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Research Article

Nonlinear Forced Vibration Analysis for Thin Rectangular Plate on Nonlinear Elastic Foundation

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Abstract: Nonlinear forced vibration is analyzed for thin rectangular plate with four free edges on nonlinear elastic foundation. Based on Hamilton variation principle, equations of nonlinear vibration motion for thin rectangular plate under harmonic loads on nonlinear elastic foundation are established. In the case of four free edges, viable expressions of trial functions for this specification are proposed, satisfying all boundary conditions. Then, equations are transformed to a system of nonlinear algebraic equations by using Galerkin method and are solved by using harmonic balance method. In the analysis of numerical computations, the effect on the amplitude-frequency characteristic curve due to change of the structural parameters of plate, parameters of foundation and parameters of excitation force are discussed.

Keywords: Amplitude-frequency characteristic, forced vibration, nonlinear elastic foundation, thin rectangular plate

INTRODUCTION

Rectangular plates are readily seen in many civil engineering applications such as highway concrete out layers, airport runways, building foundations and so forth. Researches about its mechanical behaviors have been undertaken by numerous experts, optimized Kantorovich method and analyzed the self-vibration characteristics of thickness-varying rectangular plates with one edge free by hypothesizing on its vibration mode function (Sonzogni *et al.*, 1990). Researched the vibration patterns of moderate thickness rectangular plates with multifarious foundation models: considered coupled effect of elastic foundation, a nonlinear constant load analysis for moderate rectangular plate was conducted (Xiao *et al.*, 2004), a nonlinear free vibration equation for moderate thickness cracked plates had been set and solved (Xiao *et al*., 2005), its self-vibrating amplitude-frequency characteristic curve was then analyzed; researched nonlinear vibration of disconnected thin plate on elastic foundation (Xiao and Fu, 2006) and conducted a analysis of disconnected thin rectangular plates' self-vibration on nonlinear elastic foundation (Xiao and Yang, 2011), meanwhile , the constant load characteristics of four free edges rectangular plate was discussed with consideration of nonlinear elastic foundation (Xiao and Zhong, 2009).

Established an approach to nonlinear vibration of orthotropic thin rectangular plate on elastic foundation by using orthogonal collocation and solved its nonlinear eigenvalue with iterative method (Bhaskar and Dumir, 1988). Also, there were researches of rectangular plates' vibration on linear elastic foundation (Qu and Liang, 1996) and circle plates' bifurcation and chaotic behavior on nonlinear elastic foundation (Qiu and Wang, 2003).

So far, much attention of examinations of plate vibration on elastic foundation has been drawn to the realm of free vibration, whereas forced vibration was discussed less frequently. Hence, this study presents a research of the nonlinear forced vibration characteristics for thin rectangular plates on nonlinear elastic foundation to extend a discussion of forced vibration characteristics of plates. Hamilton energy differentiation principles has been utilized for building a nonlinear forced vibrating equation of thin rectangular plates under harmonic load, later it is solved by exploiting Galerkin method and Harmonic balance method. Effect of variables like geometric and mechanical parameters of plates, response modus of foundation, varying stimulus force on amplitude-frequency characteristic curve of plates' forced vibration, their resonance characteristics were also analyzed. The outcomes serve as theoretical merit to the direction of construction programs.

CONTROL EQUATIONS

Assume a thin rectangular with length α, breadth b, thickness h and four free edges. Lateral distribution of the harmonic load is described as $q(x, y, t) = q_0(x, y) \cos \theta t$, in which $q_0 (x, y)$ is the amplitude of excitement,

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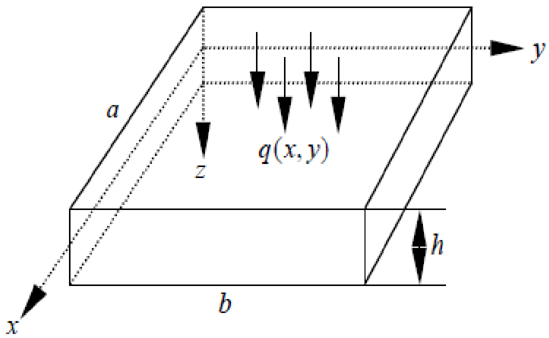

Fig. 1: Loaded thin rectangular plate

θ is the frequency of exciting force. A nonlinear Winker foundation is considered, $p = - k_1 w - k_2 w^3$ in which w, k_1 and K_2 are vertical dynamic deflection of plate, linear and nonlinear rigidity coefficients of elastic foundation, respectively. Figure 1 shows a loaded thin rectangular plate.

The total potential energy of this system is gained by $\Pi = \Pi_1 + \Pi_2 - \Pi_3$, as Hamilton energy differentiation principle interprets, when the system reaches to a steady balance, its total potential energy has the minim value, alas $\delta\Pi = 0$, or $\delta\Pi_1 + \delta\Pi_2 - \delta\Pi_3 = 0$$\Pi_1$ is the strain energy of elastic plate, its value can be obtained by:

$$\Pi_1 = \frac{1}{2}\iiint_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}) dxdydz$$

Π_2 is the study due to motor inertia, the particular value is calculated by:

$$\Pi_2 = -\iint [\rho h (u_{,tt} u + v_{,tt} v + w_{,tt} w) + \rho J (u_{,ytt} u_{,y} + v_{,xtt} v_{,x})] dxdy$$

where, P, J are the density and moment of inertia of elastic plate, respectively.

Π_3 is the study done by external force:

$$\Pi_3 = \iint_A (q + p) w dxdy + \int_C (\bar{N}_x u + \bar{N}_y v + \bar{N}_z w + \bar{M}_x v_{,x} + \bar{M}_y u_{,y}) ds$$

where, $\bar{N}_x, \bar{N}_y, \bar{N}_z, \bar{M}_x, \bar{M}_y$ are the known force and torment on the boundary of elastic plate, while $u, v, w, \alpha = \frac{\partial v}{\partial x}, \beta = \frac{\partial u}{\partial y}$, are the known displacement on the boundary.

Therefore, the motor control equations of thin rectangular plate on nonlinear elastic foundation are:

$$\begin{cases} N_{x,x} + N_{xy,y} = \rho h u_{,tt} \\ N_{xy,x} + N_{y,y} = \rho h v_{,tt} \\ Q_{x,x} + Q_{y,y} = \rho h w_{,tt} - q + p \\ -(N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy}) \\ M_{x,x} + M_{xy,y} - Q_x = \rho J v_{,xtt} \\ M_{xy,x} + M_{y,y} - Q_y = \rho J u_{,ytt} \end{cases} \quad (1)$$

where, N_x, N_y, N_{xy} membrane internal force of plate and their relationship are can be described as:

$$[N_x, N_y, N_{xy}] = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_x, \sigma_y, \sigma_{xy}] dz$$

M_x, M_y, M_{xy} are internal torques of plate whilst fitting in the following relationship:

$$[\mathrm{M}_x, \mathrm{M}_y, \mathrm{M}_{xy}] = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_x, \sigma_y, \sigma_{xy}] z\, dz;$$

Q_x, Q_y Are transversal shear which can be calculated by:

$$[Q_x, Q_y] = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_{xz}, \sigma_{yz}] dz$$

For thin rectangular plate with free edges, the boundary conditions of its surface forces are:

$$\begin{cases} x = 0, a \quad M_x = \bar{M}_x, M_{xy} = \bar{M}_{xy}, Q_x = \bar{Q}_x, \\ \qquad N_x = \bar{N}_x, N_{xy} = \bar{N}_{xy}; \\ y = 0, b \quad M_y = \bar{M}_y, M_{xy} = \bar{M}_{xy}, Q_y = \bar{Q}_y, \\ \qquad N_y = \bar{N}_y, N_{xy} = \bar{N}_{xy} \end{cases} \quad (2)$$

According to the interrelationship between thin rectangular plate's thin film stress, its bending stress and its displacement, she control equation of forced vibration of four-edge-free thin rectangular plate on nonlinear elastic foundation are:

$$\begin{cases} \phi_{,xxxx} + 2\phi_{,xxyy} + \phi_{,yyyy} = \\ Eh(w_{,xy}^2 - w_{,xx} w_{,yy}) \\ D\nabla^4 w + k_1 w + k_2 w^3 + \rho h w_{,tt} = \\ h(\phi_{,yy} w_{,xx} + \phi_{,xx} w_{,yy} - 2\phi_{,xy} w_{,xy}) + q \end{cases} \quad (3)$$

where, the flexural rigidity of plate is $D = Eh^3/12\ (1 - \mu^2)$; elastic modus and Poisson's ration are E, μ respectively; ø is the stress function.

To clarify equations, a few dimensionless parameters are introduced:

$$\begin{cases} \xi = \frac{x}{a}, \quad \eta = \frac{y}{b}, \quad W = \frac{w}{h} \\ F = \frac{\phi}{Eh^3}, \quad \lambda_1 = \frac{h}{a}, \quad \lambda_2 = \frac{h}{b} \\ K_1 = \frac{12(1-\mu^2)h}{E}k_1, K_2 = \frac{12(1-\mu^2)h^3}{E}k_2 \\ K_3 = \frac{12(1-\mu^2)h^4}{a^2b^2}, \tau = t\sqrt{\frac{E}{12\rho h^2(1-\mu^2)}} \\ Q_0 = \frac{a^3}{D}q_0, \Theta = \theta\sqrt{\frac{12\rho h^2(1-\mu^2)}{E}} \end{cases} \tag{4}$$

where, $Q = Q_0(\xi, \eta)\cos\Theta\tau$

Hence, the dimensionless control equations of forced vibration of four-edge-free thin rectangular plate are shown as:

$$\begin{cases} \lambda_1^4 F_{,\xi\xi\xi\xi} + 2\lambda_1^2\lambda_2^2 F_{,\xi\xi\eta\eta} + \lambda_2^4 F_{,\eta\eta\eta\eta} \\ +\lambda_1^2\lambda_2^2(W_{,\xi\xi}W_{,\eta\eta} - W_{,\xi\eta}^2) = 0 \\ \lambda_1^4 W_{,\xi\xi\xi\xi} + 2\lambda_1^2\lambda_2^2 W_{,\xi\xi\eta\eta} + \lambda_2^4 W_{,\eta\eta\eta\eta} \\ +K_1 W + K_2 W^3 + W_{,\tau\tau} = K_3(F_{,\eta\eta}W_{,\xi\xi} \\ +F_{,\xi\xi}W_{,\eta\eta} - 2F_{,\xi\eta}W_{,\xi\eta}) + Q \end{cases} \tag{5}$$

Its dimensionless boundary conditions are:

$$\begin{cases} \xi = 0,1 \;\; M_\xi = 0, M_{\xi\eta} = 0, Q_\xi = 0, F_{,\eta\eta} = 0, F_{,\xi\eta} = 0; \\ \eta = 0,1 \;\; M_\eta = 0, M_{\eta\xi} = 0, Q_\eta = 0, F_{,\xi\xi} = 0, F_{,\eta\xi} = 0 \end{cases} \tag{6}$$

SOLUTION TO THE EQUATION

Based on plate's boundary conditions solution to Eq. (5) is assumed in terms of separated functions:

$$\begin{cases} F(\xi,\eta,\tau) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn}(\tau)X_m(\xi)Y_n(\eta) \\ W(\xi,\eta,\tau) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} W_{pq}(\tau)\Phi_p(\xi)\Psi_q(\eta) \end{cases} \tag{7}$$

where,

$X_m = (\cosh a_m\xi - \cos a_m\xi) - b_m(\sinh a_m\xi - \sin a_m\xi)$

$Y_n = (\cosh a_n\eta - \cos a_n\eta) - b_n(\sinh a_n\eta - \sin a_n\eta)$

(m,n=1,2,3…)

$\Phi_1 = 1, \Psi_1 = 1$

$\Phi_2 = \sqrt{3}(1-2\xi), \Psi_2 = \sqrt{3}(1-2\eta)$

$\Phi_p = (\cosh a_p\xi + \cos a_p\xi) - b_p(\sinh a_p\xi - \sin a_p\xi)$

$\Psi_q = (\cosh a_q\eta + \cos a_q\eta) - b_q(\sinh a_q\eta - \sin a_q\eta)$

(p,q=3,4,5…)

The shape functions distinctively satisfy all boundary conditions

To introduce Eq. (7) to (5) the residual value is acquired, then the orthogonality relation is preset by combining Galerkin method and Vibration Eigen functions of beam, nonlinear ordinary differential equations $F_{mn}(\tau)$, $W_{pq}(\tau)$ are shown as:

$$\begin{cases} a_{1ij}^{mn}F_{mn}(\tau) + a_{2ij}^{pqst}W_{pq}(\tau)W_{st}(\tau) = 0 \\ a_{3ij}^{pq}W_{pq}(\tau) + a_{4ij}^{pq}W''_{pq}(\tau) = \\ a_{5ij}^{mnpq}F_{mn}(\tau)W_{pq}(\tau) + a_{6ij}\cos\Theta\tau \end{cases} \tag{8}$$

where, α^{mn}_{1ij} to α_{6ij} are constant coefficients whose values are mentioned elsewhere.

In general Eq. (8) are solved with harmonic balance method, thus unknown functions $F_{mn}(\tau)$, $W_{pq}(\tau)$ are expended as Cosine Fourier series for time parameter τ:

$$\begin{cases} F_{mn}(\tau) = \sum_{k=0}^{\infty} F_{mn}^{(k)}\cos k\omega\tau \\ W_{pq}(\tau) = \sum_{k=0}^{\infty} W_{pq}^{(k)}\cos k\omega\tau \end{cases} \tag{9}$$

where, coefficients $F^{(k)}_{mn}$, $W^{(k)}_{pq}$ are the k-th amplitude of harmonic waves for $F_{mn}(\tau)$ and $W_{pq}(\tau)$, respectively; ω, the dimensionless frequency of plate's forced vibration, is associated with circular frequency of forced vibration ϖ :

$$(\omega, \; \omega_0) = \sqrt{\frac{12\rho \mathrm{h}^2(1-\mu^2)}{\mathrm{E}}}(\varpi, \varpi_0)$$

where, ω_0 and ϖ_0 are dimensionless and dimensional base frequency for linear forced vibration of plate, respectively.

To combine Eq. (9) and (8) the control equation for plate's frequency of forced vibration is present as:

$$a_{3ij}^{pq}\sum_{k=0}^{\infty} M + \frac{a_{2ij}^{pqst}a_{5ij}^{mnpq}}{a_{1ij}^{mn}}\left(\sum_{k=0}^{\infty} M\right)^3 = a_{4ij}^{pq}\sum_{k=0}^{\infty} k^2\omega^2 M + a_{6ij}\cos\Theta\tau \tag{10}$$

where, $M = W^{(k)}_{pq}\cos k\omega\tau$

Steps for calculation are as followed: First off enter the geometric and mechanical parameters of plate, elastic modulus of foundation and exciting force, then using Eq. (10) to yield ω, Eq. (9) for one set values of F_{mn}, W_{pq}, $F(\xi, \eta, \tau)$ and $W(\xi, \eta, \tau)$ is calculated through Eq. (7). Finally, the amplitude-frequency curve is

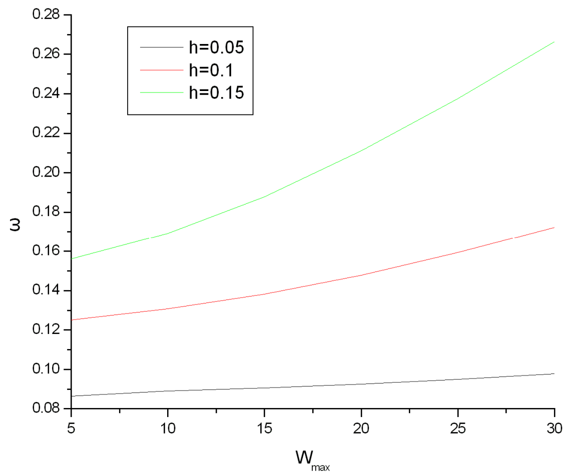

Fig. 2: Amplitude-frequency curve of various thick plates

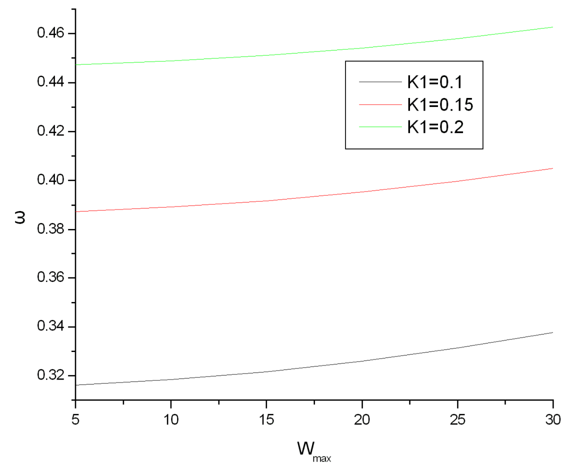

Fig. 3: Amplitude-frequency curve of different K1

illustrated for thin rectangular plate on nonlinear elastic foundation when plate's parameters and foundation's parameters vary.

EXAMPLE ANALYSIS

Without consideration of exciting force effect, this issue withdraws to issue of free vibration, whose theoretical analyses and solution were given by Xiao and Yang (2011). Those analyses verify the accurate and suitable choices of algorithm, the trial function and solution.

Assuming base soil is common cohesive soil, parameters for elastic plate are $a = 1.8$ m, $b = 1.5$ m, $h = 0.1$ m, $E = 3\times10^4$ MN/m^2, $\mu = 0.15$, $\rho = 2450$ kg/m^3; the rigid coefficients of foundation are $k_1 = \times10^2$ MN/m, $k_2 = 40$ MN/m^3; dimensionless amplitude of exciting force is $Q_0\ (\xi,\ \eta) = 0.01$, dimensionless frequency ratio is $s = \Theta/\omega = 1$

Three plates with thickness of 0.05, 0.1 and 0.15 m were selected while other parameters are identical. Figure 2 indicates the effect of thickness on Amplitude-frequency curve. When amplitude of forced vibration stay the same, the frequency of plate's forced vibration increases drastically as thickness became greater.

Figure 3 and 4 illustrate foundation parameters K1 and K2's impact on Amplitude-frequency curve of plate's forced vibration. With a stable amplitude increment of K1 and K2 lead to enhance in frequency which indicates moderate elevation of foundation's response modulus helps to increase the forced vibration frequency of plate.

Figure 5 illustrates the effect of exciting force's amplitude on Amplitude-frequency curve of plate's forced vibration. As exciting force amplitude abates from 1.5 to 0.5 while other parameter still, forced vibration frequency increases.

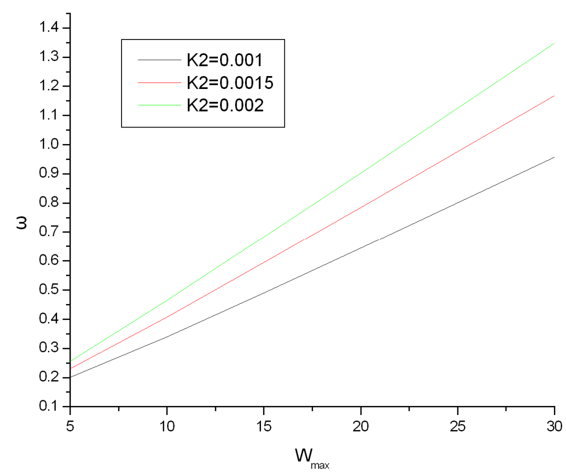

Fig. 4: Amplitude-frequency curve of different K2

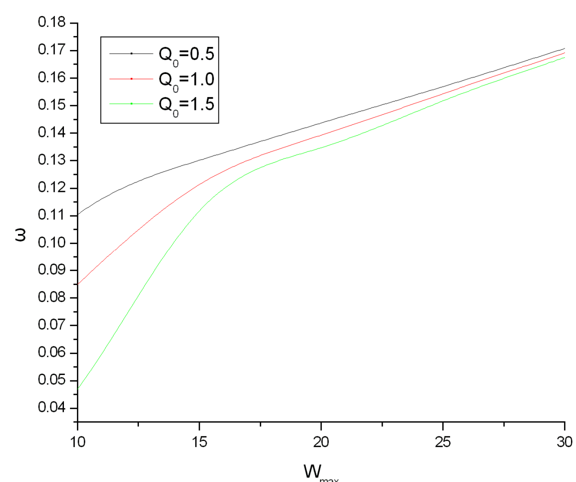

Fig. 5: Amplitude-frequency curve of different exciting force amplitude

Seen in Fig. 6 when $s = \Theta/\omega$ approaches to the value 1, namely when frequency nearly equal to

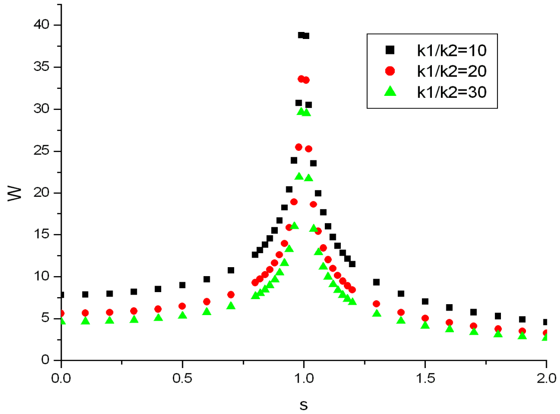

Fig. 6: Effect of exciting force frequency on amplitude-frequency curve of plate

system's fixed frequency, the amplitude of forced vibration w drastically rises, the resonance region approximately lies between s = 0.7 to 1.3. Also, as foundation's linear coefficient strengthens, nonlinear characteristic of system fades.

CONCLUSION

This study presents an analysis of harmonic-excited forced vibration of thin rectangular plate on nonlinear elastic foundation, including the consequential effects of mechanical parameters of plate, response modulus of foundation and change in exciting force on Amplitude-frequency curve.

The results show, in general, frequency of forced vibration augments along with increment of plate's amplitude. When amplitude remains stable, increments in plate's thickness and response modulus of foundation result in rise of elastic plate's forced vibration frequency; while higher amplitude of exciting force leads to lower forced vibration frequency. Especially when frequency of exciting force is 0.7 to 1.3 times of system's fixed frequency forced vibration amplitude intensively expends a clearly characteristic of resonance effect. Therefore in civil engineering application such characteristic can be exploited in structure and pavement demolition, as adjusting load frequency to system's fixed frequency.

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