

## Research Article

### Analysis of Probabilistic Harmonic Currents and Voltages of Electronic Power Converter Contributed in Distribution System

Kuo-Hua Liu

Department of Electrical Engineering, Hsiuping University of Science and Technology, 11, Gongye Rd., Dali Dist., Taichung City 41280, Taiwan

**Abstract:** Harmonic analysis is an important application to power systems and an efficient approach to evaluate the injected Total Harmonic Distortion (THD) of currents flowing into a power system due to the summation of randomly varying harmonic currents produced by AC/DC power converters is presented in this study. A probabilistic model is established to derive the magnitude of the injected harmonic currents using the expected value and variance of the harmonic current sources. This study presents a method for predicting the net harmonic current and voltage produced by a large number of electronic power converter loads at the Point of Common Coupling (PCC). Two of the more difficult aspects of applying IEEE Standard 519 are one determining an appropriate harmonic penetration Point of Common Coupling (PCC) and second establishing a harmonic model at the increasingly distributed loads test system. The estimated results obtain by simulating a distribution system show that the feasibility of the proposed method.

**Keywords:** Extension rule, parallel algorithm, probabilistic, theorem proving

## INTRODUCTION

The use of electric energy is, in developed countries around the world, a natural part of life. The trend of electrical power system is changing rapidly with the widespread use of power electronic technology in electrical power networks. Electronic power converter loads with their capacity for producing harmonic currents now constitute the most important class of nonlinear loads in the power system. Advances in semiconductor device technology have fueled a revolution in power electronics over the past decade and there is every indication that this trend will continue. Equipment includes adjustable-speed motor drives (Kaprielian and Rabbani, 1996), electronic power supplies, battery chargers (Staats *et al.*, 1997), dc motor drives (Kaprielian *et al.*, 1994) and many other rectifiers (Wang *et al.*, 1994) and inverter (Infield *et al.*, 2004) applications and so on. When electronic power converters first became commonplace, many utility engineer became quite concerned about the ability of the power system to accommodate the harmonic distortion. Because this problem of harmonic sparked the research that has eventually let too much of the knowledge about all aspects of power quality.

The phrase power quality has been widely used during the last decade and includes all aspects of events in the system that deviates from normal operation. It is however more correct to distinguish between voltage and current quality, rather than power quality.

Harmonic sources, which are electronic power converter, are generally considered to be injection sources into the linear network models. They can be represented as current injection sources.

Harmonic currents produced by nonlinear loads are injected into the supply system. These currents can interact adversely with the utility power system. The interact often gives rise to voltage and current harmonic distortion observed in many place in the system. Therefore, to limit both voltage and current harmonic distortion, IEEE Standard 519-1992 proposes to limit harmonic current injection at the Point of Common Coupling (PCC).

For simulation purposes the harmonic sources can be divided into three categories (Arrillaga *et al.*, 1985):

- Large numbers of distributed nonlinear components of small rating
- Large and continuously randomly varying nonlinear loads
- Large power converters and transmission system level power electronic devices

The study of harmonic components is conventionally based on a deterministic approach. This method has two primary defects:

- The varying characteristics of the harmonics have been neglected and the analyzed results are too conservative.

- The harmonic components in actual power systems are continuously varying with time but the selected parameters of this approach are fixed and the analyzed results can not reflect the actual variations of system characteristic.

Consequently, the deterministic approach to analyze randomly varying harmonic currents is not appropriate. To achieve a more realistic approach to analyze time-varying harmonic components is by means of probabilistic scheme. We may first consider the harmonic components of a power system as a function with certain probabilistic density distribution and then establish a random system model using probabilistic function. The characteristics of the studied system harmonics can then be mathematically expressed.

Generally speaking, it is neither capable nor practical to acquire all the actual parameters of the studied system. The main study is just to determine some representative data of the random function in order to clearly describe the characteristics of the system. The important data to describe the

characteristics of a random system are usually selected to be the location and the dispersion.

### RESEARCH STATUS SYSTEM MODEL OF HARMONIC CURRENT CREATE VOLTAGE DISTORTION

Figure 1 shows the equivalent circuit for a typical six-pulse AC-to-DC power converter with six controllable thyristors (Wang *et al.*, 1994).

Nonlinear loads appear to be sources of harmonic current in shunt with and injecting harmonic currents I<sub>line</sub> into the power system. The most common loading pattern is increasingly distributed loads shown in Fig. 2. The test system architecture is lumping loads in triangle distributed geometric configurations.

The line parameters and feeder capacity are listed in. The electronic power converter load was modeled using the model of reference. It is a significant generator of harmonic current. By decomposing the harmonic current into their real and imaginary

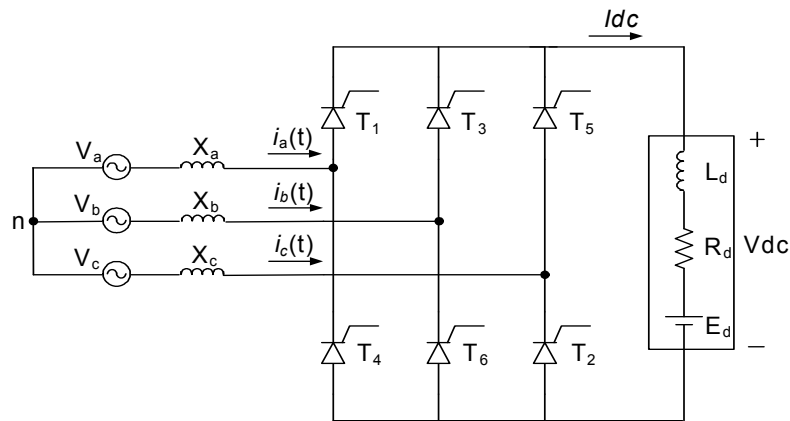


Fig. 1: Equivalent circuit of a typical six-pulse AC-to-DC converter

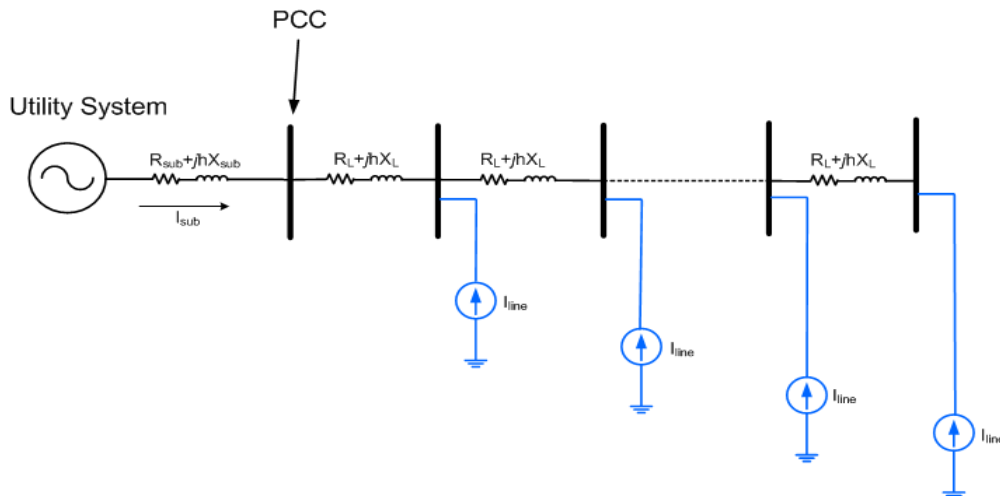


Fig. 2: The increasingly distributed loads test system

components, plotting each real and imaginary harmonic current is formulated as a function of the random variable  $\alpha$ . We define  $X_h(\alpha)$  and  $Y_h(\alpha)$  to be the values of the real and imaginary harmonic currents of order  $h$  for an individual injected harmonic current at random variable  $\alpha$ . For predicting the net harmonic current by a large number of electronic power converter loads, let the sum of X components be S where,  $S = \sum_{i=1}^N X_i$  and the sum of Y components be W where,  $W = \sum_{i=1}^N Y_i$ . It follows from the central limit theorem for sufficiently large number of phasors to be added; the Probability Density Function (PDF) of the sum approaches a normal distribution. If  $\alpha$  is assumed to be following a uniform distribution from  $\alpha_1$  to  $\alpha_2$ .

To analyze the operation of the system with nonlinear loads is to calculate the effect of each individual harmonic current as it flows through the various impedances of the distribution system. Fourier analysis tells us that the 6-pulse current drawn by the rectifier bridge (to convert AC to DC) has a fundamental frequency component plus of the odd harmonics (5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, etc.). When modeling the distribution system, we can think of each AC to DC power converter as a generator of harmonic currents. Each harmonic current injected into the power system by a non-linear load will flow through the feeder impedance, resulting in a voltage drop at that harmonic frequency. This is a simple application of Ohm's Law:

$$V_h = I_h \times Z_h \tag{1}$$

where,

- $V_h$  = h<sup>th</sup> harmonic voltage
- $I_h$  = h<sup>th</sup> harmonic current
- $Z_h$  = Feeder impedance at h<sup>th</sup> harmonic

The accumulative effect of the voltage drops at each frequency produces voltage distortion. A common term used to indicate the amount of waveform distortion is Total Harmonic Distortion, or THD. THD is expressed in percent and for power systems can be applied to both voltage and current. Voltage Total Harmonic Distortion (THD<sub>V</sub>) is defined as a root mean square of all the harmonic voltage drops and is expressed as follows:

$$THD_V = \frac{\sqrt{|V_2|^2 + |V_3|^2 + |V_4|^2 + |V_5|^2 + \dots}}{|V_1|} \times 100\% \tag{2}$$

$$= \sqrt{\frac{\sum_{n=2}^{\infty} |V_n|^2}{|V_1|^2}} \times 100\%$$

Current distortion (THDI) is a measure of the combined effect of the various harmonic currents present:

$$THD_I = \frac{\sqrt{|I_2|^2 + |I_3|^2 + |I_4|^2 + |I_5|^2 + \dots}}{|I_1|} \times 100\% \tag{3}$$

$$= \sqrt{\frac{\sum_{n=2}^{\infty} |I_n|^2}{|I_1|^2}} \times 100\%$$

### JOINTLY GAUSSIAN RANDOM VARIABLES

Two random variables of real component X and imaginary component Y are said to have a Bivariate Normal Distribution (BND) if their Joint Density Function (JDF) is of the form [10]:

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho_{xy}^2}} e^{\left(\frac{-1}{2}G(x, y; \mu_x, \mu_y, \sigma_x, \sigma_y)\right)} \tag{4}$$

where,

$$G(x, y, \mu_x, \mu_y, \sigma_x, \sigma_y) = \frac{1}{(1-\rho_{xy}^2)} \left[ \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{2\rho_{xy}(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 \right] \tag{5}$$

where,

- $\mu_x$  &  $\mu_y$  : The expected values of X and Y
- $\sigma_x$  &  $\sigma_y$  : The standard deviations of X and Y
- $\rho_{xy}$  : The correlation coefficient of X and Y

For net harmonic phasor (N), the expected value, the standard deviation for real and imaginary components and the correlation coefficient can be respectively expressed as below:

$$\bar{X} = E[X] \tag{6}$$

$$\bar{Y} = E[Y] \tag{7}$$

$$\sigma_X^2 = E\left[(X - \bar{X})^2\right] \tag{8}$$

$$\sigma_Y^2 = E\left[(Y - \bar{Y})^2\right] \tag{9}$$

$$\rho_{XY} = \frac{E\left[(X - \bar{X}) - (Y - \bar{Y})\right]}{\sigma_X\sigma_Y} \tag{10}$$

The PDF of the harmonic magnitude of the sum can be derived by transforming the rectangular component to polar coordinates [4] as below:

$$f_S(S) = \int_0^{2\pi} f_S(S \cos \theta, S \sin \theta) \quad (11)$$

where,  
 $x = s \cdot \cos \theta$   
 $y = s \cdot \sin \theta$

A useful method of summarizing the probability of the phasor magnitude of trends is to create a histogram. The cumulative frequency curve is computed to demonstrate the computation of the 95<sup>th</sup> percentile value. It is economical to consider harmonic magnitude exceeding some level. These levels can be used for comparison with the maximum levels recommended by IEEE Standard 519. The probability of the phasor magnitude K not exceeding some harmonic level K<sub>max</sub> can then be determined numerically:

$$P(K \leq K_{\max}) = cdf(K_{\max}) = \int_0^{K_{\max}} pdf(K) dK \quad (12)$$

**A computer program using Monte Carlo simulation technique:** All simulations have been performed using 40,000 trials on a personal computer.

### LOAD MODELING IN HARMONIC PENETRATION STUDIES

This study proposes the three feeders, Feeder-1, Feeder-2 and Feeder-38, under feeder length of 1-10 miles are employed and various firing-angle limits of power converters are listed in Table 1 to 4.

The utility provided 12.47 kV radial distribution feeder Voltages (V<sub>L</sub>). Voltage distortion limits for current distortion are listed in and are described in Table 5. This table applies to steady state operation of six-pulse drives and general distortion situations. This can be done provided that every no characteristic and even harmonic is less than 25% of the limits in these tables.

These generally will be met at the PCC provided the harmonic distortion limits are met. It should be noted that even if the voltage and current distortion limits are met at the PCC, they could very easily be exceeded downstream where connected equipment could be affected. Since voltage distortion is the result of harmonic currents passing through the impedance of the power system, voltage distortion will always be higher downstream where the harmonic currents are generated and where system impedance is highest. Table 6 shows the Harmonic Voltage Distortion Limits in Percent of Nominal Fundamental Frequency Voltage.

Table 1: Lower and upper firing-angle limits of 10 power converters

Converter	1	2	3	4	5	6	7	8	9	10
$\alpha_{\min}$	3°	6°	25°	10°	5°	10°	5°	15°	10°	5°
$\alpha_{\max}$	28°	85°	75°	50°	55°	65°	65°	75°	69°	80°

Table 2: Lower and upper firing-angle limits of 10 power converters

Converter	1	2	3	4	5	6	7	8	9	10
$\alpha_{\min}$	5°	5°	5°	5°	5°	5°	5°	5°	5°	5°
$\alpha_{\max}$	55°	55°	55°	55°	55°	55°	55°	55°	55°	55°

Table 3: Lower and upper firing-angle limits of 10 power converters

Converter	1	2	3	4	5	6	7	8	9	10
$\alpha_{\min}$	10°	10°	10°	10°	10°	10°	10°	10°	10°	10°
$\alpha_{\max}$	75°	75°	75°	75°	75°	75°	75°	75°	75°	75°

Table 4: Lower and upper firing-angle limits of 10 power converters

Converter	1	2	3	4	5	6	7	8	9	10
$\alpha_{\min}$	5°	20°	5°	10°	5°	10°	5°	15°	10°	5°
$\alpha_{\max}$	25°	45°	25°	30°	45°	30°	65°	75°	30°	25°

Table 5: Harmonic current distortion limits in percent of IL

$V_{\text{rms}} \leq 69 \text{ kV}$						
$I_{\text{sc}}/I_L$	$h < 11$	$11 \leq h < 17$	$17 \leq h < 23$	$23 \leq h < 35$	$35 \leq h$	TDD
<20	4.00	2.0	1.5	0.6	0.3	5.00
20<50	7.00	3.5	2.5	1.0	0.5	8.00
50<100	10.0	4.5	4.0	1.5	0.7	12.0
100<1000	12.0	5.5	5.0	2.0	1.0	15.0
>1000	15.0	7.0	6.0	2.5	1.4	20.0

Table 6: Harmonic voltage distortion limits in percent of nominal fundamental frequency voltage

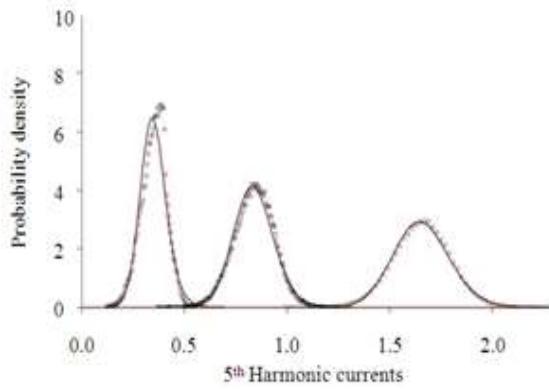
Bus voltage at PCC	Individual voltage distortion (%)	Total voltage distortion THD (%)
$V_{\text{rms}} \leq 69 \text{ kV}$	3.0	5.0
$69 \text{ kV} < V_{\text{rms}} \leq 161 \text{ kV}$	1.5	2.5
$V_{\text{rms}} > 161 \text{ kV}$	1.0	1.5

Figure 3 to 5, respectively plot the PDFs (curves) and the results of Monte Carlo simulations (circles) of the sum of the 5<sup>th</sup> harmonic currents using Feeder-1, Feeder-2 and Feeder-3.

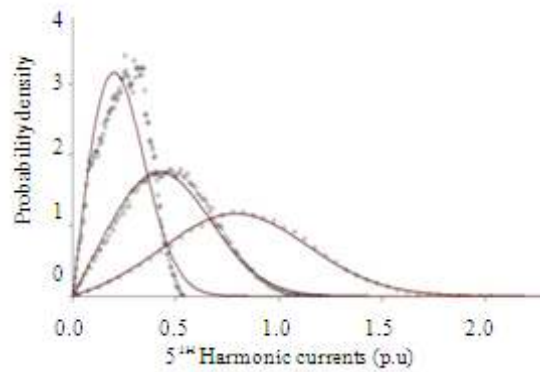
to be the given feeder capacity current  $I_L$ . Assuming that 100% of this maximum demand current is covered by Total Demand Distortion (TDD) current limits as well as individual harmonic current limits. The maximum substation current can be determined as  $I_{max} = I_L \times I_{h, limit}$ . The maximum distortion voltage can be determined as  $V_{max} = V_L \times V_{limit}$ . It should be noted that a value of the ratio  $I_{max}/I_{line}$  greater than 1 means that 100%. These ratios results indices are individual

**RESULTS**

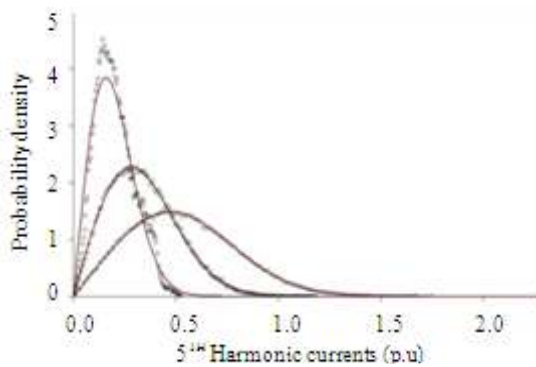
The maximum allowable substation demand current at Mahmoud and Shultz (1982), which is taken



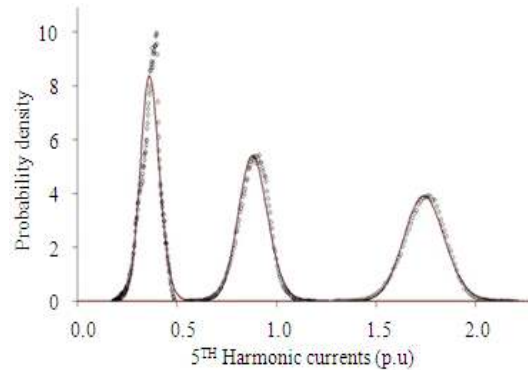
(a) Firing-angle limits using Table 1



(b) Firing-angle limits using Table 2

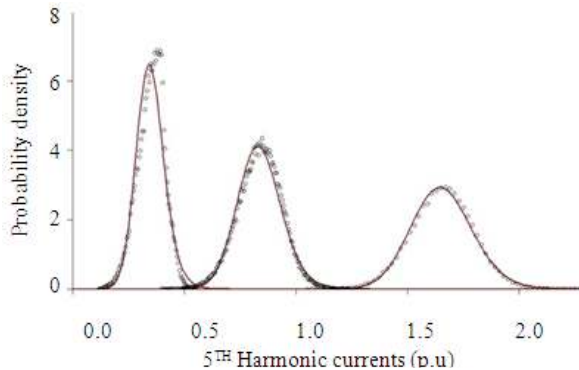


(c) Firing-angle limits using Table 3

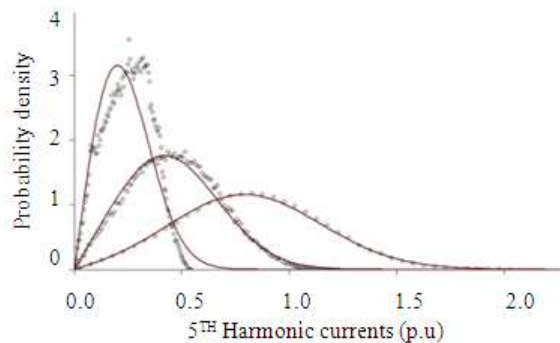


(d) Firing-angle limits using Table 4

Fig. 3: PDFs (curves) and the results of Monte Carlo simulations (circles) of the sum of the 5<sup>th</sup> harmonic currents using Feeder-1



(a) Firing-angle limits using Table 1



(b) Firing-angle limits using Table 2

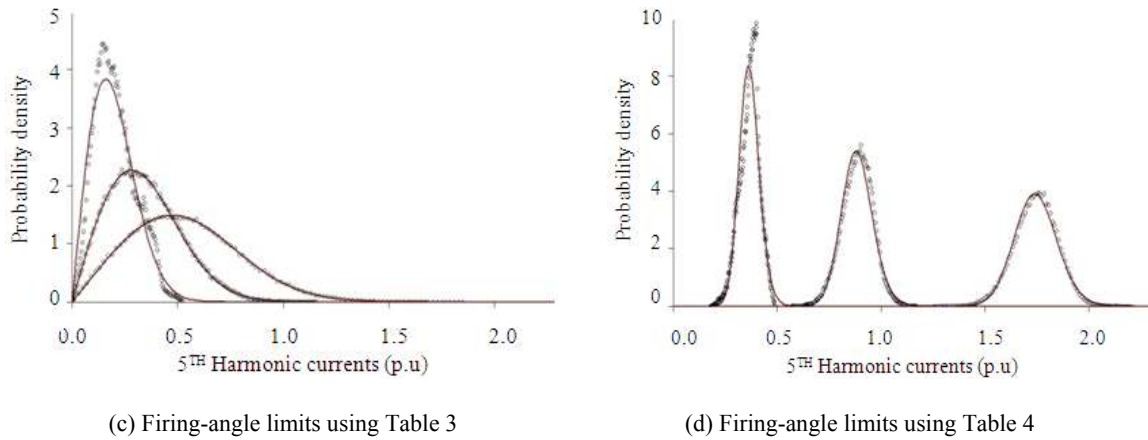


Fig. 4: PDFs (curves) and the results of Monte Carlo simulations (circles) of the sum of the 5<sup>th</sup> harmonic currents using Feeder-2

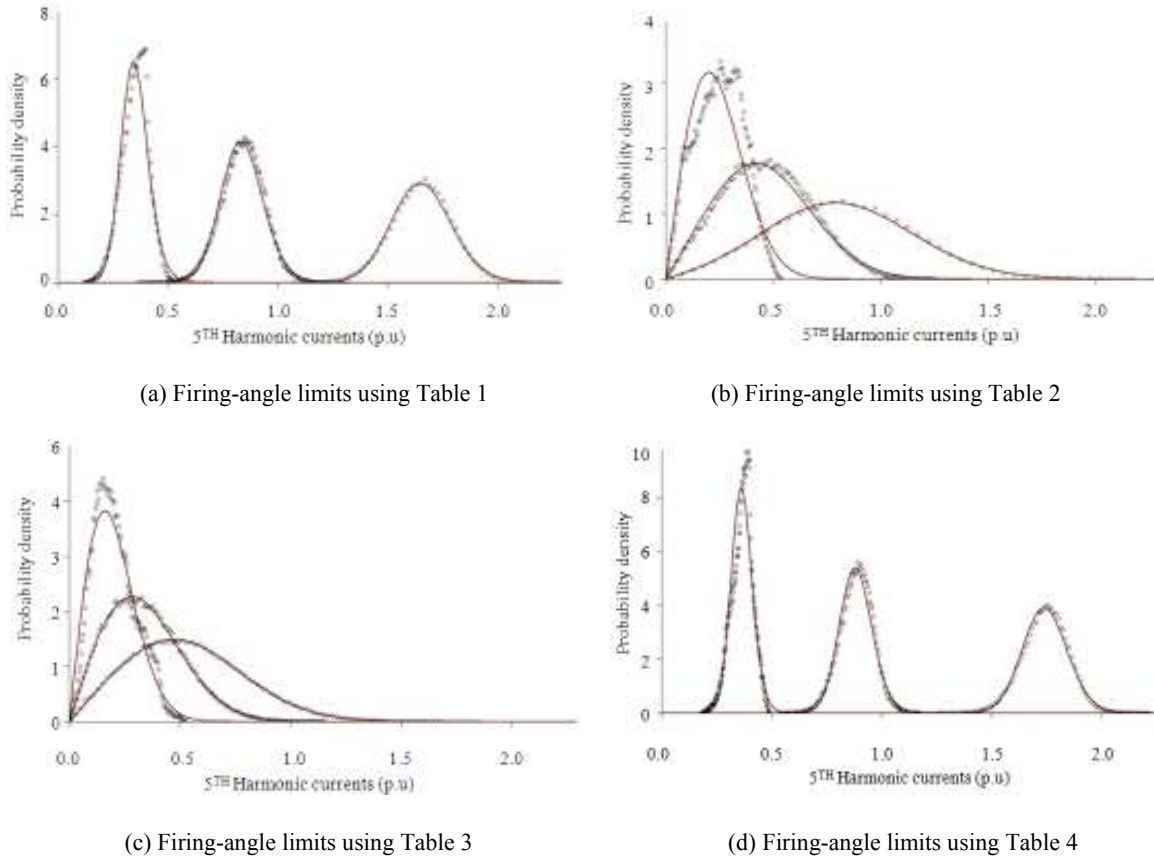


Fig. 5: PDFs (curves) and the results of Monte Carlo simulations (circles) of the sum of the 5<sup>th</sup> harmonic currents using Feeder-3.

Table 7: Allowable penetration percentage for 5<sup>th</sup> and 7<sup>th</sup> harmonics current by various firing angles of Table 1

Order	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5 <sup>th</sup>	1	100	100	100
	5	78.1258	90.3402	100
	10	51.2145	65.3692	91.7530
7 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	89.1475	100	100

Table 8: Allowable penetration percentage for 5<sup>th</sup> and 7<sup>th</sup> harmonics current by various firing angles of Table 2

Order	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	90.6548	99.8921	100
7 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	100	100	100

Table 9: Allowable penetration percentage for 5<sup>th</sup> and 7<sup>th</sup> harmonics current by various firing angles of Table 3

Order	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	97.2512	100	100
7 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	100	100	100

Table 10: Allowable penetration percentage for 5<sup>th</sup> and 7<sup>th</sup> harmonics current by various firing angles of Table 4

Order	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5 <sup>th</sup>	1	100	100	100
	5	73.8501	86.1402	99.2453
	10	39.1988	59.4892	81.3691
7 <sup>th</sup>	1	100	100	100
	5	100	100	100
	10	81.0868	99.1258	100

Table 11: Allowable penetration percentage for total harmonics current by various firing angles of Table 1

Length (miles)	Allowable penetration percentage		
	Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
1	99.6818	100	100
5	50.1252	90.5671	100
10	22.2103	85.5971	100

Table 12: Allowable penetration percentage for total harmonics current by various firing angles of Table 2

Length (miles)	Allowable penetration percentage		
	Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
1	100	100	100
5	78.5203	98.1236	100
10	35.1721	79.9945	100

Table 13: Allowable penetration percentage for total harmonics current by various firing angles of Table 3

Length (miles)	Allowable penetration percentage		
	Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
1	100	100	100
5	83.4821	100	100
10	52.7107	89.4821	100

Table 14: Allowable penetration percentage for total harmonics current by various firing angles of Table 4

Length (miles)	Allowable penetration percentage		
	Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
1	97.3541	100	100
5	47.3296	90.9820	100
10	19.1106	78.1274	99.2089

injecting harmonic currents allowable penetration percentage shows in Table 7 to 10 and total harmonic currents allowable penetration percentage shows in Table 11 to 14. Another ratio  $V_{max}/V_h$  greater than 1 means that 100% are shows in Table 15 to 18. Generally this will keep voltage distortion safely below

Table 15: Allowable penetration percentage for voltage distortion by various firing angles of Table 1

Vdist	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5%	1	100	100	100
	5	62.8347	100	100
	10	28.6793	50.9584	57.6880

Table 16: Allowable penetration percentage for voltage distortion by various firing angles of Table 2

Vdist	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5%	1	100	100	100
	5	91.1857	100	100
	10	47.5281	80.2427	86.3901

Table 17: Allowable penetration percentage for voltage distortion by various firing angles of Table 3

Vdist	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5%	1	100	100	100
	5	71.0416	100	100
	10	33.5332	59.4104	67.1388

Table 18: Allowable penetration percentage for voltage distortion by various firing angles of Table 4

Vdist	Length (miles)	Allowable penetration percentage		
		Feed-1 (%)	Feed-2 (%)	Feed-3 (%)
5%	1	100	100	100
	5	56.9387	98.4472	100
	10	25.5085	43.7650	47.8108

the maximum of 5% as recommended for sensitive loads in IEEE Standard 519-1992.

## CONCLUSION

In this study, an efficient approach to evaluate the injected harmonic currents flowing into a power system due to the summation of randomly varying harmonic currents generated by AC-to-DC power converters has been presented. The proposed method is based on a probabilistic model to derive the magnitude of the injected harmonic currents using the expected value and the standard deviation of the harmonic current sources. The specific conclusions of this study are summarized as follows:

- The harmonic components are essentially randomly varying quantities and the probabilistic model for studying harmonic components is more appropriate for actual power systems.
- The standard recommends maximum acceptable limits for both voltage and current harmonics to prevent problems that can result from heavy non-linear loading. The limits for harmonic currents are designed to minimize the amount of voltage distortion these currents would produce in the power system.

- It is found that the condition value of optimum by different firing angle and feeder capacity to take account of from a PCC with given IEEE Standard 519-1992 characteristics.

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