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Research Article

Application of Cost Allocation Concepts of Game Theory Approach for Cost Sharing Process

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Abstract: Dissatisfaction among involved parties regarding the ways of cost allocation is ordinary in the joint ventures, since each party attempts to get more interest caused by making the coalition. Various cost allocation methods such as proportional methods, some methods in cooperative game theory approach and etc have been used for the purpose of cost sharing in the joint projects. In this study the Nucleolus, Shapley value and SCRB as the cost sharing concepts in game theory approach have been used to investigate their effectiveness in fairly joint cost allocation between parties involved in constructing the joint water supply system. Then the results derived from these methods have been compared with the results of the traditional proportional to population and demand methods. The results indicated that the proportional methods may not lead to a fairly cost allocation while the Nucleolus, SCRB and the Shapley value methods can establish adequate incentive for cooperation.

Keywords: Cost allocation, cooperative game theory approach, proportional distribution, SCRB method, the Shapley Value method, the nucleolus method

INTRODUCTION

Numerous conflicts may occur in joint projects where different participants with different expectations are involved. One of the main factor causes conflicts among involved participants in a joint venture is cost. In the joint venture, dissatisfaction among involved participants regarding the ways of cost allocation is ordinary and in many practical situations, conflict regarding cost allocation arises between the involved parties. Whenever there is a joint enterprise in the particular business, there may be a question that how the group members should allocate the joint costs fairly among themselves. In some real examples such as a group of members who share a practice, the joint cost distribution can be implemented by negotiations among themselves which can lead to a particular agreement. In fact in this situation, there exists bargaining process which is under the non-cooperative game theory approach.

Cooperative game theory approach was emerged as an effective approach which takes the principle of justice into account to provide adequate incentive to motivate the parties for participation in a joint venture. Recently, the game theory approach has been used as an effective framework in decision making about some problems and conflicts in some organization. The application of this method has been extensive particularly in water resource projects (Loehman and Whinston, 1971). Since the methods from the theory of cooperative games are appropriate to such contexts like water resources development in which the main purpose is often to provide the involved users adequate data to assess the costs they would be expected to be incurred in a cooperative game (Young *et al*., 1980).

Different quantitative and qualitative methods have been proposed for conflict resolution in water resource management and some of them have been mentioned in Madani (2010): Interactive Computer-Assisted Negotiation Support system (ICANS) (Thiessen *et al*., 1998), Graph Model for Conflict Resolution (GMCR) (Fang *et al*., 1993), Shared Vision Modeling (Lund and Palmer, 1997), Adjusted Winner (AW) mechanism (Massoud, 2000), Alternative Dispute Resolution (ADR) (Wolf, 2000), Multivariate Analysis Biplot (Losa *et al*., 2001) and Fuzzy Cognitive Maps (Giordano *et al*., 2005). Fuzzy coalition was firstly proposed by Aubin in 1974, “In his definition, fuzzy coalition is an n-dimensional vector which its components are membership degree of players in the coalition” (Sadegh *et al*., 2009). Wolf (2002) presents some significant studys and case studies on the prevention and resolution of conflict (using descriptive methods) over water resources.

The advantage of these methods is to analyze which does not rely on the precise estimation; rather it is done just by point estimations. But even such point estimations are often unreliable. These shortcomings may be excluded by designing an appropriate non cooperative game and at the same time distributes costs in a cooperative and fairly manner (Young *et al*., 1980).

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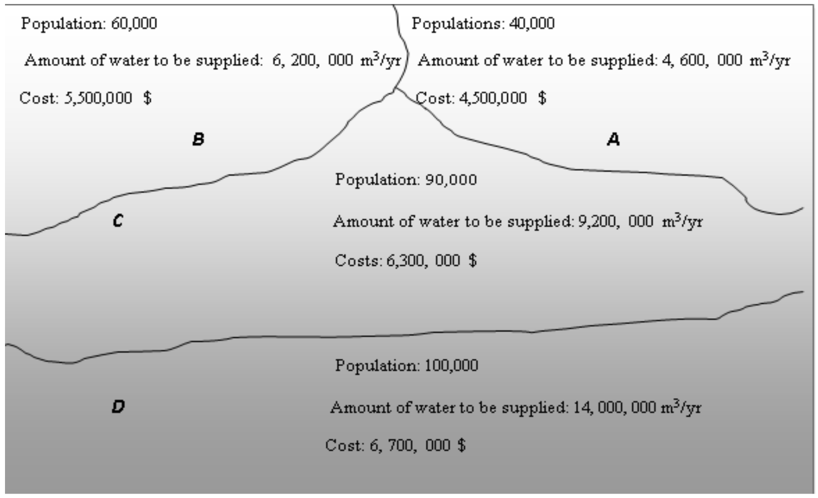

Fig. 1: Population, water demand and the cost of constructing separate water reservation system for four hypothetical municipalities A, B, C and D

The approach used in this study is traditional cooperative game theory tools which can be effective in reducing some parties' subsidization by the other parties. Game theory can recognize and clarify the behaviors of involved parties in the project to project problems and describe how interactions of different parties can lead to project evolving. This approach can lead to a fair and just distribution among the involved participants which can bring satisfaction between them. Among the most commonly used of joint cost allocation concepts of game theory approach are the Shapley value, the Core and the SCRB methods. Von Neumann and Morgenstern (1944) introduced basic concepts of cooperative game theory and Shapley (1953) introduced the Shapley value as a cooperative game concept in order to be used as a cost and benefits allocation in a coalition. Then the Shapley value method was recommended by Shubik (1962). It has been used as an effective cost distribution method to allow each party to evaluate the benefits they would expect from playing the game. In the other definition the Shapley value is the expected marginal amount contributed by a player to a coalition (Shapley and Shubik, 1973).

The Core concept as another cost allocation method in game theory approach suggests a subset solution that satisfies efficiency, individual and group rationality (Shapley and Shubik, 1973). In spite of complexity of the cost distribution process, nevertheless, in practical work, the costs of a project must usually be allocated in some way among the beneficiaries (Young *et al*., 1980).

The purpose of this study is to investigate the application of the Shapley value, the Nucleolus and SCRB concepts as the methods of cooperative game theory approach for providing a win-win situation to hypothetical involved municipalities in constructing a joint water supply system. The study intends to compare the solutions given by these methods together and also with traditional methods together. The object of these analyzes is rather just to scrutinize their behavior in practice.

RESEARCH METHODOLOGY

Cost allocation process for constructing the joint water reservation system between the four municipalities in different kind of coalitions has been done. The Shapley value, Nucleolus, SCRB and traditional methods such as proportional to population and proportional to water demand were used in this study as cost sharing methods. Different kinds of coalitions of the municipalities were considered to evaluate the effectiveness of cooperation between municipalities in different situation. The data given in this study such as population, water supply needed and the expenditure of building the water facility separately are hypothetical.

This study calculated the cost incurred to each municipality based on different kinds of coalitions made between them by cited methods to investigate the effectiveness of cooperation in reducing the cost. The results obtained by these methods were then compared together to investigate which methods can allocate the costs in a more just and fair manner between the municipalities to bring their satisfaction.

Figure 1 shows the data regarding the population, the amount of water to be supplied and the cost of constructing the water facility separately for each municipality. The geographical situation of each municipality can also be seen in Fig. 1.

COOPERATIVE GAME

A game is called cooperative, if the decision makers in a joint activity work together with forming the coalition to achieve more benefits than doing activities individually. In some cooperative games, conflict occurs during deciding on sharing the obtained dividend. And in other circumstances, conflicts may occur when decision makers should cooperate in building a joint project and then the costs of this common project are not allocated in a fairly manner. Therefore, if the involved parties do not see their portion in the total cost fairly, then they will not be willing to cooperate for joint projects.

Suppose that some decision makers are working together to build a dam. The total cost of the dam after cooperating of the players should be divided between them and each of these decision makers want to know what share of total costs is allocated to them. In this circumstance for each of the players, different percentages of the total cost can be considered and the share of each player from the total cost of construction of the dam can be indicated in a ratio vector as follows.

The following vector represents the amount of cost incurred by each player in a game with n players. In this vector it has been allocated the cost of x_i to the player i.

$X = (x_1, x_2, \ldots, x_n)$

Researchers use game theory in the allocation of costs or benefits from a cooperative game for finding a distinct ratio vector of the outcomes. This ratio vector should be not only optimized but also be equitable and fair.

Suppose that in construction of a water facility, N players cooperate together and the cost incurred for N-1 players is C (N-1). Imagine in continuance, another player as the player number n have to be added to the coalition. In this case, the cost of cooperation for constructing this common water facility will be C (N). By adding the player N, the amount of C (N)-C (N-1) will be added to the total cost of this common facility.

In most of cost allocation methods, this rate of increase in the total cost after adding the player n to the coalition incurs to this player, since this addition cost is caused by adding this player to the coalition. This addition cost due to adding the player to a coalition is called separable cost. The added costs can occur at different stages of formation of a coalition.

With high definition, the added cost regarding the addition of a subgroup of the players can also be found.

Suppose S-1 players form a subgroup of total player and cooperate together in doing the activity (S≤N). Total cost of this activity with S-1 players is equal to C(S-1). If the player s is added to this sub-group, the cost incurred by this player will be equal to C(S)-C(S-1). This cost difference is called the cost added to a subgroup of players.

Some of the methods used for allocating the costs and benefits from cooperation, especially in the management of water systems include:

- The Shapley
- The Core
- The Nucleolus
- The generalized Shapley
- The fuzzy Shapley Value
- The Nash/Nash-Harsany
- Separable Cost Remaining Benefits (SCRB) and etc

Table 1: Cost of constructing water supply under different combinations

Combination	Cost break down (million $)	Total cost (million $)
A+B+C+D	4.5+5.5+6.3+6.7	23
{A, B}+C+D	9.4+6.3+6.7	22.4
{A, C}+B+D	10.2+5.5+6.7	22.4
{A, D}+B+C	11.2+5.5+6.3	23
{A, B, C}+D	15.3+6.7	22
{A, B, D}+C	16.1+6.3	22.4
{A, C, D}+B	16.5+5.5	22
A+{B,C}+D	4.5+11.2+6.7	22.4
A+{B, D}+C	4.5+12.2+6.3	23
A+B+{C, D}	4.5+5.5+12.4	22.4
A+{B, C, D}	4.5+17.5	22
{A, B, C, D}	20.5	20.5

In this study the Shapley value, the Nucleolus and SCRB methods and two kinds of traditional cost allocation methods were investigated.

JOINT COST FUNCTIONS

According to the research methodology, municipalities A, B, C and D are considered as neighboring municipalities who are going to provide the municipal drinking water for their citizens by constructing the separate water facilities or by constructing a joint water supply facility. It is supposed that the joint supply facility is considered as a cheaper operation than the separate construction of water facility for each municipality due to economies of scale. Population and the quantity of water needed to be supplied are given in Fig. 1. After constructing the joint water supply, the problem maybe occurred is that how the costs of it should be distributed between the municipalities which bring the satisfaction of all municipalities? Table 1 shows the costs for different possible combination of municipalities in a cooperative joint water supply project. It shows that the cost of building water facility can be reduced by cooperation of all municipalities together to build a joint water supply facility.

Table 1 demonstrates that cost of the building a joint facility under cooperation of all four municipalities will be about 2.5 million dollars cheaper than the circumstance of building separate facility by each municipalities. It can be derived that cooperating A and B together to construct a joint supply water system can save about 600,000 dollars. Due to the geographical separation between A and D, joining these municipalities without including C and B will not have any substantial effect on the total cost. By joining

Table 2: Cost allocation between the municipalities using the proportion of population

Municipality	Cost allocation base on proportional to population (million $)
A	(40/29) *20.5=2.83
B	(60/290)*20.5=4.24
C	(90/290)*20.5=6.36
D	(100/290)*20.5=7.07

Table 3: Cost allocation between the municipalities using the proportion of water demand

Municipality	Cost allocation base on proportional to water demand (million $)
A	(4.6/34) *20.5=2.77
B	(6.2/34)*20.5=3.74
C	(9.2/34)*20.5=5.5
D	(14/34)*20.5=8.44

municipalities A, B and C the total cost can be saved about 1 million dollars. According to the different combination, it can be derived that the best combination regarding saving the total cost will be the cooperation and making a coalition between all four municipalities that can be saved about 2.5 million dollars of the total cost.

THE PROPORTIONAL COST ALLOCATION METHOD

One of the most common and easiest methods of cost distribution between the involved participants is the proportional cost allocation method in which incurred costs are simply allocated base on proportion of some factors such as population, demand on use and etc. One of the problems of cost allocation by this way is that the conflict may be arisen between the participants due to their self interest. And so it may reduce the incentive among involved participants for cooperation.

Table 2 indicates the cost allocation among the four municipalities in the joint venture following the study's hypothetical example based on population proportion.

However it is derived from Table 2 that municipality C and especially D hardly accept such as allocation. Since municipality D can supply the same amount of water by its own separate facility about 370,000 dollars lesser compared to the joint venture. Municipality C can also supply its necessary water with 60,000 dollars lesser that the joint venture. Indeed the municipalities C and D subsidize municipalities A and B in the coalition. It can be said that this method may not have individual rationality. Individual rationality can be achieved whenever all of the involved participants pay not more than the cost of their separate participation in building the facility. Table 3 also shows the cost allocation process using the proportional to demand.

In this way, according to Table 3, it can be derived that the principle of individual rationality does not exist for the municipality D. Municipality D by this method must pay about 1.75 million dollars more than the situation of building the separate facility on its own. By evaluating these two method, it can be derived that municipality D must pay more in the proportional to demand method compared with the proportional to population method

A fair allocation must be at least individually rational. It means that no participant should pay more in the joint venture than he would have to pay on his own. In this example the individual rationality can be achieved when $y_A \leq 4.5$ million dollars, $y_B \leq 5.5$, $y_C \leq 6.3$ and $y_D \leq 6.7$, where $y_A + y_B + y_C + y_D = 20.5$. y_A, y_B, y_C and y_D are the involved costs to municipalities A, B, C and D in the joint venture.

THE CORE AND NUCLEOLUS METHOD

Suppose in an allocation game with n players, the amount allocated to each player is shown with a ratio vector:

$$x = (x_1, x_2, \ldots, x_n) \tag{1}$$

The stability of each allocation method approach in cooperative games is based on the core. And in a cooperative game, the players will accept that allocation approach which is located within the core.

The core in a cooperative game is specified with C(N,v) (Asgari and Afshar, 2008)

In cooperative games (N,v), with n players and the cost function v, the outcome of players will be in the core if:`

$$\sum_{i=N} X_i = v(N) \tag{2}$$

where,

N = The total number of players
X_i = The cost allocated to the player i in the coalition of all players
V() = Cost function or outcome function

Equation (1) is said the performance equation. The means of performance is sharing the total cost of the common action between all players.

Equation (2) is said group rationality. The group rationality follows that if a player participates in the largest coalition of players, the amount allocated to him/her in the largest coalition is less than any amount that the player under any coalition with number of s players must pay up. For any groups that are composed only of a player, the condition is said the individual rationality.

There are two principles:

- First, where there are n independent participants {1, 2, …, n} = N and function c (S) gives the alternative costs, the condition for group rationality for a cost allocation $x = (x_1, x_2, ..., x_n)$, $\sum_N yi$ = v (N), is that:

$$\sum_S yi \le c(S) \quad (3)$$

- Second principle is related to the marginal cost. It means that any participators should not be charged less than the marginal cost of including him in the project. For example there are three participants in a joint project including M, N and P. In cooperation of all three participants, it costs 12.2 million dollars but for cooperation only M and N, it costs 9.2 million dollars. So the marginal cost of P is equal to \$12.2-9.2= \$3 million. It means that the participant P should pay at least 3 million dollars. So for establishing the marginality principle, the cost allocation x should satisfy:

$$\sum_S yi \ge c(N) - c(N-S) S<N \quad (4)$$

The group rationality is based on some goals such as giving adequate incentive to all participants to cooperate.

It is common in game theory to elucidate the two principles (3) and (4) regarding the cost savings game v. If y_i is the cost appraised for participant i and then cooperation and making a coalition lead to a saving of the amount i, then it can be said that $y_i = c(i) - x_i$.

According to (3) and (4) and based on the study's hypothetical example for four municipalities, we have:

$y_A \le 4.5$ million\$, $y_B \le 5.5$, $y_C \le 6.3$, $y_D \le 6.7$
$y_A+y_B \le 9.4$, $y_A+y_C \le 10.2$, $y_A+y_D \le 11.2$, $y_B+y_C \le 11.2$, $y_B+y_D \le 12.2$, $y_C+y_D \le 12.4$
$y_A+y_B+y_C \le 15.3$, $y_A+y_B+y_D \le 16.1$, $y_A+y_C+y_D \le 16.5$, $y_B+y_C+y_D \le 17.5$, $y_A+y_B+y_C+y_D = 20.5$ million\$

The area including this point indicates the core area which brings both individual and group rationality for municipalities. By being the core not empty, it can be concluded that the nucleolus lies in the core. Indeed, the Core solution concept refers to a 'range' of values that satisfy particular condition, but the Nucleolus solution concept recommends a unique point. The nucleolus method is based on the idea of the excess.

The least core or nucleolus is obtained by imposing the smallest uniform tax e (Young *et al.*, 1980). Therefore the nucleolus can be found by satisfying:

$$\sum_S x_i \ge v(S) - e \quad \forall\ S < N \quad (5)$$
$$\sum_N xi = v(N) \quad (6)$$

The nucleolus is the set of x calculated by (5) and (6) for this least e (Shapley and Shubik, 1973). Then, the corresponding allocation can be calculated by equation $y_i = c(i)-x_i$ for all i where c(i) is incurred cost to i in non-cooperative situation. Computing the nucleolus includes a linear program.

Table 4: The amount of x_i (million\$) for each municipality

x_A	x_B	x_C	x_D
0.4	0.7	1	0.4

Table 5: Corresponding cost allocation (in million\$) between involved municipalities based on Nucleolus method

x_A	x_B	x_C	x_D
4.1	4.8	5.3	6.3

The nucleolus solution can be achieved by following the (5) and (6). So for municipalities A, B, C and D based on (5) and (6), we have:

$x_A + x_B + x_C + x_D = 2.5 \quad x_i \ge 0$, i = A, B, C, D
$x_A + x_B \ge 0.6-e$
$x_A + x_C \ge 0.6-e$
$x_A + x_D \ge -e$
$x_B + x_C \ge 0.6-e$
$x_B + x_D \ge e$
$x_C + x_D \ge 0.6-e$

And also

$x_A + x_B + x_C \ge 1-e$
$x_A + x_B + x_D \ge 0.6-e$
$x_A + x_C + x_D \ge 1-e$
$x_B + x_C + x_D \ge 1-e$

After solving the above terms the amount of x_A, x_B, x_C and x_D can be determined. Table 4 indicates the amount of x_i calculated by the Nucleolus method.

Table 5 demonstrates the corresponding allocation calculated by equation $y_i = c(i)-x_i$ for all municipalities.

THE SHAPLEY VALUE METHOD

The Shapley value method was first introduced by Lloyd Shapley and this method for a game with n players can be demonstrated in the following formula:

$$Y_i = \sum_{\substack{s \subseteq N \\ i \in S}} \frac{(|S|-1)!(|N|-|S|)!}{|N|!} [C(S) - C(S - \{i\})], \ (i \in N) \quad (7)$$

where,

|N| I = Total number of players
|S| = The number of players in the coalition s
C (S) = The cost function for the coalition of players with s-person
C (S-{i}) = The cost function of coalition of players with s-person when player i has been deleted from the coalition

It can be said in the brief description of this method that the Shapley value is equal to the average costs of adding a player to the coalition in different stages.

The Shapley value is one of the earliest methods of cost allocation (Shapley, 1953). If i was the last player

Table 6: The cost contribution of municipalities A, B, C and D in million$ based on the Shapley value method

y_A	y_B	y_C	y_D
3.907	4.907	5.474	6.207

Table 7: Cost allocation between municipalities (in million$) using SCRB method

y_A	y_B	y_C	y_D
3.91	4.91	5.56	6.11

added to the group, his/her cost contribution to coalition S is c(S)-c(S-1). As mentioned before the Shapley value is i's average cost contribution.

So the Shapley value for municipalities A can be calculated as follows:

$$y_A = ((1! * 3!)/4! * [c(A)]) + ((1! * 2!)/4! * [c(A,B) - c(B)]) + ((1! * 2!)/4! * [c(A,C) - c(C)]) + (1! * 2!)/4! * [c(A,D) - c(D)]) + (2! * 1!)/4! * [c(A,B,C) - c(B,C)]) + (2! * 1!)/4! * [c(A,B,D) - c(B,D)]) + ((2! * 1!)/4! * [c(A.C.D) - c(C,D)]) + ((2 * 1!)/4! * [c(A,B,C,D) - c(B,C,D)])$$

= 3.907million dollars

And the cost contribution of municipalities B, C and D are 4.907, 5.474 and 6.207 million dollars, respectively.

Table 6 demonstrates the cost contribution of municipalities A, B, C and D calculated by the Shapley value method.

THE SEPARABLE COSTS- REMAINING BENEFITS (SCRB) METHOD

The SCRB method is based on interesting idea that joint costs should be distributed, more or less, in proportion to the willingness of the participant to pay (Young *et al*., 1980). It distributes costs among involved participants proportional to the benefits remained after removing the separable costs.

The proportional allocation based on this method is fulfilled after allotting each participant's marginal cost to them and then added to proportion of each participant's remaining benefit to the non-separable cost.

Assume that the participant i enter to a coalition with N-1 participants. The marginal cost or separable cost of including i is $c'(i) = c(N) - c(N-i)$. According to SCRB method the participant i's remaining benefit shown by r (i) is his/her cost in non-cooperative situation minus his/her marginal cost.

The SCRB method for a game with N player can be demonstrated in the following equation:

$$yi = c'(i) + [r(i)/\sum_N rj][c(N) - \sum_N c'(j)] \tag{8}$$

where,

- $c'(i)$ = The player i's marginal cost
- r(i) = Player i's remaining benefit
- $[\sum_N rj]$ = Cumulative sum of all players' remaining benefit
- c (N) = Incurred cost to the coalition including all players
- $[\sum_N c'(j)]$ = Cumulative sum of all players' marginal cost

So the cost allocation based on the SCRB formula for each municipality is as follows:

Marginal cost of each municipality is:

$c'(A)$ = [A, B, C, D]-[B, C, D] = 20.5-17.5 = 3 million$
$c'(B)$ = [A, B, C, D]-[A, C, D] = 20.5-16.5 = 4 million$
$c'(C)$ = [A, B, C, D]-[A, B, D] = 20.5-16.1 = 4.4 million$
$c'(D)$ = [A, B, C, D]-[A, B, C] = 20.5-15.3 = 5.2 million$

Remaining benefit of each municipality is:

r(A) = 4.5-3 = 1.5 million$
r(B) = 5.5-4 = 1.5 million$
r(C) = 6.3-4.4 = 1.9 million$
r(D) = 6.7-5.2 = 1.5 million$
$\sum = 6.4$ million $

$[C(N) - \sum_N c'(j)]$ or non-separable costs = 20.5-(3+4+4.4+5.2) =3.9 million$

So the cost allocated to each municipality is:

y_A= 3+[1.5/6.4]×3.9 = 3.91 million$
y_B = 4+[1.5/6.4]×3.9 = 4.91 million$
y_C = 4.4+[1.9/6.4]×3.9 = 5.56 million$
y_D = 5.2+[1.5/6.4]×3.9 = 6.11 million$

Table 7 demonstrates the cost contribution of municipalities A, B, C and D calculated using SCRB method.

COMPARISON OF METHODS

Figure 2 and Table 8 and 9 indicate the comparison of costs allocated to each municipality graphically and numerically respectively based on each method.

As mentioned before, the total cost of building a facility of water supply in a joint venture was 20.5 million dollars. The results in this study were obtained from two proportional allocation method including proportional to population and demand, the Shapley value, the Nucleolus and SCRB methods. The amount of cost contribution to each municipality in million dollars based on specified methods and the percentage of benefits earned by each of them are shown in Table 8 and 9 respectively. The comparison between the

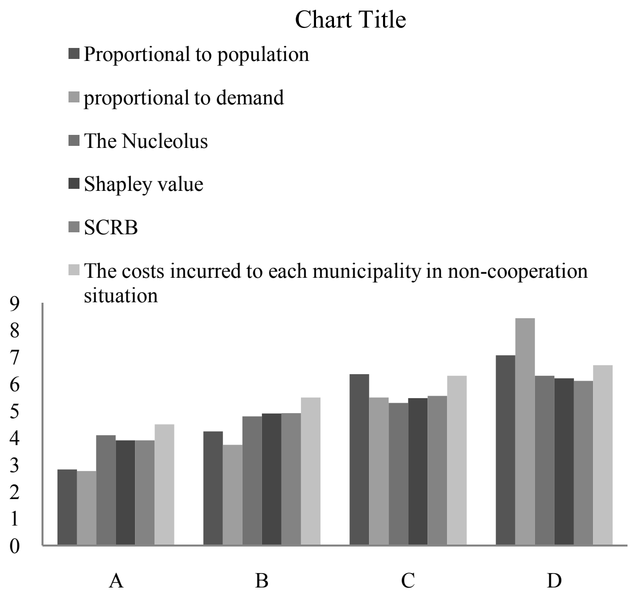

Fig. 2: Comparison between allocated costs based on different methods and cost incurred to each municipality in non-cooperation situation

Table 8: Comparison of the results based on different methods (in million$)

Allocation methods	Municipality A	Municipality B	Municipality C	Municipality D	Total incurred cost
Proportional to population	2.83	4.24	6.36	7.07	20.5
Proportional to demand	2.77	3.74	5.5	8.44	20.5
The Nucleolus	4.1	4.8	5.3	6.3	20.5
The Shapley value	3.907	4.907	5.474	6.207	20.5
SCRB	3.91	4.91	5.56	6.11	20.5

Table 9: Percentage of benefits earned by each municipality based on different concepts

Benefits (%) gained by different concepts	Municipality A (%)	Municipality B (%)	Municipality C (%)	Municipality D (%)
Proportional to population	37	22.9	-0.9	-5.5
Proportional to demand	38.4	32	12.7	-26
The Nucleolus	8.9	12.7	15.9	6
The Shapley value	13.2	10.8	13.2	7.3
SCRB	13.1	10.7	11.74	8.8

allocated costs based on different methods and the cost incurred to each municipality in noncooperation situation are also shown involved costs in the noncooperation situation are also shown in Fig. 2.

By comparing these four allocation methods, it can be derived that the proportional methods are noticeably different from the others. A comparison of the costs allocated by proportional method with non-cooperation costs shows that the proportional methods cause some participants to pay more than they would have to pay on their own. By using proportional to population method, the municipalities C and D must pay more than their separate participation, while allocation by demand penalizes D for joining to the joint venture. According to Table 9, the percentage of benefits earned by municipality C and D is -0.9% and -5.5% based on population proportion and for D is -26% based on water demand proportion. This negative rate of benefits shows that these methods may not make adequate incentive between municipalities especially C and D to join to other municipalities in one coalition, since the individual rationality has not been provided by these methods. In this method, municipalities A and B have high incentive to join to others due to high amount of benefits they will be earned.

Results based on the Nucleolus method indicate that this method is more beneficial for municipality C. Unlike proportional methods, in Nucleolus method no municipality must pay more than they would have to pay on their own. Hence it provides adequate incentive for all municipalities to join and cooperate together.

According to Shapley value method, the percentage of benefit gained for municipality A and C (13.2%) is highest compared to others. In compared to the Nucleolus method, municipality A earns more benefit and it is naturally more advantageous for it.

The SCRB method also provides more benefit to municipality A compared to other concepts. It can be derived from Table 9 and Fig. 2 that in all Nucleolus, Shapley value and SCRB methods, municipality D earns least benefit which can be due to its geographical situation.

Totally, municipality A earns most benefit by Shapley value, municipality B and C earns most benefit by the Nucleolus method and municipality D earns most benefit by the SCRB method. According to statistics, it can be concluded that the Nucleolus method may be preferable while both Shapley and SCRB method can provide adequate incentive for cooperation. By scrutinizing the amount calculated by these solution concepts, it can be derived that the Shapley value, the SCRB and the Nucleolus methods as the cost allocation methods in cooperative game theory approach take both individual and group rationality into account in the joint venture. No municipality in cooperative coalition pays more than they would have to pay on their own. So, it can provide them adequate incentive to make a coalition and participate in building a joint water supply facility.

CONCLUSION

The concepts of fairness and justice are wide enough to be discussed. Facing dissatisfaction in the cost allocation process is unavoidable due to the different kind of allocation methods. The involved participants expect to use those methods which bring the highest interest for them. One of the primary principles that should be provided in a cooperative game is individual rationality. This can be the first concept which can provide a preliminary incentive to make a coalition. According to the data analysis, the two traditional proportional methods, proportional to population and demand, cannot give adequate incentive to all involved participants. In fact these methods are not individual rational for some participants. On the other hand, the Shapley value, the Nucleolus and SCRB methods in spite of little differences in amounts, represent a fair, equitable and impartial method for allowing the financial managers to discuss over how the costs should be allocated. Regarding the individual and group rationality, all of the Shapley value and the Nucleolus and the SCRB methods establish the principle of individual and group rationality. It can be concluded from the data analysis that these cited methods can provide a fairly atmosphere between the municipalities to consent about the amount of allocated costs.

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