

Research Article

Solving a Multi-Item Finite Production Rate Model with Scrap and Multi-Shipment Policy without Derivatives

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Abstract: In real life manufacturing environments, production managers often make plan to produce m products in turn on a single machine in order to maximize the machine utilization. This study focuses on determining an optimal common production cycle time for a multi-item Finite Production Rate (FPR) model with random scrap rate and multi-shipment plan without using the derivatives. We assume that m products are manufactured in turn based on a common cycle policy on a single machine and during the processes some nonconforming items are generated randomly. These defective items cannot be reworked, thus they are scrapped at additional costs in the end of each production cycle. After the entire lot is quality assured, the delivery of finished items is under a practical multiple shipments plan. The objective is to determine an optimal common cycle time that minimizes the long-run average cost per unit time using an algebraic approach. Conventional method for solving such a problem is by the use of differential calculus on system cost function to derive the optimal policy, whereas the proposed approach is a straightforward method. It helps practitioners, who may not have sufficient knowledge of calculus, to understand and manage the real-life multi-item production systems more effectively.

Keywords: Algebraic approach, common cycle time, finite production rate, inventory, multi-item production, shipment, scrap

INTRODUCTION

For many decades, the Finite Production Rate (FPR) model has been a fundamental technique for determining the optimal lot size for items produced in-house in most manufacturing firms (Nahmias, 2009). The classic FPR model considers a production planning for single product. However, in real life manufacturing sector most vendors often make plan to produce m products in turn on a single machine in order to maximize the machine utilization. Gordon and Surkis (1975) presented a simple and practical approach to determine control policies for a multi-item inventory environment. They considered the items are ordered from a single supplier and the demands for items are subject to severe fluctuations. The time between orders can either be fixed or based on accumulating a fixed order quantity for all products. Their model balanced carrying and stock-out costs. An operational system structure is developed and a simulation procedure was used to determine the appropriate value of their inventory factor in the model. Rosenblatt and Finger (1983) considered a problem of multi-item production

on a single facility. The facility was an electrochemical machining system and the products are impact sockets of various sizes for power wrenches. A grouping procedure of the various items was adopted. A modified version of an existing algorithm was applied to insure production cycle times which are multiples of the shortest production cycle time. Byrne (1990) presented an approach to the multi-item production lot sizing problem based on the use of simulation to model the interactions occurring in the system. A search algorithm was used to adjust the lot sizes on the basis of the results of previous simulation runs, with the objective of achieving a minimum total cost solution. Studies related to addressing various aspects of multi-item production planning and optimization issues (Kohli and Park, 1994; Aliyu and Andijani, 1999; Lin *et al.*, 2006; Ma *et al.*, 2010; Rossetti and Achlerkar, 2011).

Imperfect product quality has been another special focus in recent literatures, for in real-life production environment it is inevitable to produce some defective items. Mak (1985) developed a mathematical model for an inventory system in which the number of units of acceptable quality in a replenishment lot is uncertain

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and the demand is partially captive. It was assumed that the fraction of the demand during the stock-out period which can be backordered is a random variable whose probability distribution is known. The optimal replenishment policy is synthesized for such a system. A numerical example was used to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Many studies have since been conducted to address the issues of quality assurance in production systems (Hariga and Ben-Daya, 1998; Teunter and Flapper, 2003; Wee *et al.*, 2007; Chiu *et al.*, 2009, 2011, 2012a; Lee *et al.*, 2011). In this study, all defective items produced are not repairable, thus they are scrapped at an additional disposal cost.

Another unrealistic continuous inventory issuing policy was assumed by classic FPR model to satisfy product demand. But in real world situations the multiple or periodic deliveries of finished products are often used. Schwarz (1973) examined a simple continuous review deterministic one-warehouse N-retailer inventory problem with the purpose of deciding the stocking policy, for minimizing the average system cost. Many other studies dealing with different aspects of vendor-buyer supply chain optimization issues can also be found (Hill, 1997; Sarker and Khan, 2001; Abdul-Jalbar *et al.*, 2005; Chiu *et al.*, 2011, 2012b, c).

Conventional approach for solving optimal lot sizing is by the use of differential calculus on system cost function. Recently, Grubbström and Erdem (1999) proposed an algebraic derivation to solve the Economic Order Quantity (EOQ) model with backlogging. Their algebraic approach does not reference to the first- or second-order derivatives. Similar approach was applied to solve various aspects of production and supply chain optimization (Chiu, 2008; Lin *et al.*, 2008; Chen *et al.*, 2012). This study extends such an algebraic approach to resolve a multi-item FPR model with scrap and multi-shipment policy (Chiu *et al.*, 2012d). The objective is to determine an optimal common cycle time that minimizes the long-run average cost per unit time using an algebraic approach. It may help practitioners, who have insufficient knowledge of calculus to understand the real-life multi-item production systems more effectively.

DESCRIPTION, MODELLING AND FORMULATIONS

A multi-item FPR model with random scrap rate and multi-delivery policy (Chiu *et al.*, 2012d) is reexamined here using mathematical modeling along with an algebraic derivation. Consider a FPR model where a machine can produce m products in turn, using the common production cycle policy. All items made are screened and unit inspection cost is included in unit production cost C_i . During the production of product i

(where, $i = 1, 2, \dots, m$) an x_i portion of defective items is produced randomly at a rate of d_i . All defective items cannot be repaired and thus they must be scrapped in the end of production with an additional cost C_{Si} . Under the normal operation, the constant production rate for product i , P_i satisfies $(P_i - d_i - \lambda_i) > 0$, where λ_i is the annual demand rate for product i and d_i can be expressed as $d_i = x_i P_i$. A multi-shipment policy is employed in this study. It is assumed that the finished items for each product i can be delivered to customers only if the whole production lot is quality assured in the end of production. Fixed quantity n installments of the finished batch are delivered at a fixed interval of time during delivery time t_{2i} (Fig. 1). Other variables used here include: unit holding cost h_i , the production setup cost K_i , the fixed delivery cost K_{1i} per shipment for product i and unit shipping cost C_i for product i . Additional notation also includes:

- T = The common production cycle length, to be determined
- t_{1i} = The production uptime for product i in the proposed system
- H_i = Maximum level of on-hand inventory in units for product i when regular production process ends
- Q_i = Production lot size per cycle for product i
- n = Number of fixed quantity installments of the finished batch to be delivered to customers in each cycle, it is assumed to be a constant for all products
- t_{ni} = A fixed interval of time between each installment of finished products delivered during t_{2i} , for product i
- $I(t_i)$ = on-hand inventory of perfect quality items for product i at time t
- $TC(Q_i)$ = Total production-inventory-delivery costs per cycle for product i in the proposed system
- $E[TCU(Q)]$ = Total expected production-inventory-delivery costs per unit time for m products in the proposed system
- $E[TCU(T)]$ = Total expected production-inventory-delivery costs per unit time for m products in the proposed system using common production cycle time as the decision variable

One can directly observe the following equations from Fig. 1:

$$t_{1i} = \frac{Q_i}{P_i} = \frac{H_i}{P_i - d_i} \tag{1}$$

$$t_{2i} = n t_{ni} = Q_i \left(\frac{(1 - x_i)}{\lambda_i} - \frac{1}{P_i} \right) \tag{2}$$

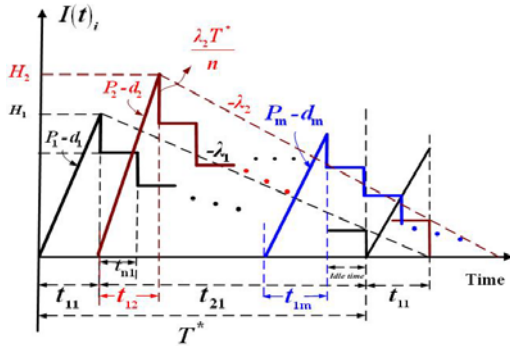


Fig. 1: On-hand inventory of perfect quality items for product i in a cycle (Chiu *et al.*, 2012d)

$$H_i = (P_i - d_i)t_{1i} = (P_i - d_i)\frac{Q_i}{\lambda_i} = (1 - x_i)Q_i \quad (3)$$

$$T = t_{1i} + t_{2i} = \frac{Q_i}{\lambda_i}(1 - x_i) \quad (4)$$

$$d_i t_{1i} = P_i x_i t_{1i} = x_i Q_i \quad (5)$$

The variable holding costs for finished products kept by vendor during t_{2i} are Chiu *et al.* (2009):

$$h \left(\frac{n-1}{2n} \right) H_i t_{2i} \quad (6)$$

Total costs per cycle $TC(Q_i)$ for m products (as shown in Eq. (7)) consists of variable production cost, setup cost, scrapped cost, fixed and variable delivery costs, holding cost during t_{1i} and holding cost for finished goods kept during t_{2i} :

$$\sum_{i=1}^m TC(Q_i) = \sum_{i=1}^m \left\{ C_i Q_i + K_i + C_{S_i} x_i Q_i + n K_{1i} + C_{T_i} Q_i (1 - x_i) + h_i \left[t_{1i} \left(\frac{H_i}{2} + \frac{x_i Q_i}{2} \right) + \frac{n-1}{2n} (H_i t_{2i}) \right] \right\} \quad (7)$$

Taking the randomness of x into account and by substituting all parameters from Eq. (1) to (6) in Eq. (7), the expected $E[TCU(Q)]$ can be obtained as Chiu *et al.* (2012d):

$$E[TCU(Q)] = E \left[\sum_{i=1}^m TC(Q_i) \right] \frac{1}{E[T]} = \sum_{i=1}^m \left\{ C_i \lambda_i \frac{1}{[1-E(x_i)]} + C_{S_i} \lambda_i \frac{E(x_i)}{[1-E(x_i)]} + \frac{K_i \lambda_i}{Q_i} \frac{1}{[1-E(x_i)]} + C_{T_i} \lambda_i + \frac{n K_{1i} \lambda_i}{Q_i} \frac{1}{[1-E(x_i)]} + \frac{h_i Q_i}{2} \left[\frac{\lambda_i}{P_i} \frac{E(x_i)}{[1-E(x_i)]} + 1 - E(x_i) - \frac{1}{n} \left(1 - E(x_i) - \frac{\lambda_i}{P_i} \right) \right] \right\} \quad (8)$$

where,

$$E[T] = Q_i (1 - E[x_i]) \frac{1}{\lambda_i}$$

One can further convert the aforementioned equation into $E[TCU(T)]$ by applying Eq. (4):

$$E[TCU(T)] = \sum_{i=1}^m \left\{ C_i \lambda_i \cdot \frac{1}{[1-E(x_i)]} + C_{S_i} \lambda_i \frac{E(x_i)}{[1-E(x_i)]} + \frac{K_i}{T} + C_{T_i} \lambda_i + \frac{n K_{1i}}{T} + \frac{h_i T \lambda_i}{2} \left[\frac{\lambda_i}{P_i} \frac{E(x_i)}{[1-E(x_i)]} \cdot \frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n} \left(1 - \frac{\lambda_i}{P_i} \frac{1}{[1-E(x_i)]} \right) \right] \right\} \quad (9)$$

THE PROPOSED ALGEBRAIC APPROACH

One notes that Eq. (9) contains one single decision variable T in different forms. Therefore, one can rearrange it as:

$$E[TCU(T)] = \sum_{i=1}^m \left\{ C_i \lambda_i \cdot \frac{1}{[1-E(x_i)]} + C_{S_i} \lambda_i \frac{E(x_i)}{[1-E(x_i)]} + C_{T_i} \lambda_i \right\} + \sum_{i=1}^m \left\{ \frac{K_i}{T} + \frac{n K_{1i}}{T} + \frac{h_i T \lambda_i}{2} \left[\frac{\lambda_i}{P_i} \frac{E(x_i)}{[1-E(x_i)]} \cdot \frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n} \left(1 - \frac{\lambda_i}{P_i} \frac{1}{[1-E(x_i)]} \right) \right] \right\} \quad (10)$$

or

$$E[TCU(T)] = \beta_1 + \beta_2 \cdot T^{-1} + \beta_3 \cdot T \quad (11)$$

where,

$$\beta_1 = \sum_{i=1}^m \left\{ C_i \lambda_i \cdot \frac{1}{[1-E(x_i)]} + C_{sr} \lambda_i \frac{E(x_i)}{[1-E(x_i)]} + C_{Ti} \lambda_i \right\} \quad (12)$$

$$\beta_2 = \sum_{i=1}^m (K_i + nK_{1i}) \quad (13)$$

$$\beta_3 = \sum_{i=1}^m \left\{ \frac{h_i \lambda_i}{2} \left[\frac{\lambda_i E(x_i)}{P_i [1-E(x_i)]} \cdot \frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n} \left(1 - \frac{\lambda_i}{P_i [1-E(x_i)]} \right) \right] \right\} \quad (14)$$

Further rearranging Eq. (14) one has:

$$E[TCU(T)] = \beta_1 + T^{-1} [\beta_2 + \beta_3 \cdot T^2] \quad (15)$$

or

$$E[TCU(T)] = \beta_1 + T^{-1} [\sqrt{\beta_2} - (T\sqrt{\beta_3})]^2 + 2T^{-1} [\sqrt{\beta_2} \cdot (T\sqrt{\beta_3})] \quad (16)$$

One notes that Eq. (16) will be minimized if the second term of the right-hand side equal zero. That is:

$$T = \frac{\sqrt{\beta_2}}{\sqrt{\beta_3}} \quad (17)$$

Substituting Eq. (13) and (14) in Eq. (17), the optimal common production cycle time T^* is obtained:

$$T^* = \sqrt{\frac{2 \sum_{i=1}^m (K_i + nK_{1i})}{\sum_{i=1}^m \left\{ h_i \lambda_i \left[\frac{\lambda_i E(x_i)}{P_i [1-E(x_i)]} \cdot \frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n} \left(1 - \frac{\lambda_i}{P_i [1-E(x_i)]} \right) \right] \right\}}} \quad (18)$$

One notes that Eq. (18) is identical to that obtained by using the conventional differential calculus method (Appendix). Applying T^* one has the $E [TCU (T)]$ as follows:

$$E[TCU(T^*)] = \beta_1 + 2\sqrt{\beta_2} \cdot \sqrt{\beta_3} \quad (19)$$

Numerical example: For the purpose of demonstrating the obtained result, the same numerical example (Chiu

et al., 2012d) is provided in this section. Reconsider that a manufacturing firm has employed a common production cycle time policy produce five products in turn on a single machine. The annual demand rates λ_i for 5 different products are 3000, 3200, 3400, 3600 and 3800, respectively. Each product can be made at different production rates P_i as 58000, 59000, 60000, 61000 and 62000 respectively. Random scrap rates during production uptime for each product follow uniform distribution over the intervals of (0, 0.05), (0, 0.10), (0, 0.15), (0, 0.20) and (0, 0.25) respectively. Scrap items for each product have different disposal costs of \$ 20, 25, 30, 35 and 40 per item, respectively. Values for other variables are as follows:

- C_i = Manufacturing cost per item for each product are \$ 80, 90, 100, 110 and 120, respectively
- K_i = Set up costs for each product are 3800, 3900, 4000, 4100 and 4200, respectively
- h_i = Unit holding costs are \$ 10, 15, 20, 25 and 30, respectively
- K_{1i} = The fixed delivery costs per shipment are \$ 1800, 1900, 2000, 2100 and 2200, respectively
- n = Number of shipments per cycle is assumed to be four (a constant)
- C_{Ti} = Unit transportation costs are \$ 0.1, 0.2, 0.3, 0.4 and 0.5, respectively

The optimal common production cycle time $T^* = 0.6662$ (years), can be obtained by applying Eq. (18) and the expected system cost per unit time for m products $E [TCU (T^*)] = \$ 2113194$, can also be computed using the Eq. (19). It is noted that these results are identical to those obtained in Chiu *et al.* (2012d).

CONCLUSION

Conventional method for solving multi-item FPR model with scrap and multi-shipment policy is by the use of differential calculus on system cost function to derive the optimal policy (Chiu *et al.*, 2012d), whereas this study proposed a mathematical modeling along with a straightforward algebraic approach. It aims at helping practitioners to understand and manage the real-life multi-item FPR systems without have to reference to differential calculus.

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APPENDIX

Derive the optimal common production cycle time for the proposed model using the differential calculus (Chiu *et al.*, 2012d). By differentiating $E [TCU (T)]$ (i.e., Eq. (9)) with respect to T gives first and second derivative, one obtains:

$$\frac{\partial E[TCU(T)]}{\partial T} = \sum_{i=1}^m \left\{ \frac{(K_i + nK_{li})}{T^2} + \frac{h_i \lambda_i}{2} \left[\frac{\lambda_i E(x_i)}{P_i [1-E(x_i)]^2} + 1 - \frac{1}{n} + \frac{1}{n} \left(\frac{\lambda_i}{P_i} \frac{1}{[1-E(x_i)]} \right) \right] \right\} \quad (A-1)$$

$$\frac{\partial^2 E[TCU(T)]}{\partial T^2} = \sum_{i=1}^m \left\{ \frac{2(K_i + nK_{li})}{T^3} \right\} \quad (A-2)$$

Equation (A-2) is resulting positive for K_i , n , K_{li} and T are all positive, thus $E [TCU (T)]$ is convex for all T different from zero.

The optimal common production cycle time T^* can be obtained by setting first derivative (i.e., Eq. (A-1)) equal to zero. Let $E_0 = [1-E(x_i)]^{-1}$ and $E_1 = E(x_i) [1-E(x_i)]^{-1}$, then:

$$\frac{\partial E[TCU(T)]}{\partial T} = \sum_{i=1}^m \left\{ \frac{(K_i + nK_{li})}{T^2} + \frac{h_i \lambda_i}{2} \left[1 + \frac{\lambda_i}{P_i} E_0 E_1 - \frac{1}{n} \left(1 - \left(\frac{\lambda_i}{P_i} E_0 \right) \right) \right] \right\} = 0 \quad (A-3)$$

$$T^* = \sqrt{\frac{\sum_{i=1}^m 2(K_i + nK_{li})}{\sum_{i=1}^m h_i \lambda_i \left[1 + \frac{\lambda_i}{P_i} E_0 E_1 - \frac{1}{n} \left(1 - \left(\frac{\lambda_i}{P_i} E_0 \right) \right) \right]}} \quad (A-4)$$

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