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Research Article

Design of EWMA Control Charts for Assuring Predetermined Production Process Quality

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Abstract: In this study, we establish the relationship model in process capability indices, the coefficient of process shift and quality target utilizing statistical tolerance technology. A new systematic method for concurrent design of process quality, Statistical Tolerance (ST) and control chart is presented. A set of standardized Process Quality Indices (PQIs) for variables is introduced for meeting the measurement and evaluation to process yield, process centering and quality loss. To develop a quality-oriented SPC approach, a standardized interface between process quality indices system and the parameter design of EWMA control charts is needed. The two-dimensional array form ST (C_p*, δ*) is presented for selecting a set of parameters (λ, L) with suitable ARL_0 and ARL_1 to realize the optimal control for a process toward a predetermined quality target. The ST (C_p*, k*) will be the guideline to form the restriction of the centerlines of the control charts for assuring the preset PQIs. If the process statistical parameters are unknown, (C_p*, k*) will be converted into the ST form for estimated values based on a required confidence probability. A case study shows the detail procedure of this concurrent design for PQIs, ST and SPC parameters.

Keywords: Concurrent design, EWMA control chart, process quality, statistical tolerance

INTRODUCTION

The Exponentially Weighted Moving Average (EWMA) control chart for the mean of a process was introduced by Roberts (1999) and it has been shown to be more efficient in detecting small in the process mean. Since EWMA charts are known to be sensitive in detecting small changes in the process mean or the process variability, it has become a popular control chart for monitoring the mean of a process.

The existing procedures for designing EWMA charts are based on the assumption of known process parameters, how to deal with the situation that the process parameters are unknown? Jones (1999) presented a design procedure for statistical designing of EWMA charts with estimated parameters, however, the way to estimate the standard deviation is considered as an incorrect practice "because it potentially combines both between-sample and within-sample variability (Lucas and Saccucci, 1990)".

The objective of this study is to develop concurrent design procedure for the EWMA chart that can assure a predetermined process quality in both conditions, i.e., the process parameters are assumed as known and not require this assumption. How to assure a predetermined process quality via the EWMA charts, especially for a high quality target? How to deal with the situation if the in controlled process mean is not exactly equal to its target value in designing EWMA chart? On the basis of research study (Zhang *et al*., 2004; Zhang, 2004), a quality-oriented ST and SPC approach that quantitatively specifies what a desired process is and how to assure it will realize concurrent designing of EWMA charts toward a predetermined quality target. To develop a quality-oriented SPC approach, a standardized interface between process quality indices system and the parameter design of control charts is needed. On the basis of the theoretical studies on the EWMA control chart, the criterion for designing an optimal EWMA chart is to make the chart have the best Average Run Length (ARL) performance. ARL_0 and ARL_1 are the main indices that determine the efficiency and sensitiveness of the control chart. ARL can be selected based on the present value of δ^* which is a standardized coefficient of process shift. At last, a case study shows the detail procedure of this concurrent design for PQIs, ST and SPC parameters.

In this study, we establish the relationship model in process capability indices, the coefficient of process shift and quality target utilizing statistical tolerance technology. A new systematic method for concurrent design of process quality, Statistical Tolerance (ST) and control chart is presented. A set of standardized Process

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Quality Indices (PQIs) for variables is introduced for meeting the measurement and evaluation to process yield, process centering and quality loss. To develop a quality-oriented SPC approach, a standardized interface between process quality indices system and the parameter design of EWMA control charts is needed. The two-dimensional array form ST (C_p*, d*) is presented for selecting a set of parameters (λ, L) with suitable ARL_0 and ARL_1 to realize the optimal control for a process toward a predetermined quality target. The ST (C_p*, k*) will be the guideline to form the restriction of the centerlines of the control charts for assuring the preset PQIs. If the process statistical parameters are unknown, (C_p*, k*) will be converted into the ST form for estimated values based on a required confidence probability. A case study shows the detail procedure of this concurrent design for PQIs, ST and SPC parameters.

QUALITY-ORIENTED STATISTICAL TOLERANCING

Two main problems of ST: There are two main problems that are needed to be solved in the existing Statistical Tolerance (ST) models and representations:

- What is the basis of the decision making for selecting a suitable ST value
- How to make the designated ST value as a helpful reference value to the actual SPC parameters design

The current standards and technique relative to ST, or the representations of ST have not completely settled the problems.

The rapid response to the requirements of customers and markets promotes the Concurrent Engineering (CE) technique in product and process design. The decision making for process quality target, SPC method, sampling plan and control chart parameter design can be done at the stage of process quality plan based on historical data and process knowledge database. Therefore, it is a reasonable trend to introduce the concepts and achievements on process quality evaluation and process capability analysis, CE and SPC techniques into process plan and tolerance design. To develop a quality-oriented SPC approach, a standardized interface between process quality indices system and the parameter design of control charts is needed.

Three process quality indices can be the functions of C_p and k, i.e., f_{Pd} (C_p, k), f_{PC} (C_p, k), and f_{Pql} (C_p, k). Thus, P_d, P_c and P_{ql} can be indicated in terms of C_p and k under the condition that the mass-manufacturing process is in controlled state, as seen in Table 1 and PQIs in terms of C_p and δ in Table 2.

The standardized PQIs value system is presented based on raisonne grading by using the geometrical series of preferred numbers and arithmetical series and the relative ST zones to assure these PQIs, i.e., quality-oriented statistical tolerance zones, can be formed based on the equations in Table 1 and 2.

Table 1: PQIS in terms of C_p and k

PQI	Computation formula	
P_d	$P_d = \{\Phi[-3C_P(1+k)] + 1 - \Phi[3C_P(1-k)]\} \times 100\%$	(1)
P_c	$P_c = (M\pm t/6)\{\Phi[C_P(1-3k)] - \Phi[-C_P(1+3k)]\}.100\%$	(2)
P_{ql}	$P_{ql} = [1/(3.C_P)^2 + k^2] \times 100\%$	(3)

Table 2: PQIs in terms of C_p and δ

PQI	Computation formula	
P_d	$P_d = [1 - \Phi(3C_p - \delta) + \Phi(-3C_p - \delta)] \times 100\%$	(4)
P_c	$P_c = (M\pm t/6)\{\Phi[C_P - \delta] - \Phi[-C_P - \delta]\}.100\%$	(5)
P_{ql}	$P_{ql} = [1 + \delta^2/(3C_P)^2] \times 100\%$	(6)

Quality-oriented statistical tolerance: The Quality-Oriented Statistical Tolerance (QOST) can be represented in the forms indicated as a set of two-dimensional array, i.e., (C_p^*, δ^*) or (C_p^*, k^*), their meanings are as follows:

$$(C_P^*, k^*) \Leftrightarrow \begin{cases} C_p \geq C_P^* \\ |k| \leq k^* \end{cases}$$

$$(C_P^*, \delta^*) \Leftrightarrow \begin{cases} C_p \geq C_P^* \\ |\delta| \leq \delta^* \end{cases}$$

CONCURRENT DESIGN OF EWMA CHARTS

On the basis of the theoretical studies on the EWMA control chart, the criterion for designing an optimal EWMA chart is to make the chart have the best Average Run Length (ARL) performance (Gan, 2001). For a desired in-control ARL_0, if one wants to detect a specified pair of changes in the process mean and variability quickly, the parameter combination of (λ, L) for the optimal design provides the desired ARL_0 and minimizes the out-of-control ARL_1 for the specified changes in the mean and variability. ARL_0 and ARL_1 are the main indices that determine the efficiency and sensitiveness of the control chart. ARL can be selected based on the preset value of δ^* which is a standardized coefficient of process shift. The two-dimensional array form ST (C_p^*, δ^*) is presented for selecting a set of parameters (λ, L) with suitable ARL_0 and ARL_1 to realize the optimal control for a process toward a predetermined quality target. The ST (C_p^*, k^*) will be the guideline to form the judgment to the centerlines of control charts if the process statistical parameters are known. If the process statistical parameters are estimated, the ST (C_p^*, k^*) will be converted into the ST form [$\hat{C}^*_{p,\ 1-\alpha}$ (min), $\hat{k}^*_{p,1-\alpha}$ (max)] based on a required confidence probability (1- α).

When process parameters are unknown, a reference sample of m subgroups of size n from an in-controlled process must be taken for computation of estimated parameters. Estimating process parameters for variable-data is summarized by Raid and Alice (2003) as follows:

As $\hat{\mu}_0$ and $\hat{\sigma}_0$ are the estimated values of μ_0 and σ_0, the confidence intervals of μ_0 and σ_0 should be considered based on a preset confidence probability 1-α by which the sample size n and subgroup number m of the reference sample are designed. The process quality target, statistical tolerance, sampling plan with confidence interval and the restrict condition to center line of EWMA control chart can be concurrently designed under the guideline of this new approach. A detail procedure with a case study is given to show the application of this new approach for concurrent design EWMA chart based on the quality-oriented statistical tolerance technique.

Case study: It is known that the specification to the whole diameter of a component: $\Phi 40\,\binom{0}{-0.016}$, quality requirements to the final grinding process emphases on process centering as well as process yield. The process parameters are unknown.

It is better to get the data from the $\bar{x}$ and S control charts for estimating process parameters. As the quality requirement to the final grinding process is quite tight restrict, monitoring both the small process shift and variability must be considered to assure the PQI. Obviously, it is better to apply the EWMA combined with charts for $\bar{x}$ and S. The EWMA chart is sensitive to small shift of process mean and $\bar{x}$ chart is sensitive to the shift of $\delta \geq 1.5$, S chart is for monitoring process variability. Concurrent design for ST and SPC based on the given PQIs is given as follows:

- Select PQI and target value of the process investigated; preset confidence probability 1-α.
  PQI: P_c (M±t/6) ≥70%, preset confidence probability 1-α = 95%.
- Decide sample size n and subgroup number m of the reference sample.
  Control chart can be used both retrospectively and prospectively for estimating process parameters. Retrospectively, a historical sample of observation with sample size n = 5 and subgroup number m = 20 is taken to define the state of an in-controlled process and estimate the unknown parameters.
  Measure and collect the data according to decided sampling plan with sample size n and subgroup number m from an in-controlled process; Estimate the process mean and standard deviation using $\bar{x}-s$ chart:

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} = 39.9918$$

$$\hat{\sigma} = \hat{\sigma}_{\bar{s}} = \frac{\bar{s}}{c_4} \approx 0.0018$$

Compute and draw the CL, UCL and LCL of the $\bar{x}-s$ chart based on the estimated process parameters $\hat{\mu}_0$ and $\hat{\sigma}_0$;

For the $\bar{x}$ chart:

UCL = $\hat{\mu}_0$ + $A_3\bar{s}$ = 39.9918 + 1.427*0.0018 = 39.9944
CL = $\hat{\mu}_0$ = 39.9918
LCL = $\hat{\mu}_0$ - $A_3\bar{s}$ = 39.9918 - 1.427*0.0018 = 39.9892

For the s chart:

UCL = $B_4\bar{s}$ = 2.089×0.0018 = 0.0038
CL = $\bar{s}$ = 0.0018
LCL = 0

Judge and confirm the process to be in-controlled state from the plotted $\bar{x}-s$ chart.

- Compute the estimated values of C_p, k and δ:

$$\hat{C}_P = \frac{USL - LSL}{6\hat{\sigma}} = 1.4815$$

$$\hat{k} = \frac{\hat{\mu} - M}{(USL - LSL)/2} = -0.025, \; \hat{\delta} = \frac{\hat{\mu} - M}{\hat{\sigma}} = -0.111$$

Let $\hat{C}^*_{p,1-\alpha}$ (min), be the lower confidence bound of observed estimates with 1-α confidence probability, the required confidence probability 1-α is selected to be 0.90, that is, α = 0.1.

$$\nu \cong f_n m(n-1), \quad f_n = 0.95 \quad for \quad n = 5$$

$$\hat{C}_{P,1-\alpha}(\min) \cong \hat{C}_P \sqrt{\chi^2_{1-\alpha,\nu} / \nu}$$
$$= 1.4815\sqrt{\chi^2_{1-0.1,76} / 76} = 1.324$$

- Determining ST (C_p^*, k^*), (C_p^*, δ^*) based on selected PQI and target value and $\hat{C}^*_{p,1-\alpha}$ (min)

$$(C_p^*, k^*) = (1.324, 0.187)$$

$$(C_p^*, \delta^*) = (1.324, 0.74)$$

Transform the ST (C_p^*, k^*) into the ST with estimated parameters [$\hat{C}^*_{p,1-\alpha}$ (min), $\hat{k}^*_{1-\alpha}$ (max)] based on the required confidence probability 1-α:

$$\hat{k}^*_{1-\alpha}(\max) = \max(0, k^* - \frac{t_{\nu,\alpha}}{3 \cdot C_p^* \cdot \sqrt{mn}})$$
$$= \max((0, 0.187 - \frac{t_{99,0.1}}{3 \cdot 1.324\sqrt{100}}) = 0.15$$

$$\hat{C}^*_{P,1-\alpha}(\min) \cong \hat{C}^*_P / \sqrt{\chi^2_{1-\alpha,\nu}/\nu}$$
$$= 1.324 / \sqrt{\chi^2_{1-0.1,76}/76} = 1.4815$$

$$[\hat{C}^*_{p,1-\alpha}(\min), \hat{k}^*_{1-\alpha}(\max)] = [1.48, 0.15]$$

- Judge and confirm that $\hat{\sigma}_0$ and $\hat{\mu}_0$ meet the following formula based on the quality requirements:

$$\begin{cases} M - 0.5 \cdot \hat{k}^*_{1-\alpha(\max)} \cdot t \le \hat{\mu}_0 \le M + 0.5 \cdot \hat{k}^*_{1-\alpha(\max)} \cdot t \\ \hat{\sigma}_0 \le \dfrac{t}{6 \cdot \hat{C}^*_{p,1-\alpha(\min)}} \end{cases}$$

$$\begin{cases} 39.9908 \le \hat{\mu}_0 \le 39.9932 \\ \hat{\sigma}_0 \le 0.002 \end{cases}$$

As $\hat{\mu} = 39.9918$; $\hat{\sigma} \approx 0.00192$ they are all meet the above formula based on the quality requirements.

- Design for EWMA, select the ARL_0, ARL_1, the parameter combination L, λ based on δ^*:

n = 5, L = 2.998, $\lambda = 0.25$, $\sqrt{n}\delta^* = 1.65$, ARL_0 = 500; $ARL_1 = 4.74 \approx 5$

- Obtain the observation data according to the planed sample size n and sampling time interval:
  As the process output is 30 pieces/h, process is stable, take sample data (n = 5) every 20 min, list the sampling data in Table 3.
- Compute and draw the CL, UCL and LCL of the EWMA chart based on the estimated process parameters $\hat{\mu}_0$ and $\hat{\sigma}_0$:
  EWMA center line and control limit: n = 5, L = 2.998, $\lambda = 0.25$

Upper control limit:

$$UCL = \hat{\mu}_0 + L\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} =$$

$$39.9918 + 0.00097 \cdot \sqrt{\left[1-(0.75)^{2i}\right]}$$

Center line: CL = $\hat{\mu}_0$ = 39.9918

Lower control limit:

$$LCL = \hat{\mu}_0 - L\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 39.9918 - 0.00097 \cdot \sqrt{\left[1-(0.75)^{2i}\right]}$$

List the statistic Z_i based on the subgroup data:

Table 3: Sampling data with n = 5

X_1	X_2	X_3	X_4	X_5	$\bar{x}_i$
39.994	39.994	39.990	39.993	39.989	39.992
39.991	39.991	39.991	39.991	39.989	39.991
39.991	39.990	39.992	39.990	39.993	39.991
39.990	39.992	39.991	39.990	39.994	39.991
39.991	39.993	39.989	39.987	39.986	39.989
39.993	39.994	39.991	39.991	39.992	39.992
39.991	39.993	39.995	39.990	39.993	39.992
39.989	39.992	39.991	39.993	39.996	39.992
39.989	39.986	39.988	39.988	39.991	39.988
39.990	39.990	39.990	39.993	39.991	39.991
39.988	39.992	39.991	39.995	39.996	39.992
39.993	39.991	39.992	39.989	39.988	39.990
39.992	39.994	39.994	39.991	39.991	39.992
39.994	39.990	39.989	39.991	39.991	39.991
39.994	39.991	39.995	39.993	39.996	39.994
39.992	39.993	39.993	39.993	39.993	39.993
39.991	39.990	39.993	39.989	39.990	39.990
39.988	39.994	39.994	39.992	39.993	39.992
39.992	39.991	39.991	39.990	39.998	39.992
39.990	39.995	39.992	39.989	39.990	39.991

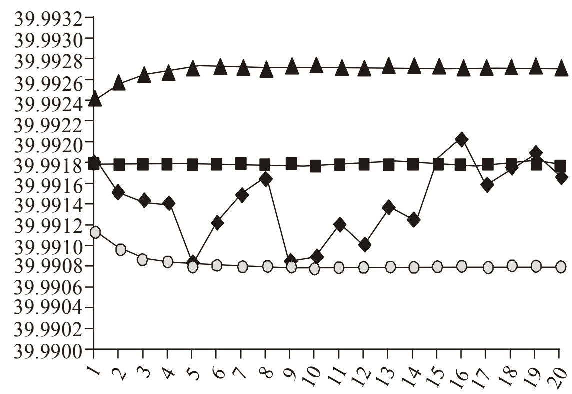

Fig. 1: The EWMA chart based on Table 3

$$Z_i = \lambda\bar{x}_i + (1-\lambda)Z_{i-1} = 0.25\,\bar{x}_i + 0.75\,Z_{i-1}$$

where, $Z_0 = \hat{\mu}_0 = 39.9918$, $\lambda = 0.25$.
The EWMA chart based on the sampling data in Table 3 is plotted in Fig. 1. The legends represent the estimated process parameters $\hat{\mu}_0$.

- Estimate the PCI, PQI and related parameters:

$\hat{\mu}_0 = 39.9914$, $\hat{\sigma}_0 = 0.00209$

$\hat{C}_p = 1.275$, $\hat{k} = -0.0719$

The lower confidence bound of C_p can be computed with 1-α_1 (α_1 = 0.1) confidence probability as follows:

$$C_{P,1-\alpha(\min)} = \hat{C}_P\sqrt{\frac{\chi^2_{\nu,1-\alpha}}{\nu}} = 1.275\sqrt{\frac{\chi^2_{76,0.90}}{76}} = 1.13$$

where, $\nu \cong f_n m(n-1) = 0.95 * 20 * 4 = 76$

When k is minus, the lower confidence bound of k can be computed with 1-α_2 (α_2 = 0.1) confidence probability as follows:

Table 4: The frequencies of sizes in each zone based on Table 3

The frequencies of sizes in lower zone (LSL~M-t/6)	The frequencies of sizes in center zone (M-t/6~ M+t/6)	The frequencies of sizes in upper zone (M+t/6~ULS)
6	78	16

$$k_{L,1-\alpha} = \hat{k} - t_{\nu,\alpha} \frac{1}{3\hat{C}_p \cdot \sqrt{mn}} = -0.110$$

The lower confidence bound of P_c (M±t/6) can be computed with $1\text{-}\alpha_1 \times \alpha_2$ confidence probability as follows:

$$P_{C,1-\alpha(\min)} = \{ \Phi[C_{P,1-\alpha(\min)}(1 - 3 \cdot k_{1-\alpha})] - \Phi[-C_{P,1-\alpha(\min)}(1 + 3 \cdot k_{1-\alpha})] \} \cdot 100\% = 71.25\%$$

This means that percentage of the sizes in center zone will be no less than 71.25% with 99% confidence probability. This estimation can be confirmed by the actual result in Table 4.

P_c = 78%>70% (the predetermined PQI).

CONCLUSION

To develop a quality-oriented SPC approach, a standardized interface between process quality indices system and the parameter design of control charts is needed. The quality-oriented statistical tolerancing is presented in three levels. The process quality target, statistical tolerance, sampling plan with confidence interval and the restrict condition to center line of EWMA control chart can be concurrently designed under the guideline of Quality-oriented statistical tolerancing and SPC technique. The predetermined PQIs can be assured via this CE approach under certain condition (process parameters are known or unknown) with a preset confidence probability.

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