2; 4: Gear 4; 5: Gear 5; 6: Output gear 6; 7: Input shaft 1; 8: Shaft 2; 9: Shaft 3; 10: Output shaft 4

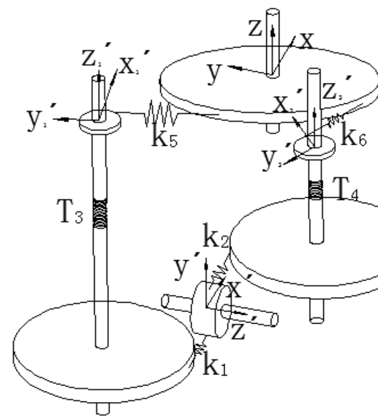

Fig. 2: Split torque mode (damping element, support element not shown)

Load sharing is affected by gear eccentric error, assembly error, bearing eccentric error and tooth thickness error.

The change of transmission error of gear 1, 2 due to manufacturing and assembly error is:

$$\begin{aligned} e_1 &= E_{g1}\sin(\omega_1 t+\beta_{g1}) + E_{b1}\sin(\omega_1 t+\beta_{b1}) \\ &+ A_{b1}\sin(\gamma_{A1}) - E_{g2}\sin(\omega_2 t+\beta_{g2})\cos\alpha_1 \\ &- E_{b2}\sin(\omega_2 t+\beta_b)\cos\alpha_1 - A_{b2}\sin(\gamma_{Ab2})\cos\alpha_1 \\ &- B_{b2}\sin\alpha_1 + \varepsilon_1 + \varepsilon_2 \end{aligned} \tag{1}$$

The change of transmission error of gear 1, 3 due to manufacturing and assembly error is:

$$\begin{aligned} e_2 &= -E_{g1}\sin(\omega_1 t+\beta_{g1}) - E_{b1}\sin(\omega_1 t+\beta_{b1}) \\ &- A_{b1}\sin(\gamma_{Ab1}) + E_{g3}\sin(\omega_3 t+\beta_{g3})\cos\alpha_1 \\ &+ E_{b3}\sin(\omega_3 t+\beta_{b3})\cos\alpha_1 + A_{b2}\sin(\gamma_{Ab2})\cos\alpha_1 \\ &+ B_{b3}\sin\alpha_1 + \varepsilon_1 + \varepsilon_3 \end{aligned} \tag{2}$$

The change of transmission error of gear 4, 6 due to manufacturing and assembly error is:

$$\begin{aligned} e_5 &= -E_{g4}\sin(\omega_2 t+\beta_{g4}) - E_{b2}\sin(\omega_2 t+\beta_{b2}) \\ &- A_{b2}\sin(\gamma_{Ab2}) + E_{g6}\sin(\omega_4 t+\beta_{g6}) \\ &+ E_{b4}\sin(\omega_4 t+\beta_{b4}) + A_{b4}\sin(\gamma_{Ab4}) + \varepsilon_4 + \varepsilon_6 \end{aligned} \tag{3}$$

The change of transmission error between gear 5, 6 due to manufacturing and assembly error is:

$$\begin{aligned} e_6 = &-E_{g5}\sin(\omega_3 t+\beta_{g5}-a_4)-E_{b3}\sin(\omega_3 t \\ &+\beta_{b3}-a_4)-A_{b3}\sin(\gamma_{Ab3}-a_4) \\ &+E_{g6}\sin(\omega_4 t+\beta_{g6}-a_4)+E_{b4}\sin(\omega_4 t+\beta_{b4} \\ &-a_4)+A_{b4}\sin(\gamma_{Ab4}-a_4)+\varepsilon_5+\varepsilon_6 \end{aligned} \tag{4}$$

E_{gi} (i = 1-6) is gear eccentric error; E_{bi} (i = 1-4) is shaft eccentric error; A_{bi} (i = 1-4) is shaft radial direction assembly error; B_{bi} (i = 2, 3) is shaft axis direction assembly error; β_{bi} (I = 1-6) is gear eccentric error phase angle; β_{bi} (i = 1-4) is shaft eccentric error phase angle; γ_{Abi} (i = 1-4) is shaft radial direction assembly error phase angle; ω_i is gear shaft rotation angular velocity; α_i (i = 1, 2) is pressure angle; a_i (i = 4) is angle of shaft 2, 4, 3.

Transmission system dynamical equations: Displacement of input shaft 1 in global coordinate system is x_i (i = 1, 2), displacement of shaft 2 in global coordinate system is x_i (i = 3 - 5), displacement of shaft 3 in global coordinate system is x_i (i = 6-8), displacement of output shaft 4 in global coordinate system is s (i = 9-10). Angel of gears θ_i (i = 1-6), mesh force and damping force is calculated in global coordinate system and corresponding local coordinate system, parameter in local coordinate system should be converting to global coordinate system.

Meshing force and damping force of gear 1, 2 is:

$$\begin{cases} F_1 = k_1 f_1[(x_1 - r_1\theta_1 - x_2 + r_2\theta_2)\cos\alpha_1 \\ +(z_1 - z_2)\sin\alpha_1 - e_1] \\ D_1 = c_1[(\dot{x}_1 - r_1\dot{\theta}_1 - \dot{x}_2 + r_2\dot{\theta}_2)\cos\alpha_1 \\ +(\dot{z}_1 - \dot{z}_2)\sin\alpha_1 - \dot{e}_1] \end{cases} \tag{5}$$

Meshing force and damping force of gear 1, 3 is:

$$\begin{cases} F_2 = k_2 f_2[(x_3 + r_3\theta_3 - x_1 - r_1\theta_1)\cos\alpha_1 \\ +(z_3 - z_1)\sin\alpha_1 - e_2] \\ D_2 = c_2[(\dot{x}_3 + r_3\dot{\theta}_3 - \dot{x}_1 - r_1\dot{\theta}_1)\cos\alpha_1 \\ +(\dot{z}_3 - \dot{z}_1)\sin\alpha_1 - \dot{e}_2] \end{cases} \tag{6}$$

Meshing force and damping force of gear 4, 6 is:

$$\begin{cases} F_5 = k_5 f_5(-x_2\sin a_1 + y_2\cos a_1 - r_4\theta_4 \\ + x_6\sin a_1 - y_6\cos a_1 + r_6\theta_6 - e_5) \\ D_5 = c_5(-\dot{x}_2\sin a_1 + \dot{y}_2\cos a_1 - r_4\dot{\theta}_4 \\ + \dot{x}_6\sin a_1 - \dot{y}_6\cos a_1 + r_6\dot{\theta}_6 - \dot{e}_5) \end{cases} \tag{7}$$

Meshing force and damping force of gear 5, 6 is:

$$\begin{cases} F_6 = k_6 f_6(-x_3\sin a_2 + y_3\cos a_2 - r_5\theta_5 \\ + x_6\sin a_2 - y_6\cos a_2 + r_6\theta_6 - e_6) \\ D_6 = c_6(-\dot{x}_3\sin a_2 + \dot{y}_3\cos a_2 - r_5\dot{\theta}_5 \\ + \dot{x}_6\sin a_2 - \dot{y}_6\cos a_2 + r_6\dot{\theta}_6 - \dot{e}_6) \end{cases} \tag{8}$$

Torque of shaft 2:

$$T_3 = k_3 f_3(\theta_4 - \theta_2) + c_3(\dot{\theta}_4 - \dot{\theta}_2) \tag{9}$$

Torque of shaft 2:

$$T_4 = k_4 f_4(\theta_5 - \theta_3) + c_4(\dot{\theta}_5 - \dot{\theta}_3) \tag{10}$$

where,

e_i (i = 1, 2, 5, 6) : Equivalent mesh errors
k_i (i = 1, 2, 5, 6) : Meshing stiffness
c_i (i = 1, 2, 5, 6) : Meshing damping
α_i (i = 1, 2) : Pressure angle

A concentrated mass dynamical equation of split torque transmission system used Newton method was established.

Since the system is semi-definite system, the removal of rigid-body displacements is necessary, therefore new generalized parameter is introduced (refer with Eq. (11):

$$\begin{cases} x_{11} = r_1\theta_1 - r_2\theta_2 \\ x_{12} = r_1\theta_1 - r_3\theta_3 \\ x_{13} = \theta_4 - \theta_2 \\ x_{14} = \theta_5 - \theta_3 \\ x_{15} = r_4\theta_4 - r_6\theta_6 \\ x_{16} = r_5\theta_5 - r_6\theta_6 \end{cases} \tag{11}$$

The parameters were unified and the original equations were transformed into 16 equations. The equations are rigid-equations, order of magnitude of stiffness, displacement was largely, stiffness was 10^{10} while displacement was 10^{-5}, it affects solution accuracy and time cost to a large extent, therefore the motion equations need dimensionless to reduce rigid of the motion equations, define dimensionless time as $\tau = w_n t$ ($\omega_n = \sqrt{k_1(I_1 r_2^2 + I_2 r_1^2)/(I_1 I_2)}$), scale of dimensionless displacement as $b_c = \frac{\bar{T}_1}{2r_1\bar{k}_1}$ ($\bar{T}_1$ as the average input torque, $\bar{k}_1$ as the average stiffness of first stage gear), in addition with scale of radian as $\vartheta = b_c/r_1$, then:

$$X_i(\tau) = x_i(t)/b_c,\ \theta_i(\tau) = \vartheta_i(t)/(b_c/r_1)$$

The final equations can be written in matrix:

$$m\ddot{x} + c\dot{x} + kx = F \tag{12}$$

Table 1: Parameters of split torque transmission

Parameter	Value
First stage pinion teeth number	32
First stage face gear teeth number	124
First stage pressure angle (°)	20
First stage modulus (mm)	1.6
Input torture (N·m)	786.48
Input speed (/min）	8000
Shaft 1 support (radial) stiffness (N/m)	1×107
Shaft 2, 3 support (axis) stiffness (N/m)	5×108
Second stage pinion teeth number	27
Second stage gear teeth number	176
Second stage pressure angle (°)	20
Second stage modulus (mm)	2.5
Output torture (N·m)	19866
Torsion stiffness (N·m/rad)	1.3×10^{6}
Shaft 2, 3 support (radial) stiffness (N/m)	2×10^{9}
Shaft 4 support (radial) stiffness (N/m)	2×10^{10}

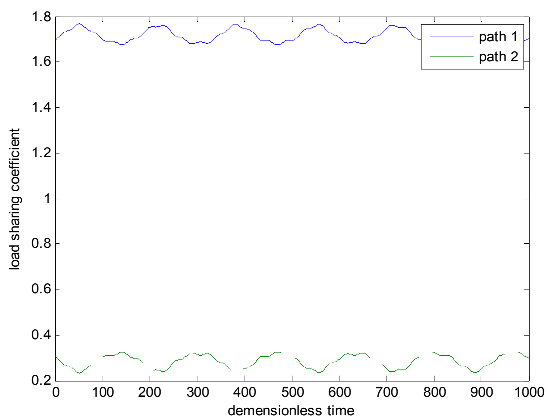

Fig. 3: Load sharing without shaft floating

x is displacement vector, X = (X_1, X_2,…, X_{14}), m is mass matrix, m = diag (m_1, m_2,…,m_{e14}), c is damping matrix, k is stiffness matrix, c and k is a function of time.

Load sharing calculated for each tooth frequency cycle is:

$$b_i = 2 \times F_i / \sum_{1}^{2} F_i \tag{13}$$

The equations can be solved by Runge-Kutta method.

CASE STUDY

Parameters utilized in computational model of split torque transmission are shown in Table 1:

Split torque transmission structure parameter was shown above. Errors was focused to find how these parameter affect the balance of the power path, all error in transmission system was set to 25 μm, if input pinion cannot float, load sharing result was shown in Fig. 3, the two power path was different obviously and it cannot reach the application requirements.

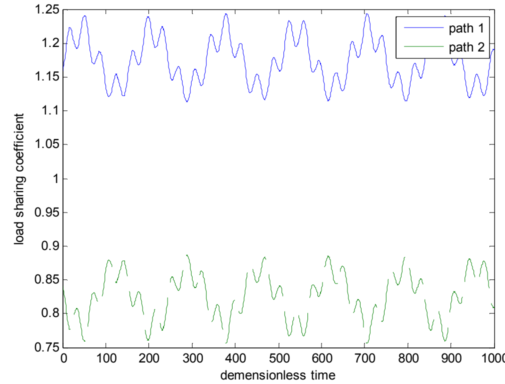

Fig. 4: Load sharing
Shaft 1 support stiffness: 5e7N/m

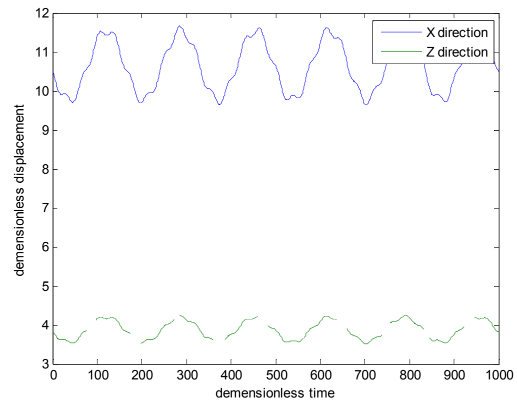

Fig. 5: Shaft 1 displacement
Shaft 1 support stiffness: 5e7N/m

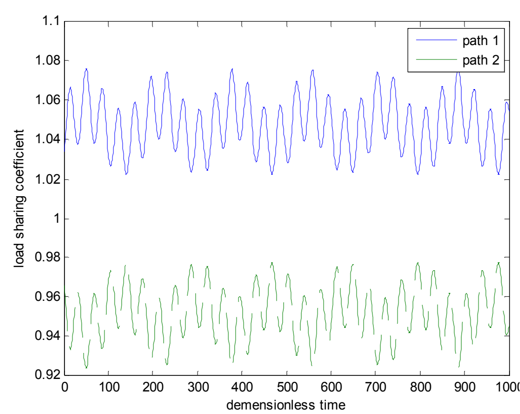

Fig. 6: Load sharing
Shaft 1 support stiffness: 1e7N/m

Input pinion floating can improve load sharing performance. When input shaft support stiffness is 5e7N/m, load sharing and input shaft displacement result was shown in Fig. 4 and 5. Decreasing input shaft support stiffness to 1e7N/m, load sharing and input shaft displacement result was shown in Fig. 6 and 7. Decreasing input shaft support stiffness, load sharing

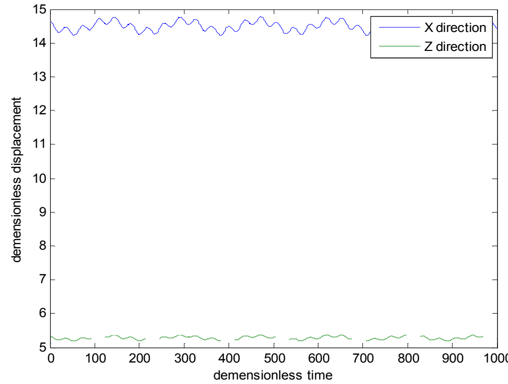

Fig. 7: Shaft 1 displacement
Shaft 1 support stiffness: 1e7N/m

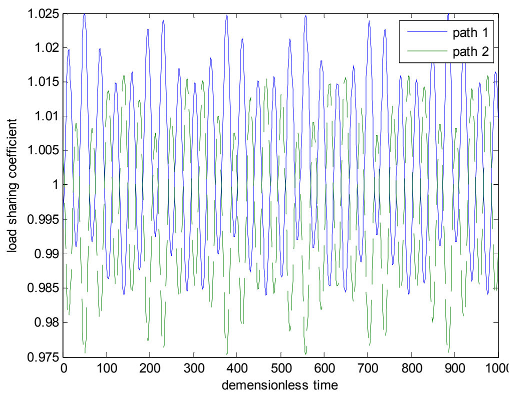

Fig. 8: Load sharing (gear 1 eccentric error)

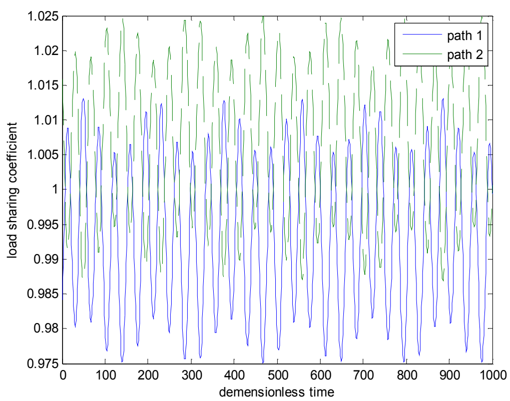

Fig. 9: Load sharing (gear 2 eccentric error)

become better and input shaft's vibration decreases, the displacement of input shaft increases. Input shaft floating can achieve load sharing.

To find how manufacturing error and assembly error affect load sharing, load sharing was calculated in one error condition (one error was set 25 μm while others was set zero). Load sharing affected by gear eccentric error were shown in Fig. 8 to 13. Gear 4 and

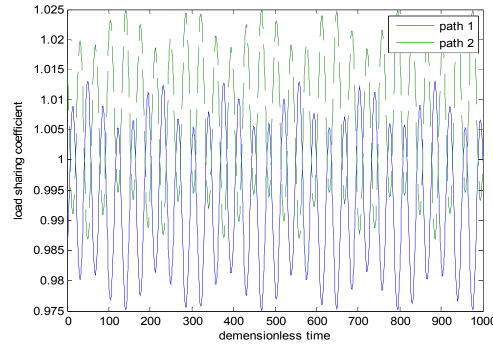

Fig. 10: Load sharing (gear 3 eccentric error)

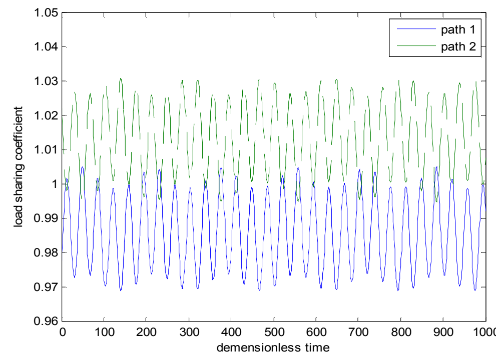

Fig. 11: Load sharing (gear 4 eccentric error)

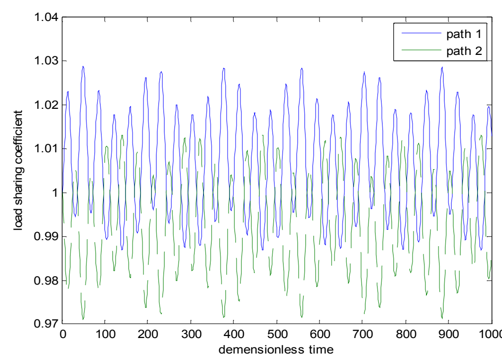

Fig. 12: Load sharing (gear 5 eccentric error)

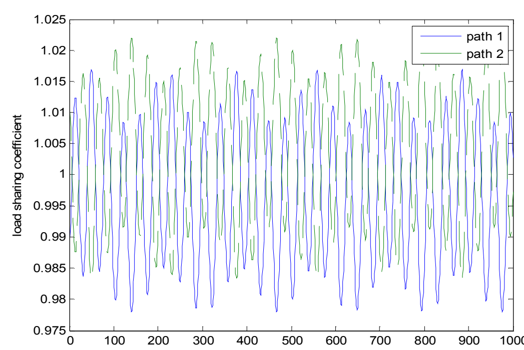

Fig. 13: Load sharing (gear 6 eccentric error)

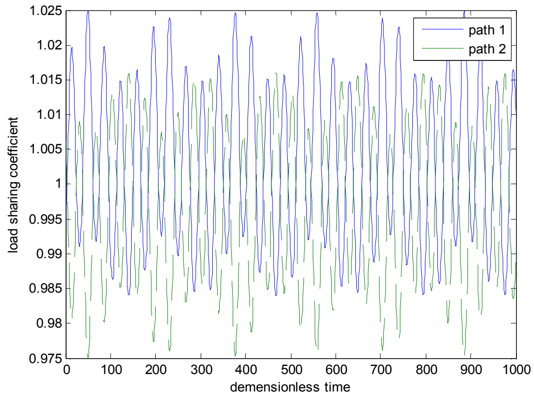

Fig. 14: Load sharing (shaft 1 eccentric error)

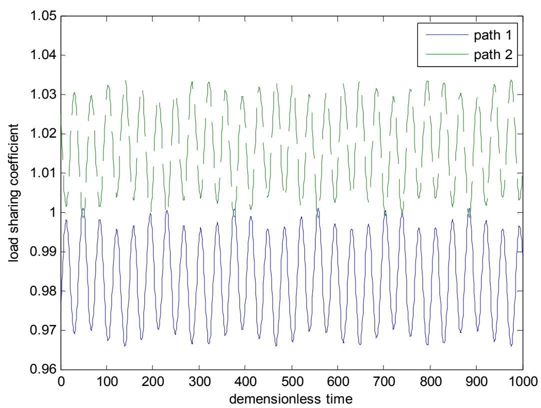

Fig. 15: Load sharing (shaft 2 eccentric error)

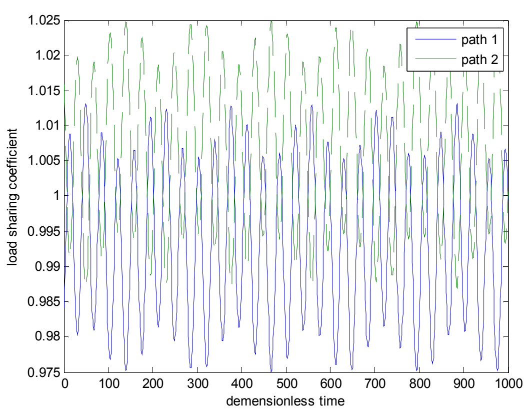

Fig. 16: Load sharing (shaft 3 eccentric error)

gear 5 eccentric error affect load sharing most, others about the same.

Load sharing affected by shaft eccentric error were shown in Fig. 14 to 16. Figure 14 and 8 had the same result because shaft 1 has only one gear on it, gear eccentric and shaft eccentric affected the contact points with the same amount. The same reason, the result of shaft 4 was omitted. The result showed shaft 2 affect load sharing most, others about the same.

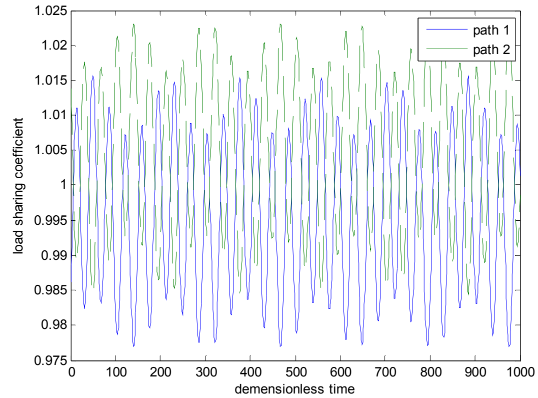

Fig. 17: Load sharing (shaft 2 axis assembly error)

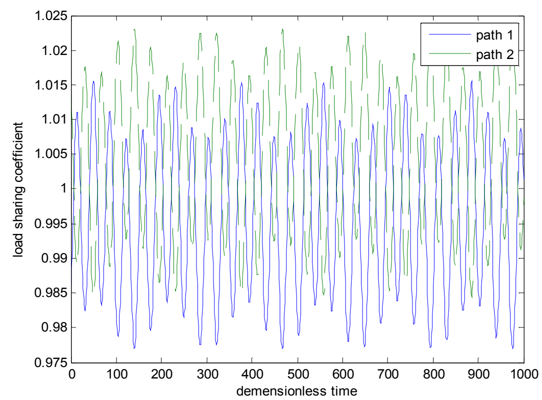

Fig. 18: Load sharing (shaft 3 axis assembly error)

Load sharing affect by shaft axis error were shown in Fig. 17 and 18. Shaft 2 and 3 axis assemble error has same affection to load sharing because of input shaft floating.

All error affect load sharing, but they affect load sharing in different ways. Load sharing was affected most by gear 4 and 5 eccentric error and shaft 2 eccentric error, which can cause the unequal by 3%.

The result shown above were time depended value, assembly error affect load sharing with constant value, which were not shown, the worst level of load sharing no more than the maximum unequal load sharing of time depended value.

Because manufacture error was random and no error can affect load sharing sharply than others, all errors must be strictly controlled.

CONCLUSION

The calculate result shows:

- Load sharing of face gear split torque transmission is affected by manufacturing error and it can be achieved by making input shaft float.
- Load sharing can be improved by decreasing input shaft support stiffness; decreasing input shaft

support stiffness can decrease transmission vibration by increasing the displacement of shaft.

- All errors affect load sharing differently, gear 4 manufacturing error and shaft 2 assemble error affect load sharing most, shaft 2 and 3 axis assemble error has same affection to load sharing because of input shaft floating.
- Load sharing was affected by all errors, one error deduced sharply cannot improve load sharing obviously.

REFERENCES

Cocking, H., 1986. The design of an advanced engineering gearbox [J]. Vertica, 10(2): 213-215.

Gu, J., Z. Fang and H. Pang, 2009a. Modeling and load analysis of spiral bevel gears power split system [J]. J. Aerosp. Power, 24(11): 2625-2629.

Gu, J., Z. Fang and J. Zhou, 2009b. Modeling and power flow analyzing for power split system of spiral bevel gears [C]. Proceedings of the International Workshop on Information Security and Application, Qingdao, China, pp: 401-404.

Heath, G.F., 1993. Advanced Rotorcraft Transmission (ART) program-final report [R]. NASA Contractor Report 191057.

Kish, J.G., 1993. Sikorsky aircraft Advanced Rotorcraft Transmission (ART) program final report [R]. NASA Contractor Report 191079.

Krantz, T.L., 1994. A method to analyze and optimize the load sharing of split-path transmissions [R]. NASA Technical Memorandum 107201.

Krantz, T.L. and I.R. Delgado, 1996. Experimental study of split-path transmission load sharing [R]. NASA Technical Memorandum 107202.

Lu, J., R. Zhu and G. Jin, 2009. Analysis of dynamic load sharing behavior in planetary gearing. J. Mech. Eng., 45(5): 85-90.

White, G., 1974. New family of high-ratio reduction gear with multiple drive paths. Proc. Instn. Mech. Engrs., 188: 281-288.

White, G., 1981. Helicopter transmission arrangements with split-torque gear trains [R]. NASA Conference Publication 2210, pp: 141-150.

White, G., 1983. Design study of a 375-kW helicopter transmission with split-torque epicyclic and bevel drive stages. J. Mech. Eng. Sci., 197(Part C): 213-224.

White, G., 1989. Split-torque helicopter transmission with widely separated engines. Proc. Instn. Mech. Engrs. , 203(G1): 53-65.

White, G., 1998. Design study of a split-torque helicopter transmission. Proc. Instn. Mech. Engrs., 212(Part G): 117-123.