

## Research Article

### Analysis of Curving Ball Based on Mathematical Model

<sup>1</sup>Chao Chen and <sup>2</sup>Bing Zhang

<sup>1</sup>Institute of Physical Education, Jilin Normal University, Siping, 136000, China

<sup>2</sup>Institute of Physical Education, Huanggang Normal University, Huangzhou, 438000, China

**Abstract:** This paper studies curving ball, in perspective of physics and biomechanics, analyzes its forces in the air, describes the curving problems in different situations and makes motion equations. It stresses the deciduous ball, makes its traces in simulated equations and compare with the real situations. We aim at providing references for teaching, training and competition and having a deeper understanding of curving ball. It also supplies more scientific theoretic basis for training teachers and popularizing such technology.

**Keywords:** Curving ball, hydrodynamics, magnus effect, sphere mathematical model

#### INTRODUCTION

With millions of joiners, football is one of the most popular sports in the world. Due to it's widely attentions, nowadays, many people are studying its technologies. In 1998, 42 of 171 goals are generated by positioning balls, and fifty percents of which are arbitrary balls. Therefore the arbitrary ball is the point. Beckham's banana ball, Cristiano and Renaldo's rapid dropping ball before the goal and bounced ball, are forcing us to study such beautiful situation (Sun and Yang, 2003).

Curving ball, also called as rotating ball, which refers to rotating ball because of the force isn't through the center. Influenced by air forces, the rotating ball will run certain curving distance. It also is the fast turning phenomenon when the ball is next to the goal or the curving motion which can fool the goalkeeper (Wang, 2001). Therefore, rotating ball is the common goal and passing ball methods, especially for positioning ball (Gai, 2003). With its rapid development, the football becomes more complex, diversified, fine and difficult. The rotating, which is the most difficult to judge, which is applied more and more widely, with more and more important effects. For example, athlete can across the keepers in curving ball; the positioning ball can be used to across the people wall to win, etc., (Chongxi, 2005).

With reference to the principle and technical keys, Chinese people have made lots of researches. Li Shuping, in the book named as motion biomechanics, clarified rotating ball's generation and traces in hydrodynamics (Ge, 1991). He said if the ball rotates, it will conduct a curving motion; Liu Dawei, from Harbin Industry University, in his book College Physics, discusses the abnormal sphere distance and indicates the reason for the rotation's curving motion in

horizontal direction relates to the air (Ge and Ye, 1992). Air is the fluid. If a football to motion in the fluid, some changes are essential (Liu, 1987). The most valuable research on curving ball is from Xiong Zhifeng, from Jiangxi Normal School. He studies Beckham's curving technology, which is the global famous football player, and explores his technical features and regulations in biomechanics and competitive abilities (Xiong, 2007). It is the reference for training, competition and researches on world-level curving technology. Therefore, curving ball includes a lot and is very important. In the following, clear analysis and discussion will be made. We aim at providing references for teaching, training and competition and having a deeper understanding of curving ball. It also supplies more scientific theoretic basis for training teachers and popularizing such technology.

#### THEORETICAL BASIS

**Sphere bernoulli's principle:** Bernoulli's principle is essential for aerodynamics. Swedish mathematician Daniel Bernoulli proposed the well-knowing principle:

$$p + \frac{1}{2} \rho V^2 = C \quad (1)$$

P = Forces in a point

$\rho$  = Density of air fluid

V = The velocity in a point

The rotating ball has great power. Its distance is different, which is caused by the surrounding air's flowing. The flowing line of the air around when the non-rotating ball motions towards the left is: the lines are symmetric, with the same flowing velocity without

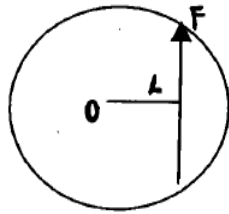


Fig. 1: Arm

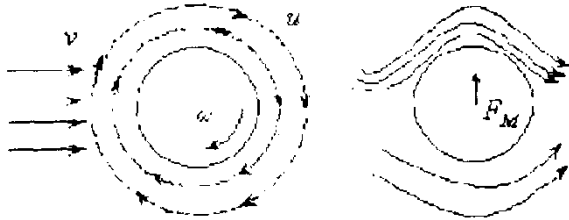


Fig. 2: Horizontal cutting surface of air flowing line distribution

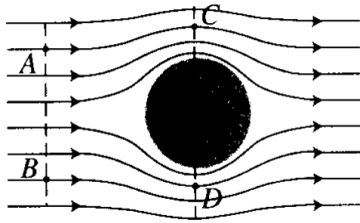


Fig. 3: Bernoulli's principle

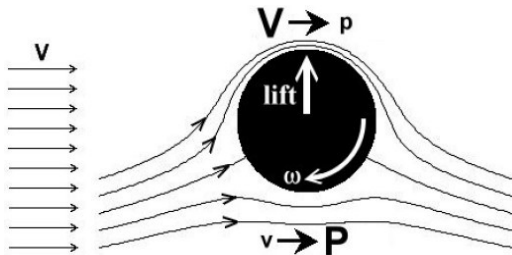


Fig. 4: Sphere's magnum effects

the difference of pressure (Wang, 2002). When the ball rotates, the rotating axis is through the sphere center and vertical to the paper, and the ball motions clockwise. The air also rotates with the ball, which will increase the flowing velocity of the air down and decreases the up flowing velocity. The greater flowing velocity is, the smaller pressure is. Compare to the non-rotating ball, the rotating ball is forced down, so its distance towards down (Li, 2007).

According to force shifting, the force (L) offsetting the sphere center will generate the force distance M and shifting force F', With reference to Fig. 1.

According to mechanic principle: rotate arm  $M = \text{force } (F) \times \text{arm } (L)$ . Then, the strength of rotation

depends on the force and arm. On the condition of constant sphere center and force, the longer arm is, the greater rotate force is.

The forces in the curving ball are weight W, air resistance  $F = c\rho AV^2/2$  (1), air resistance arm and Magnus force. To calculate conveniently, bring in coefficient  $K = c\rho A/2$ ,  $G = 8\pi\rho\omega a^3 v/3$ , then  $F = kv^2$  (2),  $L = Gv$  (Carre *et al.*, 1998). The small air resistance arm could be ignored. Due to conservations of moment of momentum,  $\omega = \omega_0$ . Formula (2) shows that, the air resistance is positive to square of its velocity, which makes the fast decreasing of horizontal velocity.

On the other hand, due to internal friction, the internal friction is positive proportion to air cling coefficient and angle velocity gradient. The air cling coefficient is small, so the decreasing of rotate angle velocity caused by friction arm is slow. When the ball advances, due to the sticky air, the air next to the surface of the sphere rotates with the ball, which generates circulated flowing surrounding the ball, shown as Fig. 2.

Long Geqi, who is from Zhejiang Normal University, in his discussion on football motion principle, shows the motion principles of curving ball and reveals that its distance is a spatial curve in Magnus effect and air resistance (Zhang, 2002).

When the rotate ball motions in the air, it will forces the air motions with the surface of ball, generating circulating flowing and pressure on the sphere, so that the difference forms. The pressure conforms to Bernoulli's principle: the faster flowing velocity is, the smaller pressure is; the slower the flowing is, the bigger pressure is Carre *et al.* (1998) With reference to Fig. 3. If fore-rotate ball motions, the up air flowing velocity will decrease because of air resistance, but the down air flowing velocity will increase. So the pressure difference is generated.

**Sphere's magnum effects:** Due to Bernoulli's principle, the ball with curving distance must rotate. The forces, which make the distance curved, are generated by rotating. Asymmetrical air fluid will be generated, generating the up force or sides force, which is vertical to the axis. The Fig. 4 shows, a rotating ball will generate rotations towards up or sides due to different rotate axis.

### MATHEMATICAL MODEL

**Forces of sphere:** Wesson (2002) indicates there are three forces on the sphere in the air, shown as Fig. 5. They are gravity, air resistance and Magnus force caused by rotating. On the condition of Fig. 5, Magus Force is opposite to gravity, which is the up force.

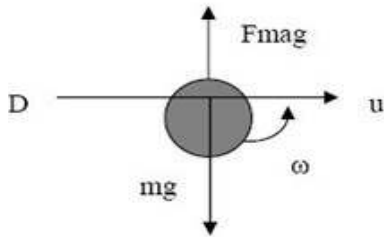


Fig. 5: Forces of rotating sphere

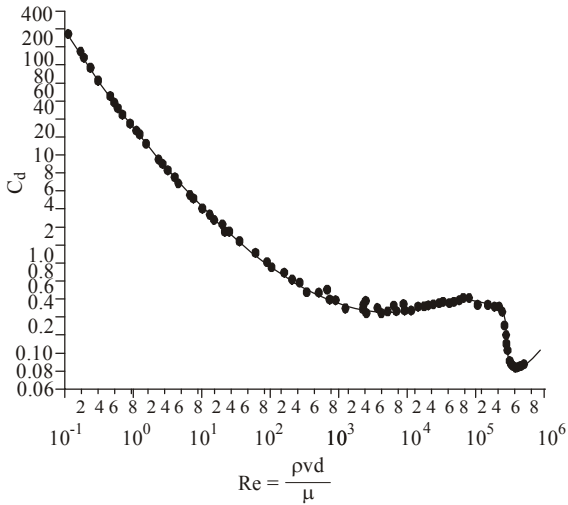


Fig. 6: Relations between resistance coefficient and sphere Reynolds number

Kreighbaum and Barthels (1996) propose, the aeromechanics depends on surface features and air area, air fluid velocity and pressure. They give the formula for any motion materials in the air:

$$D = -\frac{1}{2} C_d \rho A |v|^2 \frac{v}{|v|} \tag{2}$$

$$F_{mag} = \frac{1}{2} C_{mag} \rho A |v|^2 \frac{\omega \times v}{|\omega \times v|} \tag{3}$$

where,

- D = Air resistance
- $F_{mag}$  = Magnus force
- $C_d$  = Resistance coefficient
- $C_{mag}$  = Magnus force coefficient
- A = Shadowing area
- v = Comparative fluid velocity

**Resistance coefficient and magnus force coefficient:** Resistance coefficient is relevant to air's density, velocity and resistances of sphere shadowing area. But to the same objects, the differences of resistance coefficients are directly relevant to Reynolds number.

Carre *et al.* (1998) research found that, the Reynolds number is relevant to the smoothness of object's surface and velocity. So the faster sphere has smaller resistance.

The Anderson (2003) relation figure between resistance coefficient and Reynolds number shows  $C_d$  decreases as Re increases. At the critical point,  $C_d$  will decrease rapidly. That is because, the air fluid suddenly turning into turbulence, the flowing line separates and the resistance decreases rapidly, With reference to Fig. 6.

The same as resistance coefficient, the Magnus force is also relevant to air flowing's density, velocity and shadowing are. Non-rotating sphere's Magnus force is zero. So we only discuss the effects of rotating on Magnus coefficient.

To a rotating sphere, no matter its rotating direction, its Magnus force changes the motion distance, so that the curving ball is generated.

**Acceleration equation:** To the sphere shown in the Fig. 6, with the consideration of gravity, Magnus force and air resistance, the vector equation of motion is:

$$a = \frac{F_{mag}}{m} + \frac{D}{m} + g \tag{4}$$

Bring in  $F_{mag}$  and D then:

$$a = \frac{1}{2} \frac{C_{mag} \rho A |v|^2 \frac{\omega \times v}{|\omega \times v|}}{m} - \frac{1}{2} \frac{C_d \rho A |v|^2 \frac{v}{|v|}}{m} + g \tag{5}$$

- a = The acceleration
- m = Sphere's mass
- g = Gravity acceleration

The acceleration equation is already given. To a definite sphere, due to the uncertain  $\rho$ ,  $C_{mag}$  and  $C_d$ , we can't list a simpler equation. Under current conditions, the writer can't give more explanations and descriptions. If better conditions exist, this equation can be simulated in computer, which can show more image explanations.

Although the straight motions of curving ball can't be described, we can explain the curving ball and its relevant phenomena in such model.

### THE FUNCTION OF MODEL ON THE PHENOMENON

According to the international standards, we set the parameters as follows:

Diameter	69 cm
Mass	430 g



Fig. 7: C luo's arbitrary ball in computer simulation

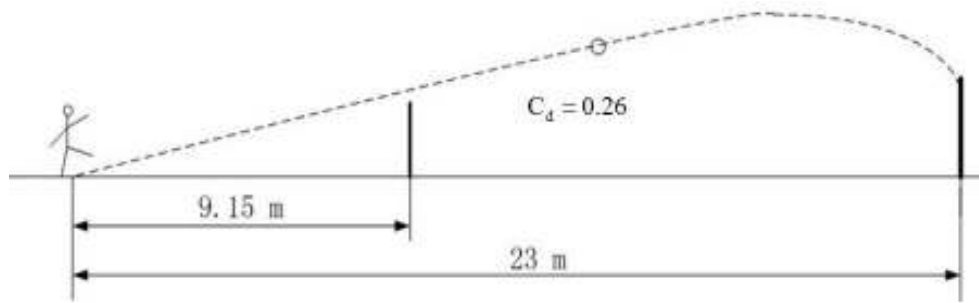


Fig. 8: Ideal elevator ball's distance

Goal's size	7.32×2.44 m
The distance between goal line and ban line	16.5 m
Air density	1.25 kg/m <sup>3</sup>

Brazil plays Didi developed elevator ball (deciduous ball), and the representation in the current football area are Pirlo, Cristiano and Ronaldo, etc. This paper takes c luo's elevator ball as an example, studies the distance of elevator ball and the reason for its dropping rapidly before the goal.

With the help of arbitrary ball in such game, we build the straight model Fig. 7.

Shown as Fig. 7, as c luo's arbitrary ball, we select his best 23-m distance to study his arbitrary ball's distance. There is nearly no sides rotations and a little outside rotation. No side rotation means any side curves. We simplify the ball's distance as a motion in the plane.

In formula (2), air resistance coefficient  $C_d$  is directly relevant to  $R_e$ . In Anderson's study,  $R_e$  is about  $2.5 \times 10^5$ . The Fig. 8 shows  $D = -0.13 \rho A |v|^2 \frac{v}{|v|}$ . So the formula (2) is simplified as:

$$C_{mag} = 2\omega R/v \quad \rho = 1.25 \text{ kg/m}^3 \quad (6)$$

In formula (3), according to paper (Qiu, 2007),  $R$  is the radius of sphere. So formula (3) is simplified as:

$$F_{mag} = \frac{\omega R}{v} \rho A |v|^2 \frac{\omega \times v}{\omega \times v} \quad (7)$$

We only study the two-dimensional motion. Compose the velocity into  $x$ ,  $y$  directions. But only consider the angle velocity in  $z$  axis:

$$v = \sqrt{v_x^2 + v_y^2}, \quad \omega = \omega_z \quad (8)$$

Papers show, the velocity of arbitrary ball can reach 120 km/h, and even 200 km/h. Imagine c luo's velocity is 100 km/h, which is mean velocity is 27 m/s. According to its feature, we imagine it doesn't rotate, which is  $\omega = 0$ . Then, we simplify the formula (5) into its simplest way:

$$a = -0.13 \frac{\rho A |v|^2 \frac{v}{|v|}}{m} + g \quad (9)$$

Take  $\rho$ ,  $A$ ,  $m$  into formula (9), then:

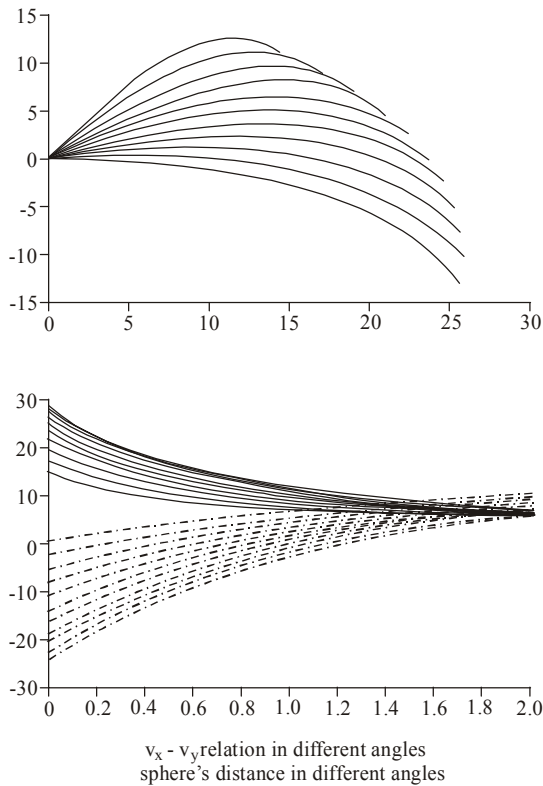


Fig. 9: Computer simulation

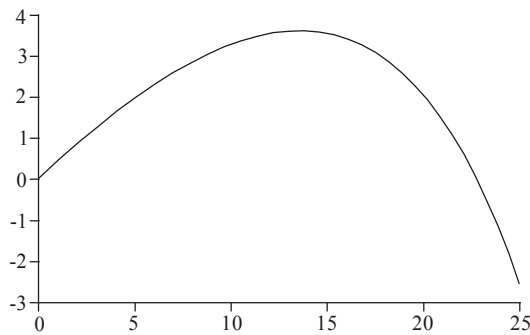


Fig. 10: Simulation distance

$$a \approx -0.05 |v| v + g \tag{10}$$

Decompose in two directions:

$$\frac{dv_x}{dt} = 0.05 v_x \sqrt{v_x^2 + v_y^2} \tag{11}$$

$$\frac{dv_y}{dt} = 0.05 v_y \sqrt{v_x^2 + v_y^2} + g \tag{12}$$

This equation can't be solved. We use the figure in the computer. Imagine the initial velocity is 30 m/s. Due to the uncertain initial angle, this simulation

considers distance situations in different initial angles, shown as Fig. 9:

We find the most suited situation among the ten distances, shown as Fig. 10.

Figure 10 is the distance in the angle 30. The ball drops in the lowest point at the 23 m, which can succeed in crossing the person wall. The distance fits the reality. Formula (11) and (12) are available.

However, Magnus force is ignored. Due to the elevator's features, the rotation is slight which makes the small Magnus force. So the distance is not influenced.

### CONCLUSION

This study analyzes and calculates the curving ball in models. It stresses the simple elevator ball (deciduous ball). Due to its special situation, Magnus force is ignored. We roughly reckon the resistance coefficient. Besides, the elevator ball's equation is given, and gets better coefficients to fits the reality.

However, there are many kinds of curving balls. For example, Beckham's curving ball, with strong side rotation, which is more complex than elevator ball. There are still many differences, which need to be studied.

In practice, practices, such as adjusting kicking strength, force direction and kicking point, etc., can help understand the features and regulations of curving ball. But theory is also essential. This paper clarifies and analyzes the mechanical principles and technical features in biomechanics and mathematical model. The fine of skills and regulation of motions should be enhanced in teaching and training. While explaining the skills, it is essential to introduce some biomechanical knowledge and mechanical principles to makes students' skills to be rationally, which can improve their thinking capability and technologies.

### REFERENCES

Anderson, J.D., 2003. Fundamentals of Aerodynamics. McGraw-Hill, Singapore.

Carre, M.J., T. Asai, T. Akatsuka and S.J. Haake, 1998. The curve kick of a football: Flight though the air. Sports Eng., 5: 193-200.

Chongxi, W., 2005. Higher Education Textbook. Ball Sports-Football, Higher Education Press, China.

Gai, H.F., 2003. Textbook for football training [M]. People Physical Education Press., Vol. 01.

Ge, L., 1991. Discussion on curving ball. Coll. Phys., 7: 26-28.

Ge, L. and W. Ye, 1992. Motion principles of rotating football. Phys. Bull., 2: 7-8.

Kreighbaum, E. and K.M. Barthels, 1996. Biomechanics: A Qualitative Approach for Studying Human Movement. 4th Edn., Allyn and Bacon, Boston, pp: 619, ISBN: 0205186513.

- Li, G., 2007. Brief analysis of the skill of upward loop pitch ball shooting. *J. Shanxi Normal School Phys. Educ.*, Vol. 5.
- Liu, D., 1987. Discussion on abnormal sphere's distance. *Coll. Phys.*, 2: 43-45.
- Qiu, Y., 2007. Measure and Analysis of Free Ball's Distance. Jan.
- Sun, C. and X. Yang, 2003. Mechanical analysis of spinning ball and non-spinning ball. *J. Xiaogan Institute*, Vol. 03.
- Wang, Z., 2001. Functions of curving ball in football match [J] *Journal of Lianyungang Teach. Coll.*, Vol. 4.
- Wang, X., 2002. *P.E Statistics and Spss* [M]. People Physical Education.
- Wesson, J., 2002. *The Science of Soccer*. Taylor & Francis, New York.
- Xiong, Z., 2007. Analysis of motion biomechanics and competitive abilities of Beckham's curving ball [J]. *J. Shanxi Normal School Phys.*, Vol. 5.
- Zhang, L., 2002. *P.E Science Methods* [M]. Higher Education Press, China.