

Research Article

High Maneuvering Target Tracking Based on Self-adaptive Interaction Multiple-Model

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Abstract: This study establishes a target motion model and an observation model under the condition of colored noise by using the Kalman filter based on an improved IMM (interactive multiple model) for maneuvering target tracking. To improve the overall performance of IMM algorithm, we proposed to combine the CV (constant velocity) and CA (constant acceleration) models with the "current" statistical model, in which its acceleration extremum is not fixed. Since the system model information is implicit in the current measurement, the Markov transition probability is computed online and real-time, so as to obtain more accurate a posteriori estimation and improve the model fusion accuracy. Monte Carlo simulations are carried out for the experiments and the results reveal that the proposed algorithm can get better performance in comparison with traditional IMM which adopts the "current" statistical model and CV-CA models.

Keywords: Interactive multiple model, Markov transition probability, Monte Carlo, target tracking

INTRODUCTION

In military and civil fields, such as missile and aero-traffic management, reliable and accurate tracking is always the main purpose of target tracking systems. In realization of maneuvering target accurate tracking, the first problem to be solved is to ensure that the target motion model matches with the actual target motion model.

After decades of continuous research by the scholars, a lot of target models and tracking algorithms have been proposed. The CV (Constant Velocity) model of uniform linear motion and the CA (Constant Acceleration) model of uniformly accelerated motion are maneuvering targets models for time-constant systems (Li and Jilkov, 2003; Xu and Wang, 2006; Wei *et al.*, 2012). These two models were the earliest, relatively simple and common models, but poorly applicable. Maneuvering acceleration model (Singer model) uses colored noises instead of white noises to describe the maneuvering control, which is more realistic (Li and Jilkov, 2003; Bilik, 2010). But it is only applicable to the uniform and accelerated targets. If the target maneuvering performance is over this range, this model will cause larger model error (Li and Jilkov, 2003; Xu and Wang, 2006).

Bar-Shalom and Birimiwal presented a CS ("current" statistical) model of maneuvering targets (Bar-Shalom and Birimiwal, 1982), firstly by adding the acceleration mean item. Secondly they used the modified Rayleigh-Markov process to describe the

maneuvering acceleration statistical characteristics of targets, reflecting the change of the target maneuvering range and intensity authentically, which was more suitable to the actual maneuvering target. Another useful method for maneuvering target tracking is the Interactive Multiple Model (IMM) algorithm (Singer, 1970; Blom and Bar-Shalom, 1988; Mazor *et al.*, 1998), which can exchange large computational resources for tracking maneuvering performance because a plurality of parallel Kalman filters are used in the model. At the same time, as the accurate transition probabilities between models are unavailable in the prior to cause usage of the IMM, the tracking precision is restricted. With the help of Singer model's thought, Mehrotra and Mahapatra proposed a Jerk model of maneuvering targets (Mahapatra and Mehrotra, 1997, 2000). It assumes that the target Jerk obeys the zero mean, one-order smooth related process and the time correlation function is in the exponential decay form. Compared with Singer model, the target acceleration change rate (Jerk) is added into the state vector. The Jerk model is a higher order and more accurate model. But in tracking of targets with step acceleration changing rate, Jerk model has steady-state deterministic error problem (Qi and Chen, 2008). In recent years, many researchers have proposed some target tracking algorithms based on nonlinear filtering such as insensitive Kalman filtering, particle filtering (Wang *et al.*, 2007; Mirosław *et al.*, 2011). These methods are not subjects to linear error or Gauss noise assumption limits, but the amount of calculation is large. In order to overcome the

deficiency of single model, many scholars put forward some improved α - β , α - β - γ filtering algorithms (Jin *et al.*, 2003; Qiao *et al.*, 2002) and some combinatorial algorithms (Lei and Han, 2006; Li *et al.*, 2007; Li and Yingmin, 2010). However, in radar applications, methods with high tracking accuracy and real-time performance for tracking high maneuvering targets may be particularly necessary for tracking the behavior of un-predictable target.

To solve the problem mentioned above, this study employs Kalman filter based on improved IMM for maneuvering target tracking. In our method, we propose to combine the CV and CA models with the "current" statistical model to improve the overall performance of IMM algorithm. Therefore the acceleration extremum is not fixed in the method. Due to the system model information that is implicit in the current measurement, the Markov transition probability is then computed online and real-timely, so that more accurate a posterior estimation and model fusion accuracy can be obtained in this way.

SYSTEM BASIC MOTION MODEL

Discrete time state equation and measurement equation: Consider the following discrete time state equation and measurement equation, which are depicted in Fig. 1:

$$X(k+1) = \Phi(k) X(k) + \Gamma w(k) \tag{1}$$

$$Z(k) = C(k) X(k) + v(k) \tag{2}$$

where, vector $X(k)$ and $Z(k)$ denote the maneuvering target motion state and measurement quantity at k moment respectively. $\Phi(k)$ and $C(k)$ denote the system process matrix and measure matrix at k moment respectively. $w(k)$ and $v(k)$ denote the system process noise and measurement noise at k moment respectively. $\Gamma(k)$ is the process noise matrix.

As the form and parameters of $\Phi(k)$ and $w(k)$ in (1) cannot be determined, maneuvering target tracking process is essentially an adaptive filtering process.

Firstly, maneuvering identification or maneuvering detection are conducted on the basis of the change of residual d . Secondly, adjusting filter gain, covariance matrix and unknown parameters according to some kind of Logic or criterion, to identify the characters of target maneuver real-timely. Finally, the state estimation value and the predicted value of the target are obtained by the filtering algorithm; thereby complete the maneuvering target tracking (Ilke, 2008).

CV (Constant Velocity) model: When the target does constant velocity motion:

$$X = [x(k) \dot{x}(k) y(k) \dot{y}(k)]^T$$

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \Gamma = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} w(k) = \begin{bmatrix} u_x(k) \\ u_y(k) \end{bmatrix}$$

where, $u_x(k)$ and $u_y(k)$ are independent Gaussian white noise with zero-mean and variances $\sigma_{ux}^2, \sigma_{uy}^2$.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, v(k) = \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix}$$

where, $v_x(k)$ and $v_y(k)$ are independent Gaussian white noise with zero-mean and variances σ^2 .

Constant acceleration model: When the target does uniform rotation movement, it can be approximated that the acceleration is constant:

$$X = [x(k) \dot{x}(k) y(k) \dot{y}(k) \ddot{x}(k) \ddot{y}(k)]^T$$

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & T & 0 & T^2/2 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Gamma = \begin{bmatrix} T^2/4 & 0 \\ T/2 & 0 \\ 0 & T^2/4 \\ 0 & T/2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$w(k) = \begin{bmatrix} u_x(k) \\ u_y(k) \end{bmatrix}$$

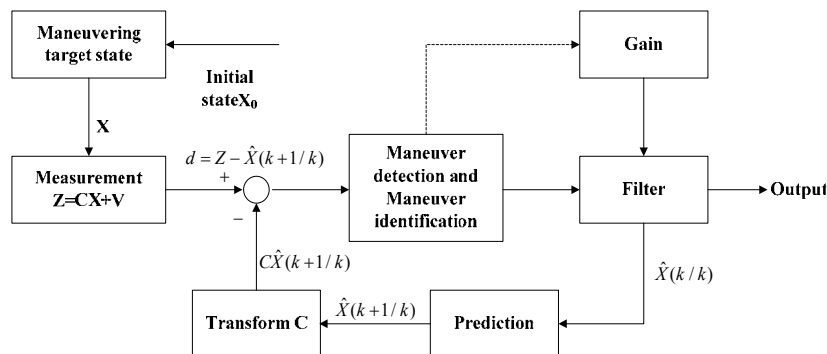


Fig. 1: Basic principle framework of the single maneuvering target tracking

where, $u_x(k)$ and $u_y(k)$ are independent Gaussian white noise with zero-mean and variances $\sigma_{ux}^2, \sigma_{uy}^2$:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{v}(k) = \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix}$$

where, $v_x(k)$ and $v_y(k)$ are independent Gaussian white noise with zero-mean and variances σ^2 .

"Current" adaptive statistical model: In one sampling period T, the relationship of target velocity increment and acceleration increment can be expressed as follows:

$$\Delta v(k) = T[a(k) + \Delta a(k)] \tag{3}$$

The predictive estimation value $\hat{x}_{k|k-1}$ of target velocity at k moment is taken as the velocity prediction estimation value from $k-1$ moment to k moment. The effect of the acceleration disturbance from $k-1$ moment to k moment on the observation value is not considered. However $\hat{x}_{k|k}$ considers the observation value at k moment, which contains the effect of the acceleration disturbance from $k-1$ moment to k moment on the observation value. So the disturbance increment relationship of the acceleration from $k-1$ moment to k moment can be described by the deviation relationship between the velocity estimate value $\hat{x}_{k|k}$ and the velocity estimate predictive value $\hat{x}_{k|k-1}$ of targets at k moment approximately, that is,

$$\Delta a(k) = \frac{1}{T} [\dot{\hat{x}}_{k|k} - \dot{\hat{x}}_{k|k-1}] \tag{4}$$

As the relationship of the maneuvering acceleration covariance and the absolute value of the acceleration disturbance increment are linear, there is a linear relationship between the acceleration increment and velocity estimation deviation at fixed sampling time. So the relationship of maneuvering acceleration covariance and velocity estimation deviation is linear, expressed as follows.

$$\sigma_{ak+1}^2 \rightarrow |\dot{\hat{x}}_{k|k} - \dot{\hat{x}}_{k|k-1}| \tag{5}$$

Take constant C as proportional coefficient, then (5) can be expressed as follows:

$$\sigma_{ak+1}^2 = C |\dot{\hat{x}}_{k|k} - \dot{\hat{x}}_{k|k-1}| \tag{6}$$

Seen from (6), when the target maneuvers, velocity estimation deviation increases, the acceleration variance also increases, so the filter gain value becomes larger. When the target doesn't maneuver, velocity estimation deviation is smaller, the acceleration variance is

smaller, there by the filter gain is not large. So the acceleration variance equation of (6) consists with its physical significance. This method adjusts the variance adaptively, which can well reflect the target motion state without the need for maneuvering detection and does not need to determine acceleration extremum a priori, so that the actual application value is high.

KALMAN FILTER OF CORRELATED MEASUREMENT NOISE

Here considering the filtering estimation problem in case of correlated measurement noise, Kalman filter basic signal model is:

$$\begin{aligned} X(k+1) &= \Phi(k+1, k) X(k) + \Gamma(k)w(k) \\ Y(k) &= C(k) X(k) \end{aligned}$$

The observation model is:

$$Z(k) = Y(k) + v(k)$$

where, $w(k)$ and $v(k)$ are Gaussian white noise with zero-mean and:

$$\begin{aligned} \text{cov } w(k) &= E\{w(k)w^T(j)\} = Q(k)\delta_{kj} \\ \text{cov } v(k) &= E\{v(k)v^T(j)\} = R(k)\delta_{kj} \\ \text{cov}\{w(k)v(j)\} &= 0 \end{aligned}$$

$$\begin{aligned} E\{X(0)\} &= m_x(0), \quad E\{X(0)w^T(j)\} = 0 \\ \text{var}\{X(0)\} &= P(0), \quad E\{X(0)v^T(j)\} = 0 \end{aligned}$$

$$\begin{aligned} E[X(k_0)] &= \mu_x(k_0), \quad \text{VAR}\{X(k_0)\} = P_x(k_0) \\ \hat{X}(k_0/k_0) &= \mu_x(k_0), \quad P_x(k_0/k_0) = P_x(k_0) \end{aligned}$$

$v_i(k)$ denotes the colored measurement noise of the i th model in IMM algorithm, regarded as a first-order Marco Cardiff sequence. A random sequence generated by that discrete white noise sequence with variance $\{\xi(k), k \in T\}$ effects on the linear system is as follows:

$$\begin{aligned} v_i(k+1) &= e^{-\beta}v_i(k) + \alpha\sqrt{1-e^{-2\beta}}\xi(k) \\ &= Q(k+1, k)v_i(k) + S(k)\xi(k) \end{aligned} \tag{7}$$

where,

$$Q(k+1, k) = e^{-\beta}, \quad S(k) = \alpha\sqrt{1-e^{-2\beta}}$$

Then the measurement equations of the i th model are:

$$Z(k) = C_i(k) X_i(k) + v_i(k)$$

which can be written as:

$$\begin{aligned} Z(k) &= C_i(k)X_i(k) + v_i(k) \\ &= C_i(k) \cdot [\Phi_i(k, k-1) \cdot X_i(k-1) + w_i(k-1)] + \\ & Q_i(k, k-1) \cdot v_i(k-1) + S(k-1) \cdot \xi(k-1) \\ &= C_i(k) \cdot \Phi_i(k, k-1) \cdot X_i(k-1) + C_i(k) \cdot w_i(k-1) + \\ & Q_i(k, k-1) [z(k-1) - C_i(k) \cdot X_i(k-1)] + S(k-1) \cdot \xi(k-1) \\ &= H_i^*(k) \cdot X_i(k-1) + Q_i(k, k-1) \cdot z(k-1) + C_i(k) \cdot w_i(k-1) + S(k-1) \cdot \xi(k-1) \end{aligned} \quad (8)$$

where,

$$H_i^*(k) = C_i(k) \cdot \Phi_i(k, k-1) - Q_i(k, k-1) \cdot C_i(k)$$

Then the measurement equation can be taken as only containing white noise sequence $w_i(k-1)$ and $\xi(k-1)$, the Kalman filter recursive steps are as follows:

Step 1: According to the previous filter value $\hat{X}(k-1/k-1)$ (or initial value $\hat{X}(0/0)$), calculating the predictive values:

$$\begin{aligned} \hat{X}(k/k-1) &= \Phi_i(k, k-1) \cdot \hat{X}(k-1/k-1) + u_i(k-1) \cdot \hat{X}(k-1/k-1) + \\ & K(k) \cdot [z(k) - Q_i(k, k-1) \cdot z(k-1) - C_i^*(k-1) \cdot \hat{X}(k-1/k-1) \\ & - C_i^*(k-1) \cdot u_i(k-1) \cdot \hat{X}(k-1/k-1)] \end{aligned} \quad (9)$$

Step 2: According to the previous filtering error variance matrix $P_{\hat{X}}(k-1/k-1)$ (or initial value $P_{\hat{X}}(0/0)$), calculating the prediction error variance matrix:

$$P(k/k) = [\Phi_i(k, k-1) - K(k) \cdot C_i^*(k-1)] \cdot P(k-1/k-1) \cdot \Phi_i^T(k, k-1) + [I - K(k) \cdot C_i(k)] Q_i(k-1) \quad (10)$$

Step 3: Calculating the Kalman gain:

$$K(k) = [\Phi_i(k, k-1) \cdot P(k-1/k-1) \cdot C_i^{*T}(k-1) + Q_i(k-1) \cdot C_i^T(k)] \cdot \begin{bmatrix} C_i^*(k-1) \cdot P(k-1/k-1) \cdot C_i^{*T}(k-1) + C_i(k) Q_i(k-1) \cdot C_i^T(k) + \\ S(k-1) \cdot S^T(k-1) \end{bmatrix}^{-1} \quad (11)$$

Step 4: Calculating the filtering estimation:

$$\hat{X}(k/k) = \hat{X}(k/k-1) + K(k) [Z(k) - C_i(k) \hat{X}(k/k-1)] \quad (12)$$

Step 5: Calculating the error variance matrix:

$$P_{\hat{X}}(k/k) = [I - K(k)C_i(k)] P_{\hat{X}}(k/k-1) \quad (13)$$

IMPROVED IMM ALGORITHM

The system equation and measurement equation of the j th model of the interactive multiple models are shown as follows:

$$\begin{aligned} X_j(k+1) &= \Phi_j(k)X_j(k) + \Gamma_j w_j(k) \\ Z_j(k+1) &= C_j(k+1)X_j(k+1) + v_j(k+1) \end{aligned}$$

where, $\Phi_j(k)$ is the state transition matrix of the j th model at k moment, $C_j(k+1)$ is the measurement transition matrix of the j th model at $k+1$ moment, $w_j(k)$ and $v_j(k+1)$ are independent Gaussian white noise with zero-mean and variances $Q_j(k)$ and $R_j(k+1)$.

Define state transfer probability matrix of system model at k moment as P_{ij} :

$$P_{ij}(k) = \frac{\Lambda_{ij}(k)P_{ij}(k-1)}{\sum_j \Lambda_{ij}(k)P_{ij}(k-1)} \quad (14)$$

P_{ij} denotes the transfer probability from the i th filter to the j th filter to filter at k moment, $i, j = 1, 2, \dots, N$:

where,

$$\begin{aligned} \Lambda_{ij}(k) &= N[\delta_{ij}(k+1) : 0, S_{ij}(k+1)] \\ &= \frac{\exp\left\{-\frac{\delta_{ij}(k)^T S_{ij}^{-1}(k) \delta_{ij}(k)}{2}\right\} P_{ij}(k-1)}{\sqrt{2 \cdot \pi \cdot S_{ij}(k)}} \end{aligned} \quad (15)$$

$\delta_{ij}(k)$ and $S_{ij}(k)$ denote measurement innovation and innovation covariance matrix respectively.

$$\delta_{ij}(k+1) = z(k+1) - C_j(k+1) \cdot [\Phi_j(k) \cdot \hat{x}_i(k|k) + B_j(k) \cdot u_j(k)] \quad (16)$$

$$\begin{aligned} S_{ij}(k+1) &= E\{\delta_{ij}(k) \cdot \delta_{ij}(k)^T\} = R_j(k+1) + C_j(k+1) \cdot \\ & \Phi_j(k) \cdot P_i(k|k) \cdot \Phi_j^T(k) \cdot C_j(k+1)^T + \\ & C_j(k+1) \cdot q_{ij}(k) \cdot C_j(k+1)^T \end{aligned} \quad (17)$$

$$q_{ij}(k) \square E[Q_i(k) | m_i(k), m_j(k+1), Z^{k+1}] \quad (18)$$

The above equations contain more accurate model of the probability distribution information $P_{ij}(k+1)$, when adjusting the interactive input of interacting multiple model algorithm, taking $P_{ij}(k+1)$ as the model transition probability matrix at next time. Therefore, the accuracy of a priori information is improved and the model fusion precision is improved.

EXPERIMENTAL RESULTS

In order to verify the effectiveness of the proposed algorithm in this study, the improved IMM algorithm is compared with the classical "current" statistical model for the maneuvering target tracking as can be seen from Fig. 2 to 4. The experiments are realized by Monte Carlo simulation test for different models, in which the sampling period is set as: $T = 1s$.

Seen from Fig. 2 and 3, because of the combination of "current" statistical model in which its acceleration extremum is not fixed with the CV/CA model in the IMM algorithm, the overall performance of IMM

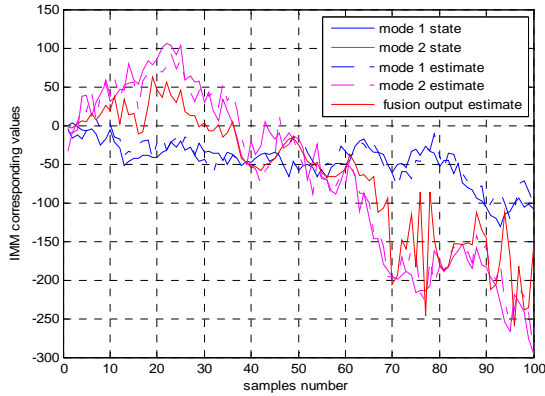


Fig. 2: Classical IMM with the model state and estimate

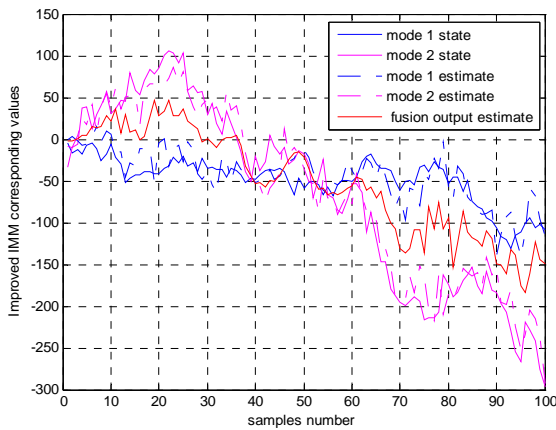


Fig. 3: The improved IMM model state and estimate

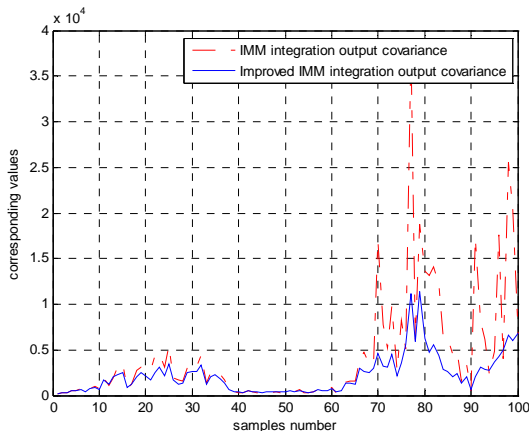


Fig. 4: The contrast of the improved algorithm and use "current" statistical model of fusion output covariance

algorithm is improved consequently. When the target doesn't maneuver nor does the low maneuvering motion, the improved algorithm performance is better than that of the classical "current" statistical model of IMM algorithm. However, when the target does large

maneuvering motion, error of improved algorithm is a little lower.

Seen from Fig. 4, the improved algorithm has better tracking effect compared with the classic "current" statistical model on maneuvering. On the non-maneuvering phase, their tracking performance is equivalent. In the motor, the improved algorithm can well suppress increment of tracking error caused by the sudden movement. When the target maneuvers, the improved algorithm uses the system model information that is implicit in the current measurement. This algorithm computes the Markov transition probability online and real-timely, therefore it can obtain more accurate a posterior estimate and improve the model fusion accuracy.

From Fig. 2, 3 and 4, we can find that this algorithm uses the Kalman filter based improved IMM for maneuvering target tracking, which combine the "current" statistical model, so its acceleration extremum is not fixed. In the algorithm the CV and CA model is to make the improved IMM have the ability to change the model set. Using the system model information that is implicit in the current measurement and the online real-timely computing Markov transition probability, more accurate a posterior estimate and the model fusion accuracy can be improved. In this way, a higher overall performance of the improved IMM algorithm can be achieved. Therefore, it is easy to see that this algorithm has better self-adaptability compared with classical IMM algorithm for different target maneuvering forms.

CONCLUSION

To overcome the acceleration extremum preset dependence problem and its fixed acceleration deficiency in the "current" statistical model, an improved interacting multiple model algorithms for maneuvering target tracking is presented in this study. The algorithm is developed by combining the CV and CA models with a "current" statistical model, which can use the implicit system model information to efficiently compute the Markov transition probability so as to improve the performance of the algorithm. The deviation between the velocity prediction estimation and velocity filtering estimation is used to adjust the acceleration variance adaptively. The Monte Carlo simulation results show that the proposed algorithm is effective and has the advantage of higher model fusion accuracy compared with the classical IMM algorithm. The adaptive tracking method in the study can be applied to track high maneuvering target because it has smaller filtering error and better performance than that of the classical IMM algorithm.

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