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Research Article

Lead Time Reduction in Budget and Storage Space Restricted Lot Size Reorder Point Inventory Models with Controllable Negative Exponential Backorder Rate

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Abstract: This study presents the mixed backorder and lost sales inventory models involving four variables; order quantity, lead time, safety factor (a discrete variable) and backorder rate. A controllable negative exponential backorder rate is considered in the proposed model. In the real market, as unsatisfied demands occur, the longer length of lead time is, the smaller proportion of backorder rate would be. Considering this reason, backorder rate is dependent on the length of lead time through the amount of shortages. The negative exponential lead time crashing cost is considered in this study. Today, the cost of land acquisition is high in most of the countries and one of the main concerns of inventory managers is to ensure that the maximum permissible storage space is enough when an order arrives. Hence, a random storage space constraint is considered, since, the inventory level is random when an order arrives. So, in this case, a chance-constrained programming technique is used to make it crisp. Moreover, another significant concern of inventory managers is how to control the maximum investment in the inventory. This study assumes the purchasing cost is paid at the time of order placing. Considering this assumption, a budget constraint is also added to the model in order to managing the maximum inventory investment. The lead time demand, first, follows a normal distribution and then, relaxes the distribution function assumption by only assuming the mean and variance of lead time demand are known and applies the minimax distribution free procedure to solve the problem. Furthermore, a numerical example is also given to illustrate the models and solution procedures.

Keywords: Chance-constrained programming technique, inventory constraints, inventory system, lead time, partial backlogging, stochastic

INTRODUCTION

The number of advantages and benefits has been associated in the efforts of control of the lead time (which is a goal of JIT inventory management philosophies that emphasizes high quality and keeps low inventory level and lead time to a practical minimum). Lead time management is a significant issue in production and operation management. In many practical situations lead time can be reduced using an added crashing cost. In other words, lead time is controllable. The crashing of lead time mainly consists of the following components: order preparation, order transit, supplier lead time and delivery time (Tersine, 1994). Decreasing lead time leads to the lower safety stock, reduction of the loss sales caused by stock out, improving the customer service level and increasing the competitive ability in business. Liao and Shyu (1991), Ben-Daya and Rauf (1994), Ouyang *et al*. (1996), Ouyang and Wu (1998) and Park (2007) considered lead time as a variable and controlled it by paying extra crashing cost and assume that the lead time can be decomposed into *n* mutually independent components which each component has a fixed crashing cost. Also, Wu *et al*. (2007) have studied on the negative exponential crashing cost and considered order quantity and lead time as variables. Besides, Gallego and Moon (1993) assume unfavorable lead time demand distribution and solved both the continuous and periodic review models with a mixture of backorder and lost sale using Minimax distribution free method.

A lot of inventory models have been considered under the assumption that shortages are allowed. One important group of these models such as Mirzazadeh *et al*. (2009) and Hariga (2010) considers that, when there is shortage, all customers wait until the arrival of the next replenishment (full backlogging case). Another situation would be to admit that all customers who are served leave the system (lost sale case). However, in many practical situations, there are customers (whose needs are not critical at that time) who are willing to wait for the next replenishment to satisfy their demands, while others do not want to or cannot wait and leave the system. These situations are modeled by considering partial backlogging in the information of mathematical models. In this case, in many real situations, during a shortage period, the longer waiting time is, the smaller the backlogging rate would be. For

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instance, for fashionable commodities and high-tech products with a product life cycle, the willingness for a customer to wait for backlogging is diminishing with the length of the waiting time. In this way, β defines the fraction of demand which is backordered during stock out period, as the function of τ, the time remaining until the next replenishment. Montgomery *et al*. (1973) proposed linear function for $\beta(\tau)$. Abad (1996) introduced exponential $\beta(\tau)$ originally, but Papachristors and Skouri (2000) referred it as exponential. Abad (1996), San Jose *et al*. (2005) and Silica *et al.* (2007) proposed rational $\beta(\tau)$. Silica *et al.* (2009) proposed mixed exponential $\beta(\tau)$ in their study. Also, Teng *et al*. (2007) have used negative exponential $\beta(\tau)$. Some of the researches consider β as a function of expected shortage quantity at the end of cycle. These studies are based upon the assumption of the larger amount of the expected shortage at the end of cycle, the smaller amount of customer can wait and hence the smaller backorder rate would be. First, Ouyang and Chaung (2001) introduced this condition in their model and some other authors generalized this assumption in their models for backorder rate (Lee, 2005; Lee *et al*., 2006, 2007; Gholami-Qadikolaei *et al*., 2012).

There are some continuous review inventory models with shortages including restrictions on inventory investment, storage space or reorder work load. Brown and Gerson (1967) proposed some models for stochastic inventory system with the limit total inventory investment. Shrady and Choe (1971) developed a model with the total time weighted shortages with the inventory investment and reorder work load constraint. Gardner (1983) prepared models for minimizing expected approximate backordered sales with the restrictions on aggregate investment and replenishment work load. Shroeder (1974) presented a constrained model by total expected annual ordering for minimizing the expected number of unit's backordered per year as the objective function. Hariga (2010) presented a stochastic full backlogging inventory system with space restriction in which the order quantity and reorder point are decision variables. Xu and Leung (2009) propose an analytical model in a two-party vendor managed system where the retailer restricts the maximum space allocated to the vendor. Bera *et al*. (2009) presented a minimax distribution free procedure for stochastic lead time and demand inventory model under budget restriction when the purchasing cost payment is due at the time of order receiving. Moon *et al*. (2012) proposed three extended models with variable capacity. First, they presented an EOQ model with random yields. Second, they developed a multi-item EOQ model with storage space and investment constraint and solved model with Lagrange multiplier method. Third, they applied a distribution free approach to the (Q, r) with variable capacity.

Ouyang and Chaung (2001) observed that the many products of well-known brand and modish goods like certain brand gum shoes and clothes may lead to a state in which clients prefer their demands to be backordered, whereas shortages happened. Doubtlessly, if the time remaining until the next replenishment exceeds, some clients avoid the backorder case. This phenomenon reveals that as shortage occurs, in the stochastic demand and deterministic lead time situations, the longer the length of lead time is the larger amount of shortages is, the smaller proportion of customers can wait and hence the smaller backorder rate would be. Therefore, the vendor have to control an appropriate length of lead time to determines a target value of backorder rate to minimizes the inventory relevant cost and increase the competitive edge in business. Consequently, they assumed that the backorder rate is dependent to the length of lead time through the amount of shortages and applied rational expected shortages level-dependent backorder rate in their model. They considered order quantity, lead time and backorder rate as the decision variables.

To our knowledge, the problem of determining optimal continuous review policies for budget and storage space constrained stochastic inventory system with deterministic variable lead time has not been explored previously. Moreover, we calculate optimal safety factor in our study whereas the previous researchers such as Ouyang and Chaung (2001), Lee (2005) and Lee *et al*. (2006, 2007) don't consider safety factor as a variable when they consider backorder rate as a function of expected shortages quantity at the end of cycle in their studies.

This study focuses on a single-item inventory system with a mixture of backorder and lost sales under budget and storage space constraints in which the order quantity, lead time, safety factor and backorder rate are decision variables. Objective is to minimize Expected Annual Cost (EAC). This study assumes the purchasing cost is paid at the time of order placing. For this reason, maximum inventory investment will occur at the time an order is placed. Considering this assumption, a budget constraint is established. Storage space constraint is random since the inventory level when an order arrives is a random variable. Hence a chance-constrained programming technique is utilized to make it crisp. This study considers negative exponential backorder rate and controllable lead time and suggested negative lead time crashing cost. This study, first assumes that the lead time demand follows a normal distribution and then relaxes the assumption about the form of the distribution function of the lead time demand and apply the mini-max distribution free procedure in order to solve the problem. A numerical example is proposed to illustrate the models and the solution procedures.

ASSUMPTIONS

The developed model is based on these assumptions:

- Shortages are allowed and partially backlogged
- Planning horizon is infinite
- Demand rate, D, is a random variable with mean $E(D)$ and standard deviation σ_D
- Inventory is continuously reviewed. The replenishments are made whenever the inventory level falls to the reorder point r
- The cost equations are approximations because inventory levels and demands are treated as continuous instead of discrete quantities
- The reorder level is larger than the mean of the lead time demand
- There are no orders outstanding at the time the reorder point is reached
- The time the system is out of stock during a cycle is small compared to the cycle length
- The purchasing cost is paid at the time of order is placed
- The reorder point r is the expected demand during lead time plus Safety Stock (SS) and $SS = k \times$ (standard deviation of lead time demand) i.e., $r = E(x) + k\sigma_x$ where k is safety factor satisfying $P(x > r) = P(z > k) = q$, z represents the standard normal random variable and q represents the allowable stock out probability during lead time
- Lead time is constant and the mean and variance of demand during lead time x is:

$$E(x) = L.E(D) and Var(x) = {\sigma_x}^2 = L.{\sigma_D}^2 \quad (1)$$

- The total crashing cost is related to the lead time by a function of:

$$C(L) = \epsilon e^{-\omega L} \quad (2)$$

here, $\epsilon \geq 0, \omega \geq 0$ are crashing cost parameters.

MODEL FORMULATION

The inventory manager places an order of amount Q when the inventory of an item reaches to the reorder level. The expected demand during shortages at the end of cycle is:

$$E(x-r)^+ = \int_r^\infty (x-r)f(x)dx \quad (3)$$

Considering partial backlogging policy, the expected number of backorder at the end of cycle is $\beta E(x-r)^+$ and the expected number of lost sale at the end of cycle is $(1-\beta)E(x-r)^+$. The expected net inventory level just before the order arrives is $r - E(x)$ $+(1-\beta)E(x-r)^+$ and the expected net inventory level at the beginning of the cycle is $Q + r - E(x)$ $+(1-\beta)E(x-r)^+$ and expected total inventory per cycle is calculated as follows:

$$\frac{Q}{D}\left[\frac{Q}{2} + r - E(x) + (1-\beta)E(x-r)^+\right] \quad (4)$$

Thus, the mathematical model of expected cost per cycle can be expressed by:

$$[A + C(L)] + h\frac{Q}{D}\left[\frac{Q}{2} + r - E(x) + (1-\beta)E(x-r)^+\right] + [\pi + \pi_\circ(1-\beta)E(x-r)^+] \quad (5)$$

Therefore, the Expected Annual Cost (EAC) is simply calculated by multiplying (5) in the expected number of cycle and model is transformed as follows:

$EAC(Q, L, \beta) =$ Ordering cost+lead time crashing cost+holding cost+stock out cost

$$EAC(Q, L, \beta) = \frac{D}{Q}[A + C(L)] + \left[\frac{Q}{2} + r - E(x) + (1-\beta)E(x-r)^+\right] \times h + \frac{D}{Q}[\pi + \pi_\circ(1-\beta)E(x-r)^+] \quad (6)$$

Perfect demand information: When the lead time demand X follows a normal probability density function (p.d.f) $f_X(x)$ with the mean of $E(x) = E(D)L$ and the standard deviation of $\sigma_x = \sigma_D\sqrt{L}$ and given that the reorder point $r = E(x) + k\sigma_x$, the expected shortages quantity at the end of cycle $E(x-r)^+$ can be expressed as follow:

$$E(x-r)^+ = \int_r^\infty (x-r)f(x)dx \quad , \ k = \frac{r - E(x)}{\sigma_x}$$

$$\rightarrow E(x-r)^+ = \sigma_x \int_k^\infty (z-k)f(z)dz$$

$$\rightarrow E(x-r)^+ = \sigma_x\left[\int_k^\infty zf(z)dz - k\int_k^\infty f(z)dz\right], \sigma_x = \sigma_D\sqrt{L}$$

$$\rightarrow E(x-r)^+ = \sigma_D\sqrt{L}\left[\int_k^\infty zf(z)dz - k(1-\varphi(k))\right]$$

$$\rightarrow E(x-r)^+ = \sigma_D\sqrt{L}\left[\int_k^\infty zf(z)dz - k\overline{\varphi(k)}\right],$$
$$\left[\int_k^\infty zf(z)dz - k\overline{\varphi(k)}\right] = U(k)$$

$$\rightarrow E(x-r)^+ = \sigma_D\sqrt{L}U(k) \geq 0 \quad (7)$$

Therefore, the expected annual cost (6) is transformed as follows:

$$EAC(Q,L,\beta) = \frac{D}{Q}[A + C(L)] + h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + (1-\beta)\sigma_D\sqrt{L}U(k)\right] + \frac{D}{Q}\left[[\pi + \pi_\circ(1-\beta)]\sigma_D\sqrt{L}U(k)\right] \quad (8)$$

We consider backorder rate as a variable which is dependent on the length of lead time through the amount of shortages. It means that when shortages happen, the larger amount of shortages through the deterministic lead time is, the smaller ratio of client can wait and therefore, the smaller backorder would be. Thus, backorder will be function in terms of the expected shortage quantity which can be expressed as follows:

$$\beta = \alpha e^{-\nu E(x-r)^+} \quad (9)$$

where, $(0 \leq \alpha \leq 1, \nu \geq 0)$ are backorder parameters.

By using negative exponential backorder rate and crashing cost, our model is transformed as follow:

$$EAC(Q,L) = \frac{D}{Q}(A + \epsilon e^{-\omega L})$$
$$+h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right]$$
$$+\frac{D}{Q}\left[\left[\pi + \pi_\circ\left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\right]\sigma_D\sqrt{L}U(k)\right] \quad (10)$$

In this study, we consider storage space constraint which is dependent on the maximum inventory size. This constraint ensures that even the inventory position reaches to the maximum level, the maximum available space is still enough for it. Therefore, this constraint is random. Thus, the form of storage space is as follow:

$$P\{f(Q + r - x) \leq F\} \geq \gamma$$

$$\rightarrow P\left\{x \geq Q + r - \frac{F}{f}\right\} \geq \gamma$$

$$\rightarrow P\left\{z \leq \frac{Q + r - \frac{F}{f} - \left(E(x) - (1-\beta)E(x-r)^+\right)}{\sigma_D\sqrt{L}}\right\} \leq 1 - \gamma$$

$$\rightarrow Q + r - \frac{F}{f} - \left(E(x) - (1-\beta)E(x-r)^+\right)$$
$$-\sigma_D\sqrt{L}z_{1-\gamma} \leq 0$$

$$\rightarrow Q + r - \frac{F}{f} -$$
$$\left(E(x) - \left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right)$$
$$-\sigma_D\sqrt{L}z_{1-\gamma} \leq 0$$

or

$$f\left(Q + k\sigma_D\sqrt{L} + \left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right)$$
$$-F - f\sigma_D\sqrt{L}z_{1-\gamma} \leq 0 \quad (11)$$

Also, in this study, a constraint on the maximum inventory investment has been considered. Warehousing inventory causes to lose the opportunity of investments in the other places and system managers would like to control it by considering this limitation on the inventory system. In this study, we assume that the purchasing costs are paid at time of order placing. Considering this assumption, the maximum inventory investment will occur at time of order placing. With this assumption, we establish a limitation on maximum inventory investment. Thus, form of budget constraint is as follow:

$$c(Q + r) \leq B$$

$$\rightarrow c(Q + E(x) + k\sigma_x) \leq B \quad (12)$$

Therefore, our model is reduced to:

$$EAC^N(Q,L) = \frac{D}{Q}(A + \epsilon e^{-\omega L}) + h$$
$$\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right]$$
$$+\frac{D}{Q}\left[\left[\pi + \pi_\circ\left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\right]\sigma_D\sqrt{L}U(k)\right]$$

Subject to:

$$f\left(Q + k\sigma_D\sqrt{L} + \left(1 - \alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right)$$
$$-F - f\sigma_D\sqrt{L}z_{1-\gamma} \leq 0$$

$$c\left(Q + E(D)L + k\sigma_D\sqrt{L}\right) - B \leq 0$$

$$Q \geq 0, L \geq 0 \quad (13)$$

We can solve this model with Lagrange multiplier method. Therefore, the Lagrange function will be in this form:

$$EAC(Q,L,\lambda_1,\lambda_2) = \frac{D}{Q}(A+\epsilon e^{-\omega L}) + h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right] + \frac{D}{Q}\left[\left[\pi + \pi_\circ\left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\right]\sigma_D\sqrt{L}U(k)\right] + \lambda_1\left[f\left(Q + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right) - F - f\sigma_D\sqrt{L}z_{1-\gamma}\right] + \lambda_2\left[c\left(Q + E(D)L + k\sigma_D\sqrt{L}\right)\right] \quad (14)$$

To minimize the above unconstrained function, the Kuhn-Tucker conditions for the minimization of a function subject to two inequality constraints are applied:

$$\frac{\partial EAC(Q,L,\lambda_1,\lambda_2)}{\partial Q} = 0 \quad (15)$$

$$\frac{\partial EAC(Q,L,\lambda_1,\lambda_2)}{\partial L} = 0 \quad (16)$$

$$\lambda_1\left[f\left(Q + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\sigma_D\sqrt{L}U(k)\right) - F - f\sigma_D\sqrt{L}z_{1-\gamma}\right] = 0 \quad (17)$$

$$\lambda_2\left[c\left(Q + E(D)L + k\sigma_D\sqrt{L}\right)\right] = 0 \quad (18)$$

The partial derivatives are:

$$-\frac{D}{Q^2}(A+\epsilon e^{-\omega L}) + \frac{h}{2} - \frac{D}{Q^2}\left\{\left[\pi + \pi_\circ\left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\right]\sigma_D\sqrt{L}U(k)\right\} + \lambda_1 f + \lambda_2 c = 0 \quad (19)$$

Therefore, Q is obtained as follow:

$$Q^N = \left[\frac{D\left\{(A+\epsilon e^{-\omega L}) + \left\{\begin{matrix}\left[\pi+\pi_\circ\left(1-\alpha e^{-\nu\sigma_D\sqrt{L}U(k)}\right)\right] \\ \sigma_D\sqrt{L}U(k)\end{matrix}\right\}\right\}}{\frac{h}{2}+\lambda_1 f+\lambda_2 c}\right]^{\frac{1}{2}} \quad (20)$$

$$-\frac{D}{Q}\epsilon\omega e^{-\omega L} + \left[\frac{\sigma_D U(k)}{2\sqrt{L}}\left(1-\alpha e^{-\tau(L)}\right) + \frac{\alpha\nu\sigma_D{}^2[U(k)]^2}{2}e^{-\tau(L)}\right]\left(h + \frac{D}{Q}\pi_\circ + \lambda_1 f\right) + \frac{\sigma_D}{2\sqrt{L}}\left(hk + \frac{D}{Q}U(k)\pi + \lambda_1 kf + \lambda_2 kc + z_{1-\gamma}\right) + \lambda_2 cE(D) = 0 \quad (21)$$

Where $\tau(L) = \nu\sigma_D\sqrt{L}U(k)$ and optimal β^N is:

$$\beta^N = \alpha e^{-\nu\sigma_D\sqrt{L}U(k)} \quad (22)$$

The solution procedure has been stated with these is as follows:

Step 1: Input the values A, B, F, k, h, c, f, α, v, ϵ, $\omega, \pi_\circ, \pi$ and γ.
Step 2: We obtained from partial derivative Eq. (15), Q(19). Put Q in Eq. (16), (17) and (18).
Step 3: Obtain λ_1, λ_2 and L by solving Eq. (16), (17) and (18), simultaneously.
Step 4: Put λ_1, λ_2 and L in Eq. (19) and find Q.
Step 5: Put $L, Q, \lambda_1, \lambda_2$ in Eq. (14) and find EAC.
Step 6: Obtain EAC^k in terms of different k and $U(k)$ and stop when:

$$EAC^k < Min\{EAC^{k-\xi}, EAC^{k+\xi}\}$$

$$\left|EAC^k - EAC^{k-\xi}\right| \le \varepsilon$$

$$\left|EAC^k - EAC^{k+\xi}\right| \le \varepsilon$$

Partial demand information: In many practical situations, the probability distributional information of lead time demand is often quite limited. Therefore, we relax the assumption about the normal distribution demand by only assuming that the lead time demand X has given finite first two moment (and hence, mean and variance are also given); i.e., the p.d.f. f_X of X belongs to the class $\mathcal{F}$ of p.d.f.'s with mean $E(x) = E(D)L$ and variance $Var(x) = \sigma_x{}^2 = L.\sigma_D{}^2$. Now we want to use the minimax distribution free procedure to solve this problem. For this purpose, we need the following proposition which was asserted by Gallego and Moon (1993):

Proposition 1: For any $f_X \in \mathcal{F}$:

$$E(D-Q)^+ \le \frac{(\sigma^2+(Q-\mu)^2)^{1/2}-(Q-\mu)}{2} \quad (23)$$

where,
Q = Overcapacity
D = Random variable with mean μ and standard deviation σ

$$E(x-R)^+ \le \frac{1}{2}\left[\sqrt{Var(x) + (R-E(x))^2} - (R-E(x))\right] = \frac{1}{2}\left(\sqrt{1+k^2}-k\right)\sigma_D\sqrt{L} \quad (24)$$

Then, from the definition of β and inequality above (22) we have:

$$\beta = \alpha e^{-\nu E(x-r)^+} \rightarrow \beta \ge \alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)} \quad (25)$$

With the definition of β and negative exponential crashing cost expected annual cost per unit time is changed as follow:

$$EAC(Q,L) \le \frac{D}{Q}(A+\epsilon e^{-\omega L}) + h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right] + \frac{D}{Q}\left[\pi + \pi_\circ\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\right]\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right) \quad (26)$$

With using chance-constrained programming technique which is explained in proposition random storage space constraint is transformed to crisp constraint which is given below:

Preposition 2 (chance-constrained): The chance-constrained programming technique can be used to solve problems involving constraints with the finite probability of being violated. This technique originally developed by Charnes and Cooper (1959). Considering γas the probability of non-violation of the constraint, then the constraint can be written as:

$$P\{f(Q+r-x) \le F\} \ge \gamma$$

$$\rightarrow \gamma \le P\{fx + F \ge f(Q+r)\}$$

With using Markov inequality:

$$P(x \ge a) \le \frac{E(x)}{a}$$

$$\rightarrow \gamma \le P\{fx + F \ge f(Q+r)\} \le \frac{E(fx+F)}{f(Q+r)} = \frac{f\left(E(x)-(1-\beta)E(x-r)^+\right)+F}{f(Q+r)}$$

$$\rightarrow \gamma \le \frac{f\left(E(x)-(1-\beta)E(x-r)^+\right)+F}{f(Q+r)}$$

$$\rightarrow \gamma f(Q+r) - fE(x) + (1-\beta)E(x-r)^+ - F \le 0$$

$$\rightarrow \gamma f(Q+E(x)+k\sigma_x) - fE(x) + f\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right) - F \le 0 \quad (27)$$

Budget constraint is as follow:

$$c(Q+r) \le B$$

$$\rightarrow c(Q+E(x)+k\sigma_x) \le B \quad (28)$$

With using minimax distribution free procedure, our model reduced to:

$$Min\left\{Max\, EAC(Q,L) = \overline{EAC(Q,L)}\right\}$$

$$\overline{EAC(Q,L)} = \frac{D}{Q}(A+\epsilon e^{-\omega L}) + h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right] + \frac{D}{Q}\left\{\left[\pi + \pi_\circ\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\right]\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right\}$$

Subject to:

$$\gamma f\left(Q+E(D)L+k\sigma_D\sqrt{L}\right) - fE(D)L + f\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right) - F \le 0$$
$$c\left(Q+E(D)L+k\sigma_D\sqrt{L}\right) - B \le 0$$
$$Q \ge 0, L \ge 0 \quad (29)$$

We can solve this model with Lagrange multiplier method. Therefore, the Lagrange function will be:

$$EAC(Q,L,\lambda_1,\lambda_2) = \frac{D}{Q}(A+\epsilon e^{-\omega L}) + h\left[\frac{Q}{2} + k\sigma_D\sqrt{L} + \left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right] + \frac{D}{Q}\left\{\left[\pi + \pi_\circ\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\right]\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right\}$$
$$+\lambda_1\left[\gamma f\left(Q+E(D)L+k\sigma_D\sqrt{L}\right) - fE(D)L + f\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right) - F\right]$$
$$+\lambda_2\left[c\left(Q+E(D)L+k\sigma_D\sqrt{L}\right) - B\right] \quad (30)$$

The Kuhn-Tucker conditions will be used to minimize the above unconstrained function subject to two inequality constraints:

$$\frac{\partial EAC(Q,L,\lambda_1,\lambda_2)}{\partial Q} = 0 \quad (31)$$

$$\frac{\partial EAC(Q,L,\lambda_1,\lambda_2)}{\partial L} = 0 \quad (32)$$

$$\lambda_1\left[\gamma f\left(Q + E(D)L + k\sigma_D\sqrt{L}\right) - fE(D)L + f\left(1 - \alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\left(\sqrt{1+k^2} - k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right) - F\right] = 0 \quad (33)$$

$$\lambda_2\left[c\left(Q + E(D)L + k\sigma_D\sqrt{L}\right) - B\right] = 0 \quad (34)$$

Partial derivatives are:

$$-\frac{D}{Q^2}\left\{\left[\pi + \pi_\circ\left(1 - \alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\right]\left(\sqrt{1+k^2} - k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right\} - \frac{D}{Q^2}\left(A + \epsilon e^{-\omega L}\right) + \frac{h}{2} + \lambda_1\gamma + \lambda_2 c = 0 \quad (35)$$

From the above partial derivative, (30), Q is obtained as follow:

$$Q^F = \left[\frac{D\left\{\left(A+\epsilon e^{-\omega L}\right)+\left\{\left[\pi+\pi_\circ\left(1-\alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)}\right)\right]\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)\right\}\right\}}{\frac{h}{2}+\lambda_1\gamma f+\lambda_2 c}\right]^{\frac{1}{2}} \quad (36)$$

$$-\frac{D}{Q}\epsilon\omega e^{-\omega L} + \left[\frac{\sigma_D\left(\sqrt{1+k^2}-k\right)}{2\sqrt{L}}\left(1 - \alpha e^{-\vartheta(L)}\right) + \frac{\alpha\nu{\sigma_D}^2\left[\left(\sqrt{1+k^2}-k\right)\right]^2}{2}e^{-\vartheta(L)}\right]\left(h + \frac{D}{Q}\pi_\circ + \lambda_1 f\right) +$$
$$\frac{\sigma_D}{2\sqrt{L}}\left(hk + \frac{D}{Q}\left(\sqrt{1+k^2} - k\right)\pi + \lambda_1 kf + \lambda_2 kc + z_{1-\gamma}\right) + \lambda_2 cE(D) = 0 \quad (37)$$

where, $\vartheta(L) = \nu\left(\sqrt{1+k^2} - k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)$ and optimalβ^F is obtained as follow:

$$\beta^F = \alpha e^{-\nu\left(\sqrt{1+k^2}-k\right)\left(\frac{\sigma_D\sqrt{L}}{2}\right)} \quad (38)$$

The solution procedure is as follow:

Step 1: Input the values of $D, A, B, F, k, h, c, f, \alpha, \nu, \epsilon, \omega, \pi_\circ, \pi$ and γ
Step 2: Obtain Q(33) from the partial derivative Eq. (29). Put Q in Eq. (30), (31) and (32)

Table 1: Optimal EAC in terms of different safety factor

k^F	$(Q^F, L^F(weeks), \lambda_1, \lambda_2)$	EAC^F	k^N	$(Q^N, L^N(weeks), \lambda_1, \lambda_2)$	EAC^N
1.50	(89.47, 1.84, 0.093, 0.00)	3069.29	1.00	(74.88, 2.59, 0.119, 0.00)	2941.64
2.00	(87.20, 2.11, 0.090, 0.00)	3009.78	1.20	(73.13, 2.96, 0.115, 0.00)	2851.49
2.20	(86.27, 2.18, 0.090, 0.00)	3000.07	1.40	(71.57, 3.19, 0.113, 0.00)	2801.64
2.40	(85.35, 2.22, 0.090, 0.00)	2996.30	1.60	(70.25, 3.30, 0.114, 0.00)	2783.47
2.44	(85.16, 2.23, 0.091, 0.00)	2996.16	1.64	(70.01, 3.31, 0.114, 0.00)	2782.77
2.45	(85.12, 2.23, 0.091, 0.00)	2996.15	1.65	(70.01, 3.32, 0.114, 0.00)	2782.76
2.46	(85.23, 2.23, 0.091, 0.00)	2996.16	1.66	(69.09, 3.32, 0.114, 0.00)	2782.77
2.50	(84.89, 2.23, 0.091, 0.00)	2996.30	1.70	(69.67, 3.32, 0.115, 0.00)	2783.30
2.70	(83.98, 2.25, 0.092, 0.00)	2999.55	1.90	(68.66, 3.30, 0.117, 0.00)	2797.64
3.00	(82.65, 2.26, 0.095, 0.00)	3011.17	2.10	(67.79, 3.22, 0.121, 0.00)	2819.64
3.50	(80.55, 2.21, 0.010, 0.00)	3044.06	2.50	(66.34, 3.01, 0.130, 0.00)	2886.32

Table 2: Optimal values in terms of different backorder parameters

α	$(Q^N, L^N(weeks), k^N, \beta^N, \lambda_1, \lambda_2)$	EAC^N	$(Q^F, L^F(weeks), k^F, \beta^F, \lambda_1, \lambda_2)$	EAC^F	$EVAI$
$\nu = 0$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	312.03
0.2	(69.22, 3.19, 1.84, 0.20, 0.119, 0.00)	2825.01	(84.88, 2.12, 2.51, 0.20, 0.100, 0.00)	3088.18	263.17
0.4	(69.54, 3.24, 1.76, 0.40, 0.117, 0.00)	2811.28	(85.58, 2.32, 2.30, 0.40, 0.093, 0.00)	3032.04	220.76
0.6	(69.88, 3.30, 1.67, 0.60, 0.115, 0.00)	2794.24	(86.56, 2.56, 2.05, 0.60, 0.086, 0.00)	2957.26	163.02
0.8	(70.41, 3.38, 1.54, 0.80, 0.112, 0.00)	2771.99	(87.67, 2.84, 1.80, 0.80, 0.077, 0.00)	2870.70	98.710
1.0	(71.12, 3.49, 1.37, 1.00, 0.109, 0.00)	2740.31	(89.39, 3.21, 1.47, 1.00, 0.065, 0.00)	2766.49	26.180
$\nu = 0.5$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(69.18, 3.19, 1.85, 0.19, 0.119, 0.00)	2825.43	(84.72, 2.06, 2.57, 0.16, 0.101, 0.00)	3109.61	284.18
0.4	(69.45, 3.23, 1.78, 0.38, 0.118, 0.00)	2812.48	(85.06, 2.18, 2.46, 0.32, 0.096, 0.00)	3059.23	246.75
0.6	(69.76, 3.28, 1.70, 0.57, 0.116, 0.00)	2797.02	(85.44, 2.30, 2.35, 0.47, 0.091, 0.00)	3006.34	209.32
0.8	(70.12, 3.34, 1.61, 0.75, 0.113, 0.00)	2778.20	(85.86, 2.41, 2.24, 0.63, 0.086, 0.00)	2950.95	172.75
1.0	(70.52, 3.41, 1.51, 0.92, 0.111, 0.00)	2754.83	(86.27, 2.53, 2.14, 0.76, 0.080, 0.00)	2893.06	138.23
$\nu = 1.0$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(69.18, 3.19, 1.85, 0.18, 0.118, 0.00)	2825.82	(84.58, 2.02, 2.62, 0.13, 0.102, 0.00)	3118.41	252.59
0.4	(69.41, 3.23, 1.79, 0.36, 0.118, 0.00)	2813.57	(84.75, 2.09, 2.56, 0.26, 0.098, 0.00)	3078.43	264.86
0.6	(69.68, 3.27, 1.72, 0.54, 0.116, 0.00)	2799.36	(84.91, 2.16, 2.51, 0.39, 0.095, 0.00)	3037.66	238.30
0.8	(69.96, 3.32, 1.65, 0.71, 0.114, 0.00)	2782.76	(85.12, 2.23, 2.45, 0.51, 0.091, 0.00)	2996.15	213.39
1.0	(70.23, 3.37, 1.58, 0.87, 0.112, 0.00)	2763.33	(85.35, 2.29, 2.39, 0.63, 0.087, 0.00)	2953.98	190.65
$\nu = 5.0$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(69.11, 3.17, 1.87, 0.14, 0.121, 0.00)	2828.37	(84.35, 1.93, 2.71, 0.03, 0.105, 0.00)	3148.91	320.54
0.4	(69.23, 3.19, 1.84, 0.28, 0.119, 0.00)	2819.81	(84.28, 1.91, 2.74, 0.06, 0.104, 0.00)	3140.19	320.38
0.6	(69.29, 3.21, 1.82, 0.41, 0.118, 0.00)	2810.84	(84.21, 1.89, 2.77, 0.10, 0.104, 0.00)	3131.35	320.51
0.8	(69.41, 3.23, 1.79, 0.53, 0.117, 0.00)	2801.50	(84.14, 1.88, 2.80, 0.13, 0.103, 0.00)	3122.40	320.90
1.0	(69.48, 3.25, 1.77, 0.65, 0.116, 0.00)	2791.81	(84.03, 1.86, 2.84, 0.17, 0.102, 0.00)	3113.34	321.53
$\nu = 10.0$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(69.03, 3.16, 1.89, 0.10, 0.120, 0.00)	2830.54	(84.35, 1.94, 2.71, 0.00, 0.106, 0.00)	3156.21	325.67
0.4	(69.07, 3.17, 1.88, 0.21, 0.119, 0.00)	2824.52	(84.35, 1.93, 2.71, 0.01, 0.106, 0.00)	3154.87	330.35
0.6	(69.10, 3.18, 1.87, 0.31, 0.119, 0.00)	2818.44	(84.33, 1.92, 2.72, 0.01, 0.105, 0.00)	3153.51	335.07
0.8	(69.10, 3.19, 1.86, 0.41, 0.118, 0.00)	2812.31	(84.27, 1.91, 2.74, 0.02, 0.105, 0.00)	3152.12	339.81
1.0	(69.11, 3.20, 1.86, 0.51, 0.117, 0.00)	2806.14	(84.22, 1.89, 2.76, 0.02, 0.105, 0.00)	3150.70	344.56
$\nu = 50.0$					
0.0	(68.95, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(68.95, 3.15, 1.91, 0.01, 0.121, 0.00)	2835.90	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.62
0.4	(68.92, 3.15, 1.92, 0.02, 0.121, 0.00)	2835.30	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	322.22
0.6	(68.87, 3.15, 1.93, 0.04, 0.121, 0.00)	2834.68	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	322.84
0.8	(68.83, 3.14, 1.94, 0.05, 0.121, 0.00)	2834.05	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	323.47
1.0	(68.79, 3.14, 1.95, 0.07, 0.121, 0.00)	2833.39	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	324.13
$\nu = 100.0$					
0.0	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(68.95, 3.16, 1.91, 0.00, 0.121, 0.00)	2836.46	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.4	(68.95, 3.16, 1.91, 0.00, 0.121, 0.00)	2836.43	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.6	(68.96, 3.16, 1.91, 0.00, 0.121, 0.00)	2836.39	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.8	(68.96, 3.16, 1.91, 0.00, 0.121, 0.00)	2836.36	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
1.0	(68.96, 3.16, 1.91, 0.00, 0.121, 0.00)	2836.33	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
$\nu = \infty$					
0.0	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.2	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.4	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.6	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
0.8	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03
1.0	(68.94, 3.15, 1.91, 0.00, 0.121, 0.00)	2836.49	(84.38, 1.95, 2.69, 0.00, 0.106, 0.00)	3157.52	321.03

Table 3: Optimal values in terms of different Maximum Available Space (MAS)

MAS	$(Q^F, L^F(weeks), k^F, \beta^F, \lambda_1, \lambda_2)$	EAC^F	$(Q^N, L^N(weeks), k^N, \beta^N, \lambda_1, \lambda_2)$	EAC^N	$EVAI$
14000	(92.23, 2.23, 2.48, 0.51, 0.066, 0.00)	2917.86	(76.21, 3.44, 1.67, 0.71, 0.085, 0.00)	2683.54	234.32
13500	(88.65, 2.23, 2.47, 0.51, 0.078, 0.00)	2953.94	(73.08, 3.38, 1.66, 0.71, 0.098, 0.00)	2729.46	224.48
13000	(85.12, 2.23, 2.45, 0.51, 0.091, 0.00)	2996.15	(69.96, 3.32, 1.65, 0.71, 0.114, 0.00)	2782.76	213.39
12500	(81.59, 2.22, 2.43, 0.51, 0.105, 0.00)	3045.29	(66.84, 3.25, 1.64, 0.71, 0.132, 0.00)	2844.45	200.84
12000	(78.11, 2.22, 2.40, 0.51, 0.122, 0.00)	3102.29	(63.74, 3.18, 1.63, 0.71, 0.150, 0.00)	2915.80	186.49

Table 4: Optimal values in terms of other MII and MAS and different crashing cost parameter (ω)

ω	$(Q^F, L^F(weeks), k^F, \beta^F, \lambda_1, \lambda_2)$	EAC^F	$(Q^N, L^N(weeks), k^N, \beta^N, \lambda_1, \lambda_2)$	EAC^N	$EVAI$
0.5	(111.89, 1.35, 2.35, 0.00, 0.049)	2962.92	(95.04, 2.86, 1.66, 0.00, 0.062)	2667.41	295.51
0.75	(108.31, 1.59, 2.43, 0.00, 0.044)	2837.17	(97.18, 2.68, 1.68, 0.00, 0.044)	2545.74	291.43
1	(108.88, 1.53, 2.48, 0.00, 0.035)	2750.84	(100.39, 2.42, 1.69, 0.00, 0.031)	2480.10	270.74

Step 3: Obtain λ_1, λ_2 and L by solving Eq. (30), (31) and (32), simultaneously
Step 4: Put λ_1, λ_2 and L in Eq. (34) and find$Q$
Step 5: Put $L, Q, \lambda_1, \lambda_2$in Eq. (28) and find EAC
Step 6: Obtain EAC^k in terms of different k and stop when:

$$EAC^k < Min\{EAC^{k-\xi}, EAC^{k+\xi}\}$$

$$|EAC^k - EAC^{k-\xi}| \leq \varepsilon$$

$$|EAC^k - EAC^{k+\xi}| \leq \varepsilon$$

NUMERICAL EXAMPLE

In order to illustrate our solution procedure, let us consider an inventory system with the following data:

$$D = 600\,unit/year\,, A = 200\$\,, H = 20\$, E(D) = 11\,unit/week, \sigma_D = 3\,unit/week\,,\ \pi = 50\$\,, \pi_\circ = 100\$\,, \epsilon = 156, \omega = 0.75, \alpha = 0.8, \nu = 1, f = 150\,, c = 100\$\,, \gamma = 0.92\,, \varepsilon = 0.01, Z_{1-\gamma} = -1.4$$

Maximum inventory investment(MII) = 14000 \$,

Maximum available space (MAS) = 13000M^2

In Table 1, we consider different values for safety factor (k) and obtain (EAC, Q, L) in partial and perfect demand information. The results show that EAC is convex for different amounts of safety factor in both partial and perfect demand. In this example, storage space constraint is binding. The expected annual cost for partial demand information is 2996.15 and perfect demand information is 2782.76. Consequently, the Expected Value for Additional Information ($EVAI$) is obtained as follow:

$$EAV^I = EAC^F - EAC^N = 2996.15 - 2782.76 = 213.39$$

In Table 2, we consider different values for backorder parameters (α, ν). The results reveal that with increasing α and decreasing $\nu(\alpha \neq 0\ \&\ \nu \neq \infty)$, the expected annual cost (EAC) will be decreased and backorder rate will be increased accordingly.

In Table 3, we consider different values for Maximum Available Space (MAS). The results show that the larger amount of MAS, the smaller EAC for both partial and perfect demand information, would be.

In Table 4, we consider other values for maximum available space ($MAS = 17500$) and maximum inventory investment ($MII = 13500$). So, in this case, the budget constraint is binding. The results are obtained in terms of different crashing cost parameter ($\omega = 0.5, 0.75, 1$). It can be see that the larger amount of ω, the smaller amount of EAC would be.

CONCLUSION

This study determines optimal continuous review policies for multi constrained single-item mixed inventory backorder and lost sales with controllable lead time in partial and perfect lead time demand distribution information environments. Backorder rate is dependent on the length of lead time through the amount of shortages. This study assumes that purchasing cost is paid at time of order placing. Considering this reason, a budget constraint is established in the proposed model. A random storage space limitation also is considered in this study. The lead time demand, first, assumes normal distribution and then, removes this assumption by only assuming that the first and second moments of probability distribution of lead time demand are known. In the latter case, minimax distribution free procedure and chance constrained programming technique is used to minimize the objective function.

For the future research, we suggest these directions for the model:

- Modifying model by fuzzifying D, annual demand or X, lead time demand
- Using other constraints like reorder workload
- Considering defective items
- Assuming periodic review policy

NOTATIONS

The following notations have been used in this study:

Q = Order quantity
r = Reorder point
k = Safety factor
L = Length of lead time
β = The fraction of demand which is backordered during stock out period
D = Average demand per year
π = Stockout cost per unit short
π_0 = Marginal profit per unit
c = Purchasing cost per unit
f = Space used per unit
h = Holding cost per year per unit
A = Fixed ordering cost per order
ϵ, ω = Total crashing cost parameters (□ $\geq 0, \omega \geq 0$)
α, ν = Backorder parameters $(0 \leq \alpha \leq 1, \nu \geq 0)$
$C(L)$ = Total lead time crashing cost per order
B = Maximum inventory investment

F = Maximum available space
x = Demand during lead time
x^+ = Maximum value of x and 0
$E(\cdot)$ = Mathematical expectation

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