

Research Article

Improving Nonlinear Coordinated Control of Non-ideal STATCOM and Excitation of Salient-pole Generator

Xia Zhu and Jie Wang

Department of Electrical Engineering, Shanghai Jiaotong University, Dongchuan Road 800, Shanghai 200240, China

Abstract: The complexity of controlling is analyzed when the phase difference of voltage and current at the access point of STATCOM is in arbitrary angle β which is not in the ideal state of 90° for the coordinated control of detailed model of the salient-pole generators and STATCOM that is one of the core devices of the FACTS. The characteristics of single machine infinite-bus system including STATCOM are analyzed by establishing the nonlinear system model with algebraic constraint equations of one salient-pole generator and STATCOM and the detail control method of the excitation and STATCOM devices is designed by adopting geometric feedback linearization theory combined with classical linear quadratic optimal control. The method of design in this study expands the applied range of geometric linearization theory used in differential-algebraic systems and makes the coordinated control of excitation and STATCOM more feasible for practical engineering applications, which makes up for the insignificance of the coordinated control research of salient-pole generator excitation and STATCOM.

Keywords: Algebraic constraint equations, coordinated control, nonlinear systems, salient-pole generators, STATCOM

INTRODUCTION

Modern power system is a typical strong nonlinear complex dynamic system and it has been subjected to various random unpredictable disturbances. The compensation devices widely used presently response slowly to the sudden increase in load, which makes the grid lack of the necessary support of dynamic reactive power, thus, the system is prone to be short of reactive power once it has been disturbed and it raises the voltage stability problem. The Flexible AC Transmission Systems (FACTS) technology based on high-power power electronics is widely used in the power industry to enhance the stability and security of power system and to increase the transfer capacity and efficiency; and simultaneously save energy and improve power quality and other aspects.

Static var compensator (STATCOM) is one of the core devices of the FACTS, compared with the traditional compensation devices, it is a small device which has exceptional low voltage characteristic and quick response and thus it becomes the research focus of today's reactive power control area. The application of STATCOM greatly improves the power system reliability, security and stability and it brings enormous economic and social benefits to the electricity industry (Wang and Chen, 2009; Wang *et al.*, 2003).

Meanwhile, as a traditional means of control, the generator excitation has played a critical and important role in the power system stability control (Rahim *et al.*, 2002; Wang, 1999). Therefore, in order to ensure safe and stable operation of complex power system, it is an important research topic at current that how to coordinately control the generator excitation and FACTS devices (Hiskens and Hill, 1989; Chatterjee *et al.*, 2007). The coordinated control of generator excitation and FACTS systems has caused people to pay attention and has been extensively studied (Geethalakshmi and Dananjayan, 2009; Xie *et al.*, 2001; Mithulananthan *et al.*, 2003; Kuiava *et al.*, 2009; Faried *et al.*, 2009; Padiyar and Prabhu, 2006; Du *et al.*, 2011; Singh *et al.*, 1990; Jiang *et al.*, 2011). But so far, the researches related to the salient-pole generator and STATCOM controller are few in domestic and foreign literature. In particular, it is the lack of analysis of salient-pole generator excitation and STATCOM coordinated control.

The robust controller is designed independently without considering the interconnection of the excitation controller comprehensively (Lesieutre *et al.*, 1999); a decentralized coordinated control strategy of generator excitation and FACTS is proposed (Senjyu *et al.*, 1996), however, this method applies only in the use of FACTS devices which utilize no-dynamic process and controllable impedance model, instead of

Corresponding Author: Xia Zhu, Department of Electrical Engineering, Shanghai Jiaotong University, Dongchuan Road 800, Shanghai 200240, China

This work is licensed under a Creative Commons Attribution 4.0 International License (URL: <http://creativecommons.org/licenses/by/4.0/>).

STATCOM devices with first-order delay model; the strong nonlinear characteristics of the power system is considered more comprehensively, the state space model including STATCOM is established for the single Machine Infinite Bus System (SMIB) and the four coordinated control strategies are designed through the application of optimal control theory and differential algebra algorithm, the theory and algorithm of information structure constrained linear quadratic optimal control (Xie *et al.*, 2002). However, this design above uses a CAD model and it assumes that the STATCOM terminal voltage vector is always in the same direction with d-axis, thus it is difficult to apply to the actual systems engineering for these limits above; The design process of salient-pole generator excitation and STATCOM coordination controller is introduced (Gu and Wang, 2006), although the design method is easy to be realized in actual project implementation, it is based on non-salient-pole generators ($x_d = x_q$) to establish the system model and it assumes that the voltage vector of STATCOM at access point is always in the same direction with d-axis; The coordinated controller is also designed about the salient-pole generator excitation and STATCOM, but the premise of its assumptions is that the phases of voltage and current are vertical to each other (Li and Wang, 2011). In the actual system, in order to achieve different control for the inductive and capacitive of STATCOM, the phase difference is usually less than 90° , thus the design also has its limitations in theoretical and practical applications.

On the basis of aforementioned research work, for the coordinated control of detail salient-pole generator model ($x_d \neq x_q$) and STATCOM, which is one of core devices of the FACTS, the complexity of control is analyzed when β is arbitrary instead of the ideal state of 90° in this study. We take δ_l is an arbitrary constant; the nonlinear system model consisted by the salient-pole generator and STATCOM with constraint algebraic equations is established. The detail coordinated control method of the excitation and STATCOM devices is given. The design method expands the applied range of geometric linearization theory and controller and makes the STATCOM model and control method more feasible for practical engineering applications by using geometric feedback linearization control theory combined with the classic linear-quadratic optimal control method (Wang and Chen, 2009; Wang *et al.*, 2003), which makes up for the insignificance of technology research on coordinately controlling salient-pole generator excitation and STATCOM. The transient simulation results show the rationality and effectiveness of the method and the control algorithm proposed in this study.

DESCRIPTION OF THE STATCOM MODEL

Take voltage bridge circuit of STATCOM as an example, its operating principle can be illustrated in Fig. 1, In order to facilitate the study, we use the first-

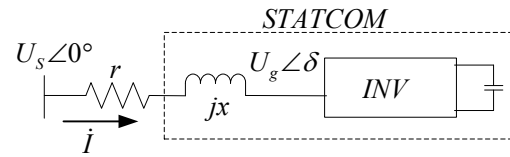


Fig. 1: Single-phase equivalent circuit diagram of the voltage bridge circuit

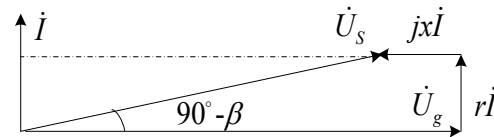


Fig. 2: Vector schematic of the voltage bridge circuit

order delay controllable reactive current source model as the model of STATCOM (Ni and Snider, 1997), which means that the device acts as a shunt current source to the grid and its internal structure is ignored. In addition, we also assume that the frequencies of the output current and voltage at access point are the same. Fig. 1 shows the Single-phase equivalent circuit diagram of the voltage bridge circuit. And Fig. 2 shows the vector schematic of the voltage bridge circuit.

STATCOM has two operating cases: The capacitive conditions and the inductive conditions. STATCOM will absorb capacitive reactive power from the system and provide reactive support for the system if it is in the capacitive operating mode; otherwise, STATCOM will absorb inductive reactive power from the system when it is in the inductive conditions.

For the reason of connecting inductive reactance, the electric grid needs to provide active power to compensate the active power loss in the STATCOM circuit and maintain the stability of the capacitor voltage on the DC side. In addition, it will only be an ideal state that the voltage U and S and current I^* are mutually perpendicular at access point of STATCOM in system grid. As far as we know, most of the existing literature only considered the ideal state currently, but phase difference of STATCOM output voltage U and g and current I are still in a difference of 90° , we take the phase difference of U and S and I as β and we will analyze the case of fault restoration system in different angles β .

The establishment of STATCOM model: Consider the SMIB installed with a STATCOM device at a point of the transmission line in this system shown in Fig. 3, the generator model is presented as the third-order salient-pole model in the controller design: the excitation voltage E_{fd} of generator and the u_s in STATCOM model are taken as the control inputs of the dynamic system, we assume P_m constant while the influence of speed governor isn't considered. Fig. 3

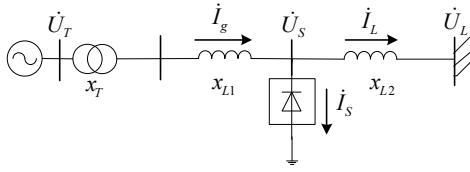


Fig. 3: The equivalent circuit of a SMIB with STATCOM device

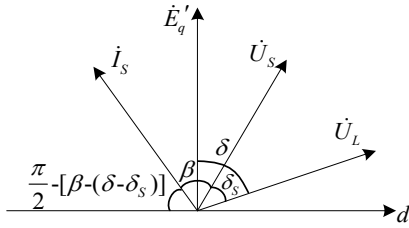


Fig. 4: System vector diagram

shows the equivalent circuit of a SMIB with STATCOM device.

The system vector diagram is showed in Fig. 4, vector E and q always is assumed to keep the same direction with q axis, d-axis lags 90° behind the q-axis. δ is the angle between E and q and U and L, δ_s is the angle between vector U and L at the node of STATCOM and infinite bus system voltage vector U and L .

Consider the generator rotor motion equation and electromagnetic dynamic equations as follows:

$$\frac{d\delta}{dt} = \omega - \omega_0; \quad (1a)$$

$$\frac{d\omega}{dt} = -\frac{D}{2H}(\omega - \omega_0) - \frac{\omega_0}{2H}(P_e - P_m) \quad (1b)$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{d0}}[E_{fd} - E'_q - (x_d - x'_d)I_{gd}] \quad (1c)$$

$$0 \leq E_{fd} \leq 5$$

The dynamic equation of reactive current of STATCOM can be expressed as Eq.1d:

$$\frac{dI_s}{dt} = \frac{1}{T_s}(u_s - I_s) - 0.1 \leq u_s \leq 0.1 \quad (1d)$$

We have Eq.2a to Eq.2d from Fig. 3:

$$I_g = I_L + I_s \quad (2a)$$

$$U_T = U_L + jx_{L2}I_L + j(x_T + x_{L1} + x_T)I_g \quad (2b)$$

$$U_s - U_L = jx_{L2}I_L = jx_{L2}(I_g - I_s) \quad (2c)$$

$$U_s e^{j\left(\frac{\pi}{2} - (\delta - \delta_s)\right)} - U_L e^{j\left(\frac{\pi}{2} - \delta\right)} = jx_{L2} \left(I_g + I_s e^{-j\left[\frac{\pi}{2} - (\beta - \delta + \delta_s)\right]} \right) \quad (2d)$$

Combine it with equation:

$$U_T = x_q I_{gq} + j(E'_q - x_d' I_{gd})$$

we can get the relationship of other system variables:

$$I_{gd} = \frac{E'_q - U_L \cos \delta - I_s X_{L2} \sin(\beta - \delta + \delta_s)}{x_T + x_{L1} + x_{L2} + x_d'} \quad (3a)$$

$$I_{gq} = \frac{U_L \sin \delta + I_s X_{L2} \cos(\beta - \delta + \delta_s)}{x_T + x_{L1} + x_{L2} + x_q} \quad (3b)$$

$$P_e = [E'_q + (x_q - x_d')I_{gd}]I_{gq} \quad (3c)$$

and the following differential algebraic equations are also obtained:

$$0 = U_s \sin(\delta - \delta_s) - \gamma_q U_L \sin \delta - \gamma_q x_{L2} I_s \cos(\beta - \delta + \delta_s) \quad (3d)$$

$$\underline{\Delta} \sigma_1(\delta, \omega, E'_q, I_s; U_s, \delta_s)$$

$$0 = U_s \cos(\delta - \delta_s) - \gamma_d U_L \cos \delta - (1 - \gamma_d)E'_q + \gamma_d x_{L2} I_s \sin(\beta - \delta + \delta_s)$$

$$\underline{\Delta} \sigma_2(\delta, \omega, E'_q, I_s; U_s, \delta_s) \quad (3e)$$

in which:

$$\gamma_d = \frac{x_d' + x_{L1} + x_T}{x_d' + x_{L1} + x_{L2} + x_T}; \quad \gamma_q = \frac{x_q + x_{L1} + x_T}{x_q + x_{L1} + x_{L2} + x_T}$$

Finally, we expressed the algebra Eq. (3d) and Eq. (3e) in the form of implicit function as follows: $0 = \sigma_1(x, z)$; $0 = \sigma_2(x, z)$, where the state variables are:

$$\mathbf{x} = (\delta, \omega, E'_q, I_s)^T \quad (4a)$$

and the algebraic variables are:

$$\mathbf{z} = (U_s, \delta_s)^T \quad (4b)$$

Thus the Eq. (1) and Eq. (4) represent dual-input dual-output differential algebraic control system which consists of four state variables and two algebra variables.

Discussion of the algebraic equations: For algebra Eq. (3d) and Eq. (3e), they can be further arranged as follows:

$$A_{11} \sin(\delta - \delta_s) + A_{12} \cos(\delta - \delta_s) = B_1 \quad (5a)$$

$$A_{21} \sin(\delta - \delta_s) + A_{22} \cos(\delta - \delta_s) = B_2 \quad (5b)$$

where

$$\begin{aligned} A_{11} &= U_s - \gamma_q x_{L_2} I_s \sin \beta; \quad A_{12} = -\gamma_q x_{L_2} I_s \cos \beta; \\ A_{21} &= \gamma_d x_{L_2} I_s \cos \beta; \quad A_{22} = U_s - \gamma_d x_{L_2} I_s \sin \beta; \\ B_1 &= \gamma_q U_L \sin \delta; \quad B_2 = (1 - \gamma_d) E'_q + \gamma_d U_L \cos \delta, \end{aligned}$$

since

$$\begin{aligned} \Delta &\triangleq \begin{vmatrix} U_s - \gamma_q x_{L_2} I_s \sin \beta & -\gamma_q x_{L_2} I_s \cos \beta \\ \gamma_d x_{L_2} I_s \cos \beta & U_s - \gamma_d x_{L_2} I_s \sin \beta \end{vmatrix} \quad (6) \\ &= (U_s - \xi_\beta x_{L_2} I_s)^2 + \gamma_d \gamma_q x_{L_2}^2 I_s^2 - \xi_\beta^2 x_{L_2}^2 I_s^2 \\ &= \begin{pmatrix} U_s & x_{L_2} I_s \end{pmatrix} \begin{bmatrix} 1 & -\xi_\beta \\ -\xi_\beta & \gamma_d \gamma_q \end{bmatrix} \begin{pmatrix} U_s \\ x_{L_2} I_s \end{pmatrix} \end{aligned}$$

take $\xi_\beta = (\gamma_d + \gamma_q) / 2 \sin \beta$, if the matrix $\begin{bmatrix} 1 & -\xi_\beta \\ -\xi_\beta & \gamma_d \gamma_q \end{bmatrix}$ in Eq.6 is reversible, then the equation followed should be satisfied: $\gamma_d \gamma_q \neq \xi_\beta^2$, under this condition, the Eq. (5) is solvable. Of course, it can also be shifted to eliminate the U_s in algebraic equations and then we have:

$$\begin{aligned} 0 &= \sigma^*(\delta, \omega, E'_q, I_s; \delta_s) \quad (7) \\ &= \sin(\delta - \delta_s) [\gamma_d U_L \cos \delta + (1 - \gamma_d) E'_q + (1 - \gamma_d) I_s \\ &\quad E'_q + \gamma_d x_{L_2} I_s \sin(\beta - \delta + \delta_s)] \\ &\quad - \cos(\delta - \delta_s) [\gamma_q U_L \sin \delta + \gamma_q x_{L_2} I_s \cos(\beta - \delta + \delta_s)] \end{aligned}$$

If the following equation isn't equal to zero, namely,

$$\begin{aligned} \frac{\partial \sigma^*}{\partial \delta_s} &= (\gamma_q - \gamma_d) [x_{L_2} I_s \sin(\beta - 2\delta + 2\delta_s) - U_L \sin \delta \sin(\delta - \delta_s)] \quad (8) \\ &\quad - \gamma_d U_L \cos \delta_s - (1 - \gamma_d) E'_q \cos(\delta - \delta_s) \end{aligned}$$

then the algebraic Eq. (3) is meaningful, δ_s and U_s can be obtained in turn.

Nonlinear controller design: Consider the general MIMO nonlinear differential algebraic system (Wang *et al.*, 2003):

$$\dot{x} = f(x, z) + \sum_{i=1}^m g_i(x, z) u_i \quad (9a)$$

$$0 = \sigma(x, z) \quad (9b)$$

$$y = h(x, z) \quad (9c)$$

$h = (h_1, L, h_m)^T$. Assume that the M relative degree of system (9) satisfies $r = r_1 + L + r_m \leq n$ (Wang *et al.*, 2003), then $M_h M_f^h h(x, z) = 0$ ($k \in Y_1^{r-2}$) holds and $M^g M^{r-1} h(x, z) \neq 0$ holds, by using the definitions of M derivative and M brackets (Wang *et al.*, 2003), we can choose a coordinate transformation mapping $E_\Phi = \Phi(x, z) \in \mathbb{R}^n$ as follows:

$$\begin{aligned} w_i &= \varphi_{1,i}(x, z) = M_f^{i-1} h_1(x, z), \quad i \in N_1^{r_1} \\ w_{r_1+j} &= \varphi_{2,j}(x, z) = M_f^{j-1} h_2(x, z), \quad j \in N_1^{r_2} \dots \\ w_{r_1+r_2+L+r_{m-1}+k} &= \varphi_{m,k}(x, z) = M_f^{k-1} h_m(x, z) \\ k &\in N_1^{r_m} \end{aligned}$$

which makes the Jacobi matrix of the mapping vector

$$\Phi(x, z) = [\varphi_{1,1}(x, z), L, \varphi_{m,r_m}(x, z)]^T \quad (10)$$

is nonsingular at some operating point (x_0, z_0) of system (9), then the original system (9) can be transformed into a normal form.

If the rank of the following matrix satisfies

$$\text{rank} \left[M_{g_i} M_f^{r_j-1} h_j(x, z) \right]_{m \times m} = m \quad (11)$$

then we can find the control inputs u_1, L, u_m , such that the standard form of system for the equilibrium point is asymptotically stable.

For the purpose of the design, the controller design must simultaneously meet the requirements of the stability of generator power angle and the voltage stability of STATCOM. According to the actual situation, we define the outputs as: $y_1 = h_1(x, z) = \delta$; $y_2 = h_2(x, z) U_s$, then the original system (1) is a dual-input dual-output nonlinear system which can be expressed as follows (Wang *et al.*, 2003):

$$\dot{x} = f(x, z) + \sum_{i=1}^2 g_i(x, z) u_i \quad (12a)$$

$$0 = \sigma(x, z) \quad (12b)$$

$$y = h(x, z) \quad (12c)$$

where $0 = \sigma(x, z)$ is the system algebraic equation and

$$f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2H}(\omega - \omega_0) - \frac{\omega_0}{2H}(P_e - P_m) \\ \frac{1}{T_{d0}}[-E'_q - (x_d - x'_d)I_{gd}] \\ -\frac{1}{T_s}I_s \end{pmatrix}, g_1 = \begin{pmatrix} 0 \\ 0 \\ g_{11} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_{d0}} \\ 0 \end{pmatrix},$$

$$g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ g_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_s} \end{pmatrix}, x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \delta \\ \omega - \omega_0 \\ E'_q \\ I_s \end{pmatrix}, u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} E_{fd} \\ u_s \end{pmatrix}$$

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \delta \\ U_s \end{pmatrix}, z = \begin{pmatrix} U_s \\ \delta_s \end{pmatrix}$$

It should be notice that other intermediate variables can be obtained by Eq. (2) and Eq. (3) in the above formula. According to nonlinear differential algebraic geometry linearization theory in Wang *et al.* (2003), we can choose coordinate transformation for system (12) as follows:

$$\xi_1 = h_1(x, z) = \delta \tag{13a}$$

$$\dot{\xi}_1 = M_f h_1 + M_g h_1 u = \dot{\delta} = \omega - \omega_0 = \xi_2 \tag{13b}$$

$$\dot{\xi}_2 = M_f \xi_2 + M_g \xi_2 u = -\frac{D}{2H}(\omega - \omega_0) - \frac{\omega_0}{2H}(P_e - P_m) = \xi_3 \tag{13c}$$

$$\dot{\xi}_3 = M_f \xi_3 + M_g \xi_3 u = u_1^* \tag{13d}$$

$$\dot{\xi}_4 = \dot{I}_s = u_2^* \tag{13e}$$

The system (12) can be rearranged as:

$$\dot{\xi}(x, z) = A \xi(x, z) + B u^* \tag{14}$$

in which:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix};$$

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)^T; u^* = (u_1^*, u_2^*)^T$$

Set $\xi = \Phi(x, z)$, according to the system controllable criterion, rank(B, AB, A²B, A³B) = 4, it's known that system (14) consisted by new state variables is fully controllable, where the Jacobi matrix of $\Phi(x, z) = [\xi_1(x, z), \xi_2(x, z), \xi_3(x, z), \xi_4(x, z)]^T$ at point (x_0, z_0) is:

$$J_\Phi = E_\sigma[(x, z)]|_{(x_0, z_0)} @ \frac{\partial \Phi}{\partial x} - \frac{\partial \Phi}{\partial z} \left(\frac{\partial \sigma}{\partial z} \right)^{-1} \frac{\partial \sigma}{\partial x} \tag{15}$$

Since

$$\frac{\partial \Phi}{\partial x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\partial \xi_3}{\partial \delta} & \frac{\partial \xi_3}{\partial \omega} & \frac{\partial \xi_3}{\partial E'_q} & \frac{\partial \xi_3}{\partial I_s} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

$$\frac{\partial \Phi}{\partial z} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial \xi_3}{\partial \delta_s} & 0 \end{pmatrix}^T \tag{17}$$

$$\frac{\partial \sigma}{\partial x} = \begin{pmatrix} \frac{\partial \sigma_1}{\partial \delta} & \frac{\partial \sigma_1}{\partial(\omega - \omega_0)} & 0 & \frac{\partial \sigma_1}{\partial I_s} \\ \frac{\partial \sigma_2}{\partial \delta} & \frac{\partial \sigma_2}{\partial(\omega - \omega_0)} & \frac{\partial \sigma_2}{\partial E'_q} & \frac{\partial \sigma_2}{\partial I_s} \end{pmatrix} \tag{18}$$

For notation simple, we assume $(\partial \sigma / \partial z)^{-1}$ exists and note that:

$$\left(\frac{\partial \sigma}{\partial z} \right)^{-1} = \begin{pmatrix} \frac{\partial \sigma_1}{\partial U_s} & \frac{\partial \sigma_1}{\partial \delta_s} \\ \frac{\partial \sigma_2}{\partial U_s} & \frac{\partial \sigma_2}{\partial \delta_s} \end{pmatrix}^{-1} @ \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \tag{19}$$

also, because:

$$J_\Phi = E_\sigma \Phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{20}$$

where $\Gamma_{33} = \partial \xi_3 / \partial E'_q - (\partial \xi_3 / \partial \delta_s) \gamma_{22} (\partial \sigma_2 / \partial E'_q)$ and:

$$\frac{\partial \xi_3}{\partial E'_q} = -\frac{\omega_0}{2H} \frac{\partial P_e}{\partial E'_q} = -\frac{\omega_0}{2H} \left(1 + \frac{x_q - x_{d'}}{x_{T L_1 L_2 d'}} \right) I_{gq} \tag{21}$$

$$= -\frac{\omega_0 U_L \sin \delta + X_{L_2} I_s \cos(\beta - \delta + \delta_s)}{2H x_{T L_1 L_2 d'}} \tag{22}$$

$$\frac{\partial \xi_3}{\partial \delta_s} = \frac{\omega_0}{2H} [E'_q + (x_q - x_{d'}) I_{gd}] \frac{x_{L_2} I_s \sin(\beta - \delta + \delta_s)}{x_{T L_1 L_2 q}}$$

$$+ \frac{\omega_0}{2H} (x_q - x_{d'}) \frac{x_{L_2} I_s \cos(\beta - \delta + \delta_s)}{x_{T d' L_1 L_2}} I_{gq}$$

$$\begin{aligned}
 &= \frac{\omega_0 (x_q - x_d') x_{L2} I_S}{2H x_{Td'1L2} x_{T1L2q}} \{ \cos(\beta - \delta + \delta_s) [U_L \sin \delta + I_S X_{L2} \cos(\beta - \delta + \delta_s)] \\
 &+ [\frac{x_{T1L2q}}{x_q - x_d'} E_q' - U_L \cos \delta - X_{L2} I_S \sin(\beta - \delta + \delta_s)] \sin(\beta - \delta + \delta_s) \} \\
 &= \frac{\omega_0 (x_q - x_d') x_{L2} I_S}{2H x_{Td'1L2} x_{T1L2q}} \{ U_L \sin(2\delta - \beta - \delta_s) + X_{L2} I_S \cos(2\delta - 2\beta - 2\delta_s) \} \\
 &+ \frac{\omega_0 x_{L2} I_S E_q'}{2H x_{T1L2q}} \sin(\beta - \delta + \delta_s) \\
 &\frac{\partial \sigma_2}{\partial E_q'} = 1 - \gamma_d' \tag{23}
 \end{aligned}$$

Through the analysis, we know that Jacobi matrix J_Φ is non-singular at point (x_0, z_0) if $\Gamma_{33} \neq 0$, that is

$$\begin{aligned}
 \Gamma_{33} &= \frac{\partial \xi_3}{\partial E_q'} - \frac{\partial \xi_3}{\partial \delta_s} \gamma_{22} (1 - \gamma_d') \tag{24} \\
 &= -\frac{\omega_0}{2H x_{T1L2q}} \{ X_{L2} I_S [\cos(\beta - \delta + \delta_s) + (1 - \gamma_d') \gamma_{22} E_q' \sin(\beta - \delta + \delta_s)] \\
 &+ U_L \sin \delta + (1 - \gamma_d') + \frac{(x_q - x_d') \gamma_{22} x_{L2} I_S}{x_{T1L2q}} [U_L \sin(2\delta - \beta - \delta_s) \\
 &+ X_{L2} I_S \cos(2\delta - 2\beta - 2\delta_s)] \}
 \end{aligned}$$

For general power system, $0 < \delta < \pi/2$ holds, by calculating, we know that $\partial \sigma / \partial \delta_s \neq 0$ and $\Gamma_{33} \neq 0$, thus further Jacobi matrix J_Φ is non-singular, so the original system can be geometric feedback linearized.

The role of the control input: For system (12), by the use of nonlinear optimal control method (Hiskens and Hill, 1989), the generalized quadratic performance index of the system is:

$$\min J = \int_0^\infty (x^T Q_1 x + u^T Q_2 u) dt \tag{25}$$

we may wish to choose matrices $Q_1 = \text{diag}(3600, 9000, 2500, 6400)$ and $Q_2 = \text{diag}(1, 1)$, then the control vector v can be obtained to make the performance index function J_{\min} available to the extreme value:

$$v^* = -Q_2^{-1} B^T P^* \xi = -K^* \xi \tag{26}$$

namely,

$$K^* = Q_2^{-1} B^T P^* \tag{27}$$

which can be obtained is that

$$P^* = \begin{bmatrix} 15448 & 2730 & -50 & 0 \\ 2730 & 16922 & -309 & 0 \\ -50 & -309 & -55 & 0 \\ 0 & 0 & 0 & 80 \end{bmatrix}$$

The corresponding optimal feedback gain matrix is:

$$K^* = Q_2^{-1} B^T P^* = \begin{bmatrix} -50 & -309 & -55 & 0 \\ 0 & 0 & 0 & 80 \end{bmatrix} \tag{28}$$

The optimal control can be expressed as:

$$u_1^* = 50 \xi_1 + 309 \xi_2 + 58.6 \xi_3 \tag{29a}$$

$$u_2^* = -80 \xi_4 \tag{29b}$$

For the design of dual-input dual-output system (1) and (13), the optimal controls are as follows expressions:

$$u_1^* = \xi_3^* = F_1(x, z) + a_{11}(x, z)u_1 + a_{12}(x, z)u_2 \tag{30a}$$

$$u_2^* = \xi_4^* = F_2(x, z) + a_{22}(x, z)u_2 \tag{30b}$$

where, $a_{21}(x, z) = 0$. We can obtain the control inputs from Eq. (30) as follows:

$$u_1 = \frac{\Omega_1}{\Delta}; u_2 = \frac{\Omega_2}{\Delta} \tag{31}$$

where,

$$\begin{aligned}
 \Delta &= a_{11}(x, z)a_{22}(x, z) \\
 \Omega_1 &= (v_1 - F_1(x, z))a_{22}(x, z) - (v_2 - F_2(x, z))a_{12}(x, z) \\
 \Omega_2 &= (v_2 - F_2(x, z))a_{11}(x, z)
 \end{aligned}$$

in which

$$\Delta = a_{11}(x, z)a_{22}(x, z) \neq 0; u_1 = E_{fd}, u_2 = u_s$$

SIMULATION RESULTS

A simulation for the SMIB system in Fig. 5 is presented to evaluate the effectiveness of the controller proposed in this study. The system parameters used in

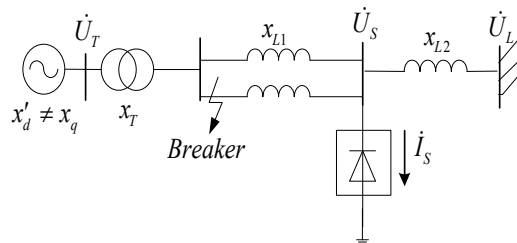


Fig. 5: Experiment simulation systems

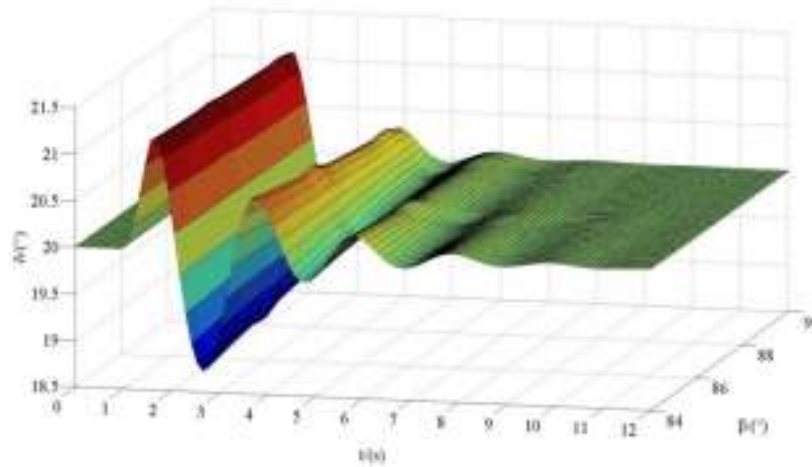


Fig. 6: The power angle response curve of generator

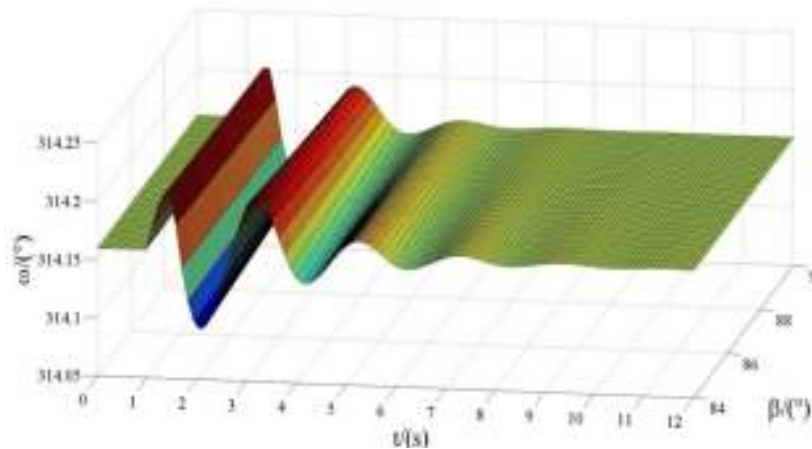


Fig. 7: Angular rate response curve

simulation are as follows: $x_d = 0.76$ pu, $x'_d = 0.66$ pu, $x_q = 0.76$ pu, $H = 10$ S, $D = 5$ pu, $x_T = 0.1$ pu, $x_{L1} = 0.2$ pu, $x_{L2} = 0.3$ pu, $P_m = 0.9$ pu, $T'_{d0} = 10$ S, $U_L = 1$ pu, $\omega_0 = 314.15$ rad/S; STATCOM: $S = 0.12$ pu, $-0.1 \leq u_s \leq 0.1$.

Analysis: for the initial operating equilibrium point (1.5714, 314.1593, 1.0408, 0.011) in this single machine infinite bus system, we can know that $\partial\sigma^*/\partial\delta_s \neq 0$ for Eq. (8), the algebraic equations have meaning, δ_s and U_s can be obtained in turn, the solution of original system exists and the simulation results are shown in Fig. 6 to 9.

For STATCOM, we have:

$$S = P + jQ = \dot{U} \dot{I}$$

$$= \frac{U_s^2}{2R} \sin 2\left(\frac{\pi}{2} - \beta\right) + j \frac{U_s^2}{R} \sin^2\left(\frac{\pi}{2} - \beta\right) \quad (32)$$

where β is close to 90° , normally, $(\pi/2) - \beta = 0 \square \sim \pm 6^\circ$, thus we have:

$$P^* = \frac{1}{R^*} \sin^2\left(\frac{\pi}{2} - \beta\right) \approx \frac{1}{R^*} \left(\frac{\pi}{2} - \beta\right)^2 \quad (33)$$

$$Q^* = \frac{1}{2R^*} \sin 2\left(\frac{\pi}{2} - \beta\right) \approx \frac{1}{R^*} \left(\frac{\pi}{2} - \beta\right) \quad (34)$$

The power angle response curve of generator is shown in Fig. 6; and the angular rate response curve is shown in Fig. 7 and 8 represents the bus voltage response curve where STATCOM is accessed; Fig. 9 shows the transient potential response curve.

When the output is rated reactive power:

$$P_N^* = \frac{1}{R^*} \left(\frac{\pi}{2} - \beta_N\right)^2 \quad (35)$$

$$\frac{1}{R^*} \left(\frac{\pi}{2} - \beta_N\right) = Q_N^* = 1 \quad (36)$$

$$\left|\frac{\pi}{2} - \beta_N\right| = P_N^* = R_N^* \quad (37)$$

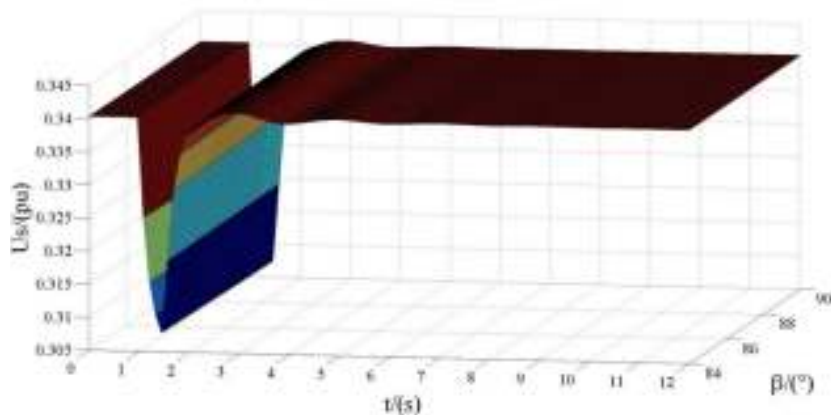


Fig. 8: Bus voltage response curve where STATCOM is accessed

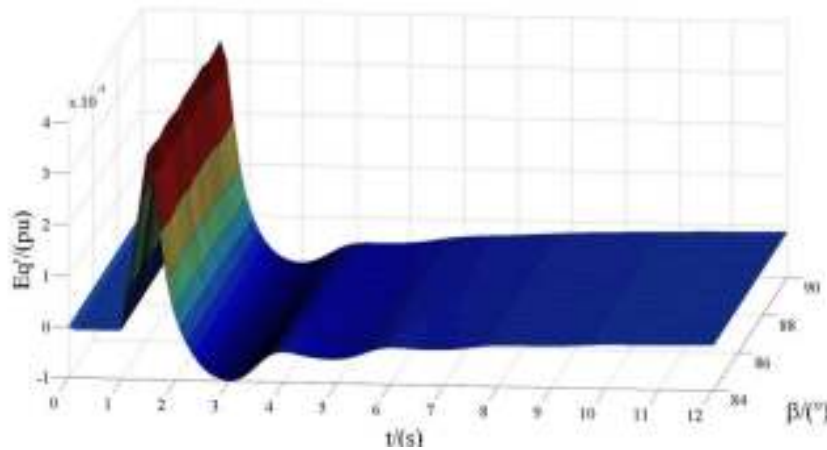


Fig. 9: Transient potential response curve

The size change of β is determined by the rated losses. In actual operating, $(\pi/2)-\beta$ is a very small absolute value of the angle and varies within a few degrees in plus or minus (Dai and Chen, 2009). It can be seen that the reactive power output in the steady state of the STATCOM device is proportional to the angle of control value. It may cause much reactive power output of STATCOM if β emerges small change (Canizares, 2000; Narne *et al.*, 2011).

It can be seen from the waveform that the system responses faster when β is closer to 90° , but the voltage will be a serious distortion. By the working principle of STATCOM, we can know that β will change if control the size of the filtering inductance L . The smaller L is, the smaller the electro-magnetic time constant of the converter circuit is, then DC containers can complete energy exchange between the power sources in less time and the dynamic response of reactive power compensation current is faster. In addition, if L is small, then its power electronic devices have low pressure requirements, simple structure, small size, low cost and faster dynamic response. However, when the filter inductor is small, the harmonic content of the output

reactive current will increase, the ability of anti-supply voltage disturbance of the devices would be poor and its requirements for control system will be higher.

CONCLUSION

Previous researches about STATCOM all are based on the ideal work conditions, though the design is simple and easy to be realized, it has limitations if it is used into actual engineering design. The problems of coordinate control of STATCOM device under non-ideal state and traditional excitation mode of salient-pole generator are studied in this study. The influence of non-vertical at arbitrary angle β for the effectiveness of the control under actual operating state is discussed. Using the methods of geometric feedback exact linearization, zero dynamic design principle and classic quadratic optimal control, we design nonlinear coordination controller of STATCOM and generator excitation, which can make the dynamic system asymptotically stable, this controller can simultaneously meet the requirements of generator excitation and the stability of STATCOM voltage. Take SMIB system as

an example, using the method proposed in this study, we design the nonlinear excitation controller. It can be seen from the curve of simulation results that this control scheme not only perform well in stabling the system work angle and voltage and it works out differently in dealing with improving the transient stability and also dynamic performance when STATCOM operates in different state angle β . Therefore, the simulation results prove the effectiveness and practicality of the model and the coordinated control method proposed in this study.

NOMENCLATURE

D	= The damping coefficient
H	= Inertial coefficient
T'_{do}	= Inertia time constant of the generator
\dot{U}_s, \dot{I}	= The voltage and current at access point of STATCOM in System grid
\dot{I}_s	= STATCOM reactive current output
β	= The phase difference of \dot{U}_s , and \dot{I}_s of STATCOM
U_L, δ_L	= The voltage amplitude value in the infinite bus system at the reference point and the corresponding phase angle
u_s	= Taken as the control input (The meaning of the symbol u_s will be explained in Eq.1d)
E_{fd}	= The excitation voltage of generator
E'_q	= The generator transient electromotive force
P_m	= The input mechanical power
\dot{U}_T	= The generator terminal voltage
\dot{U}_L	= The infinite bus voltage vector
X_T	= The equivalent reactance of the generator
X_{L1}, X_{L2}	= The equivalent reactances of the transmission line
$X(x_1, x_2, \dots, x_n)^T$	= The state vector $\mathbf{x} \in \mathbb{R}^n$
$Z(z_1, z_2, \dots, z_l)^T$	= The algebraic vector $\mathbf{z} \in \mathbb{R}^l$
$y(y_1, y_2, \dots, y_m)^T$	= The output vector $\mathbf{y} \in \mathbb{R}^m$
$\mathbf{u}_i (\mathbf{i} \in \mathbb{N}_1^m)$	= Input vector ($\mathbf{i} \in \mathbb{N}_k^p$ means $i = k, k+1, \dots, p, k \leq p$) L, p, K ≤ p)
$f(x, z), g_i(x, z)$	= The n, m and l dimension smooth vector fields
$\Phi(x, z)$	= The Jacobi matrix at point (x_0, z_0) $\Phi(x, z) = [\xi_1(x, z), \xi_2(x, z), \xi_3(x, z), \xi_4(x, z)]^T$
$Q_1(t) \in R^{n \times n}$	= Positive semi-definite state weighting matrix
$Q_2(t) \in R^{r \times r}$	= Positive definite control weight matrix

v^*	= The optimal control vector
K^*	= The optimal feedback gain matrix
P^*	= The solution of Riccati matrix equation $A^T P + PA - PBQ^{-1}B^T P + Q_1 = 0$
L	= The filter inductance of STATCOM

ACKNOWLEDGMENT

This study was supported by National Natural Science Foundation of China (No. 61074042).

REFERENCES

- Canizares, C.A., 2000. Power flow and transient stability models of FACTS controllers for voltage and angle stability studies. Proceeding of the IEEE Winter Meeting on Power Engineering-Society, 1: 1447-1454.
- Chatterjee, K., D.V. Ghodke, A. Chandra and K. Al-Haddad, 2007. Simple controller for STATCOM-based var generators. IET Power Electron., 2: 192-202.
- Dai, W.J. and J.J. Chen, 2009. Transient and steady-state analysis for STATCOM mathematical model. 2nd International Conference on Power Electronics and Intelligent Transportation System (PEITS), 3: 271-275.
- Du, Y., J.H. Su and X.Z. Yang, 2011. A coordinated DC voltage control strategy for H-bridge cascaded STATCOM. 4th International Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT), pp: 461-465.
- Faried, S.O., R. Billinton and S. Aboreshaid, 2009. Probabilistic technique for sizing FACTS devices for steady-state voltage profile enhancement. IET Gener. Transm. Dis., 3: 385-392.
- Geethalakshmi, B. and P. Dananjayan, 2009. A combined multipulse-multilevel inverter based STATCOM for improving the voltage profile and transient stability of power system. Int. J. Power Electron., 1: 267-285.
- Gu, L.H. and J. Wang, 2006. Nonlinear coordinated of excitation and STATCOM of power systems. Electr. Pow. Syst. Res., 77: 788-796.
- Hiskens, I.A. and D.J. Hill, 1989. Energy functions, transient stability and voltage behavior in power systems with nonlinear loads. IEEE Trans. Power Syst., 4: 1525-1533.
- Jiang, J.G., D. Teng and C. Lin, 2011. Simulation analysis of control method in cascade H-bridge static synchronous compensator. Proc. CSU-EPSCA, 23(1): 98-102.
- Kuiava, R., R.A. Ramos and N.G. Bretas, 2009. Control design of a STATCOM with energy storage system for stability and power quality improvements. IEEE International Conference on Industrial Technology (ICIT 2009), Gippsland, VIC, pp: 1-6.

- Lesieutre, B.C., P.W. Sauer and M.A. Pai, 1999. Existence of solutions for the network/load equations in power systems. *IEEE Trans. Circuits Syst.*, 46: 1003-1011.
- Li, S.F. and J. Wang, 2011. Excitation stabilization control design of salient-pole generators with statcom of power systems. *Trans. China Electrotec. Soc.*, 26: 173-180.
- Mithulananthan, N., C.A. Canizares and G.J. Rogers, 2003. Comparison of PSS, SVC and STATCOM controllers for damping power system oscillations. *IEEE Trans. Power Syst.*, 18: 786-792.
- Narne, R., J.P. Therattil and L. Sahu, 2011. Dynamic stability enhancement using self-tuning static synchronous compensator. *International Conference on Process Automation, Control and Computing (PACC)*, pp: 1-5.
- Ni, Y. and L. Snider, 1997. STATCOM power frequency model with VSC charging dynamics and its application in the power system stability analysis. *4th International Conference on Advances in Power System Control, Operation and Management (IEE Conf. Publ. No. 450)*, 1: 119-124.
- Padiyar, K.R. and N. Prabhu, 2006. Design and performance evaluation of subsynchronous damping controller with STATCOM. *IEEE T. Power Deliver.*, 21: 1398-1405.
- Rahim, A.H.M.A., S.A. Al-Baiyat and H.M. Al-Maghrabi, 2002. Robust damping controller design for a static compensator. *IEE Proc.-Gener. Transm. Distrib.*, 149: 491-496.
- Senjyu, T., M. Molinas and K. Uezato, 1996. Multi-machine power system stabilization with FACTS equipment applying fuzzy control. *Proceedings of the 35th Conference on Decision and Control*, 2: 2202-2207.
- Singh, S.P., B. Singh and M.P. Jain, 1990. Performance characteristics and optimum utilization of a cage machine as capacitor excited induction generator. *IEEE T. Energy Convers.*, 5: 679-685.
- Wang, H.F., 1999. Phillips-Heffron model of power systems installed with STATCOM and applications. *IEE Proc.-Gener. Transm. Distrib.* 146: 521-527.
- Wang, J. and C. Chen, 2009. *Bifurcation and Stability Control of the Structure Preserving Electric Power System*. Science Press.
- Wang, J., C. Chen and M.L. Scala, 2003. Periodic solution in multimachine power systems affected by perturbation of nonlinear loads. *IEEE T. Circuits Syst. I*, 50: 1363-1369.
- Xie, X.R., W.J. Cui and Y.L. Tang, 2001. Robust adaptive control of STATCOM'S reactive current. *Proceeding of the CSEE*, 21: 35-39.
- Xie, X.R., G.G. Yan and W.J. Cui, 2002. STATCOM and generator excitation: Coordinated and optimal control for improving dynamic performance and transfer capability of interconnected power systems. *1st International Conference and Exhibition on Transmission and Distribution in the Asia Pacific Region*, 1: 190-194.