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Research Article

Buyback Contract Coordinating Supply Chain Incorporated Risk Aversion

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Abstract: This research studies the buyback contract of a supply chain system composed of a risk-neutral supplier and a risk-averse retailer. The buyback contract is divided into two cases, the credit for all unsold goods and the credit for a partial return of goods, which are theoretically analyzed and simulated numerically respectively. The results show that when the retailer is risk averse, the supply chain system is able to achieve coordination. The buyback price is an increasing function of and the buyback ratio is also an increasing function of, while the wholesale price is a decreasing function of the risk aversion.

Keywords: Buyback contract, risk aversion, supply chain coordination

INTRODUCTION

Theoretical analysis has drawn the conclusion that contracts such as the buyback contract, the revenue-sharing contract and the quantity-flexibility contract can coordinate supply chain and optimize the profit of the whole supply chain and those of each chain member (Cachon, 2003). However, in the practice the coordination of supply chain often lose efficiency. One of the primary reasons is the risk-neutral assumption that the prior researchers have supposed. But, the risk attitude has a big influence. For example, Luca *et al*. (2012) considered the pricing problem when a risk-averse retailer facing an uncertainty demand and pointed out that the price by a risk-averse retailer was lower than that by a risk-neutral retailer. Thus, it is necessary to investigate the risk attitude of the supply chain participants. This study studies the buyback contract for the supply chain with a risk-neutral supplier and a risk-averse retailer.

Pasternack (1985) firstly discussed the return policy by developing a model of fixed price and used optimal pricing and return policy to ensure channel coordination. It found that the partial return policy with full price could ensure channel coordination in single retailers and single supplier system, while the full return policy with part price could make the multi-retailers system channel coordination. Arcelus *et al*. (2012) evaluated the pricing and ordering policies of the retailer facing a price-dependent stochastic demand under different degrees of risk tolerance. Choi and Chiu (2012) explored the Mean-Downside-Risk (MDR) and Mean-Variance (MV) newsvendor models under both the exogenous and endogenous retail pricing decision cases and showed that the analytical solution for both the MDR equaled that of MV problems. Choi and Ruszczynski (2011) considered a multi-product newsvendor using an exponential utility function and proved that when this ratio approached zero the risk-averse solution converged to the corresponding risk-neutral solution, while when the product demands were positively (negatively) correlated the risk aversion leaded to a lower (higher) optimal order quantities than the solution with independent demands. Ozgun and Chen (2011) probed a risk-averse retailer, formally modeled the risk aversion by adopting the Conditional-Value-at-Risk (CVaR) decision criterion and drawn the conclusion that the manufacturer's preferred rebate type depended on whether the retailer is risk neutral or sufficiently risk averse. Chiu *et al*. (2011) carried out a MV analysis under TSR contracts and found that the supplier could coordinate the channel with flexible TSR contracts. Wu and Wang (2010) analyzed the impact of risk aversion on the manufacturer's decisions, obtained results that characterized the explicit relationship between the manufacturer's risk attitude and his optimal decision. Wei and Choi (2010) explored the use of a Wholesale Pricing and Profit Sharing scheme (WPPS) for coordinating supply chain under the MV decision framework and showed that there existed an unique equilibrium of the Stackelberg Game with WPPS in the decentralized case. Wang and Webster (2009) used a utility function to describe the decision-making behavior of a risk averse retailer, concluded that a risk averse retailer's order quantity would be less than a arbitrarily small volume when sales prices was higher than a threshold value. Choi and Li (2008) proposed an MV formulation for a single supplier and a single retailer,

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studied both cases of centralized and decentralized supply chains and illustrated how a return policy could be applied for managing the supply chain to address the issues such as channel coordination and risk control. Lin and Zhang (2010) discussed the effect of buyback price and the market uncertainty on decision variables and the objective function and gave the determinant conditions of the size of the optimal order quantity for the retailer when the demand satisfied the second-order stochastic dominance criteria and then carried out numerical simulation analysis, which showed that the buyback price had the effect that should not be neglected on retailer's inventory decision-making. Qin and Zhao (2011) analyzed the effect of supplier's loss aversion on production capacity and the profits of supply chain members in the consignment contract, which founded that the profit of the supply chain members is a decreasing function of the loss aversion. This study will design full and partial buyback contract to coordinate the supply chain system composed of a risk-neutral supplier and a risk-averse retailer, respectively.

NOTATIONS AND MODEL

- D>0 = The market demand during the selling season
- F(x) = The distribution function of the demand, which is differentiable, strictly increasing. Let $\bar{F}(x)=1-F(x)$ and F(0) = 0
- F(x) = The density function of the demand. Let $\mu=E(D)$. Then, $\mu=\int_0^{\infty} xf(x)dx$
- p = The retail price
- w = The wholesale price
- c = The supplier's production cost per unit
- q = The order quantity of the retailer
- b = The price that the supplier pays the retailer per remaining at the end of the season
- λ = The proportion that the supplier buy back the remaining at the end of the season
- k_r = The risk aversion coefficient. Especially, when k_r = 0, the retailer is risk neutral

Moreover, the net salvage value of a unit is supposed to be 0 and the case of stock out is impossible. It is clear that $p>w>c$, $b<w$, $\lambda<1$.
Then, the retailer's expected sales is:

$$\begin{aligned} S(q) &= \int_0^q xf(x)dx + \int_q^{\infty} qf(x)dx \\ &= q - \int_0^q F(x)dx \\ &= \int_0^q [1-F(x)]dx \\ &= \int_0^q \bar{F}(x)dx \end{aligned} \tag{1}$$

The expect inventory of the retailer is:

$$\begin{aligned} I(q) &= \int_0^q (q-x)f(x)dx \\ &= q - S(q) \end{aligned} \tag{2}$$

The profit of the supply chain is:

$$\Pi_T = pS(q) - cq \tag{3}$$

Substituting (3) into (1), gets:

$$\Pi_T = p\int_0^q \bar{F}(x)dx - cq \tag{4}$$

Obviously, the supply chain optimal order quantity q is:

$$q_T^* = \arg\max_q \Pi_T = p\int_0^q \bar{F}(x)dx - cq \tag{5}$$

namely,

$$\bar{F}(q_T^*) = \frac{c}{p} \tag{6}$$

or

$$q_T^* = \bar{F}^{-1}(\frac{c}{p}) = F^{-1}(\frac{p-c}{p}) \tag{7}$$

Supply chain coordination means that in order to achieve the maximum of the whole profit of the supply chain, the supplier provides a proper contract to the retailer to make the retailer's order quantity equals the above q_T^*.

FULL BUYBACK CONTRACT

Under the full buyback contract, the supplier charges the retailer w per unit and pays the retailer b per unit remaining at the end of the season.

The risk-neutral retailer: The retailer's expected profit is:

$$\Pi_r(q,w,b) = (p-b)\int_0^q \bar{F}(x)dx - (w-b)q \tag{8}$$

The decision problem of the retailer is:

$$\begin{aligned} q_r^* &= \arg\max_q \Pi_r(q,w,b) \\ &= (p-b)\int_0^q \bar{F}(x)dx - (w-b)q \end{aligned} \tag{9}$$

namely,

$$\overline{F}[q_r^*(q,w,b)] = \frac{w-b}{p-b} \tag{10}$$

or

$$q_r^*(q,w,b) = \overline{F}^{-1}\left(\frac{w-b}{p-b}\right) \tag{11}$$

Comparing (7) and (11), it can be proved that in order to achieve supply chain coordination, the wholesale price w and buyback price b must meet:

$$\frac{w-b}{p-b} = \frac{c}{p} \tag{12}$$

by which there is 0<θ<1 simultaneously meeting:

$$p\text{-}b = \theta p \tag{13}$$

and

$$w\text{-}b = \theta c \tag{14}$$

Substituting (13) and (14) into (8):

$$\Pi_r(q,w,b) = \theta pS(q) - \theta cq \tag{15}$$

On the basis of (5):

$$\Pi_r(q,w,b) = \theta\Pi_T(q) \tag{16}$$

In summary, with the full buyback contract, the retailer's expected profit is a linear function of the centralized supply chain profit. Then, in the pursuit of the maximum expected profit, the retailer also achieves the maximum expected profit of the supply chain. Therefore, the full buyback contract which satisfies the conditions (12) or (13) and (14) can achieve supply chain coordination.

The risk-averse retailer*:* The participants in the supply chain have different risk attitude. In the supply chain system which composed with a supplier and a retailer, the small and medium-sized retailers may prefer risk aversion.
In this case, the retailer's utility function is:

$$U_r = E(\Pi_r) - k_r Var(\Pi_r) \tag{17}$$

where, $U_r \geq 0$.
The retailer's expected profits is:

$$\begin{aligned} E(\Pi_r) &= \Pi_r(q,w,b) \\ &= (p-b)\int_0^q \overline{F}(x)dx - (w-b)q \end{aligned} \tag{18}$$

where,

$$\begin{aligned} Var(\Pi_r) &= \int_0^q [px - wq + (q-x)b]^2 f(x)dx \\ &+\int_q^\infty (pq-wq)^2 f(x)dx \\ &-[(p-b)\int_0^q \overline{F}(x)dx - (w-b)q]^2 \\ &= (p-b)^2\{2q\int_0^q F(x)dx - 2\int_0^q xF(x)dx \\ &-[\int_0^q F(x)dx]^2\} \end{aligned} \tag{19}$$

by which:

$$\begin{aligned} \frac{\partial Var(\Pi_r)}{\partial q} &= (p-b)^2\{2\int_0^q F(x)dx + 2qF(q) \\ &-2qF(q) - 2F(q)\int_0^q F(x)dx\} \\ &= 2(p-b)^2\overline{F}(q)\int_0^q F(x)dx \end{aligned} \tag{20}$$

So, the first derivative of U_r about q is:

$$\begin{aligned} \frac{\partial U_r}{\partial q} &= \frac{\partial\left[E(\Pi_r) - k_r Var(\Pi_r)\right]}{\partial q} \\ &= \frac{\partial E(\Pi_r)}{\partial q} - k_r\frac{\partial Var(\Pi_r)}{\partial q} \\ &= (p-b)\overline{F}(q) - (w-b) \\ &-2k_r(p-b)^2\overline{F}(q)\int_0^q F(x)dx \\ &= (p-b)\left[1-F(q)\right]\left[1-2k_r(p-b)\int_0^q F(x)dx\right] \\ &-(w-b) \end{aligned} \tag{21}$$

Let (21) = 0. Then,

$$F(q) = 1 - \frac{w-b}{(p-b) - 2k_r(p-b)^2\int_0^q F(x)dx}$$

namely,

$$q^* = F^{-1}\left(1 - \frac{w-b}{(p-b) - 2k_r(p-b)^2\int_0^q F(x)dx}\right) \tag{22}$$

When q* of (22) equals that of (7), the supply chain coordination is realized.

According to (22), if the other values remain unchanged, q^* will be reduced gradually with the increasing of k_r. Therefore, in order to keep q^* be the optimal, the wholesale price w should decrease with k_r and the buyback price b should increase with k_r.

PARTIAL BUYBACK CONTRACT

Under the partial buyback contract, the supplier sells products to the retailer in the wholesale price w, then retailer sells goods to customers in the retail price p and finally the supplier buy back the goods not selling in a certain proportion of λ in the wholesale price of w, where 0<λ<1.

The risk-neutral retailer: The expected profit of the retailer is:

$$\Pi_r(q,w,\lambda) = \int_0^q \left[px - wq + \lambda(q-x)w\right] f(x)dx + \int_q^\infty (p-w)qf(x)dx = (p-w)q - p\int_0^q F(x)dx + \lambda w\int_0^q F(x)dx \tag{23}$$

The decision problem of the retailer is:

$$q_r^*(q,w,\lambda) = \arg\max_q \Pi_r(q,w,\lambda) = (p-w)q - p\int_0^q F(x)dx + \lambda w\int_0^q F(x)dx \tag{24}$$

Let first order derivative equal zero. Then:

$$\frac{\partial \Pi_r(q,w,\lambda)}{\partial q} = (p-w) - pF(q) + \lambda wF(q) = 0$$

namely:

$$q_r^*(q,w,\lambda) = F^{-1}\left(\frac{p-w}{p-\lambda w}\right) \tag{25}$$

In order to coordinate the supply chain, the wholesale price w and the buyback proportion λ must meet:

$$\frac{p-w}{p-\lambda w} = \frac{p-c}{p} \tag{26}$$

namely,

$$\lambda^* = \frac{pw-pc}{pw-wc} \tag{27}$$

According to (27), the optimal buyback proportion λ is an increasing function of the wholesale price w.

The risk-averse retailer: The expected profit of the retailer is:

$$E(\Pi_r) = \Pi_r(q,w,\lambda) = (p-w)q - p\int_0^q F(x)dx + \lambda w\int_0^q F(x)dx \tag{28}$$

where,

$$Var(\Pi_r) = \int_0^q \left[px - wq + \lambda(q-x)w\right]^2 f(x)dx + \int_q^\infty (p-w)^2 q^2 f(x)dx - [(p-w)q - p\int_0^q F(x)dx + \lambda w\int_0^q F(x)dx]^2 = (p-\lambda w)^2\{2q\int_0^q F(x)dx - 2\int_0^q xF(x)dx - [\int_0^q F(x)dx]^2\} \tag{29}$$

by which:

$$\frac{\partial Var(\Pi_r)}{\partial q} = (p-\lambda w)^2[2\int_0^q F(x)dx + 2qF(q) - 2qF(q) - 2F(q)\int_0^q F(x)dx] = 2(p-\lambda w)^2\overline{F}(q)\int_0^q F(x)dx \tag{30}$$

So, the first derivative of U_r about q is:

$$\frac{\partial U_r}{\partial q} = \frac{\partial\left[E(\Pi_r) - k_r Var(\Pi_r)\right]}{\partial q} = \frac{\partial E(\Pi_r)}{\partial q} - k_r\frac{\partial Var(\Pi_r)}{\partial q} = (p-w) - pF(q) + \lambda wF(q) - 2k_r(p-\lambda w)^2\overline{F}(q)\int_0^q F(x)dx = (p-w) - [1 - 2k_r(p-\lambda w)\int_0^q F(x)dx](p - \lambda w)F(q) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx \tag{31}$$

Let (31) = 0. Then,

$$[1-2k_r(p-\lambda w)\int_0^q F(x)dx](p-\lambda w)F(q) = (p-w) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx$$

namely,

$$F(q) = \frac{(p-w) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx}{(p-\lambda w) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx} \tag{32}$$

So, the optimal order quantity is:

$$q^* = F^{-1}\left(\frac{(p-w) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx}{(p-\lambda w) - 2k_r(p-\lambda w)^2\int_0^q F(x)dx}\right) = F^{-1}\left(\frac{\dfrac{(p-w)}{2k_r(p-\lambda w)^2\int_0^q F(x)dx} - 1}{\dfrac{(p-\lambda w)}{2k_r(p-\lambda w)^2\int_0^q F(x)dx} - 1}\right) \tag{33}$$

When q* of (33) equals to that of (7), supply chain coordination is achieved.

Table 1: The effect of the risk aversion k_r on the buyback price b in full buyback contract

k_r	0	1	2	3	4
b	0.0333	0.0362	0.0387	0.0408	0.0427
k_r	5	6	7	8	9
b	0.0444	0.0459	0.0473	0.0485	0.0496

Table 2: The effect of the risk aversion k_r on the wholesale price w in full buyback contract

k_r	0	1	2	3	4
w	0.0598	0.0579	0.0559	0.0540	0.0521
k_r	5	6	7	8	9
w	0.0502	0.0482	0.0463	0.0444	0.0425

Table 3: The effect of the risk aversion k_r on the buyback proportion λ in the partial buyback contract

k_r	0	1	2	3	4
λ	0.5556	0.6040	0.6451	0.6807	0.7120
k_r	5	6	7	8	9
λ	0.7398	0.7649	0.7876	0.8084	0.8275

Table 4: The effect of the risk aversion k_r on the wholesale price w in the partial buyback contract

k_r	0	1	2	3	4
w	0.0602	0.0573	0.0539	0.0502	0.0460
k_r	5	6	7	8	9
w	0.0411	0.0353	0.0281	0.0189	0.0058

Because (p-w) < (p-nw), when the other values remain unchanged, q* will be reduced gradually with k_r. Therefore, in order to keeping q* be the optimal, the wholesale price w should decrease with k_r and the buyback proportion b should increase with k_r.

NUMERICAL EXAMPLES

In order to give a more direct reflection of the above, a numerical simulation analysis using MATLAB 7.0 is given as follows. Let p = 0.1, c = 0.04, F~ (100, 1^2).

If w = 0.06, by substituting the parameters into the model of full buyback contract, Table 1 and Fig. 1 are gained.

It can be drawn that in order to coordinate the supply chain, the buyback price b should increase with the risk aversion k_r. Especially, for a risk neutral retailer, the buyback price b reaches the minimum.

If b = 0.033, by substituting the parameters into the model of full buyback contract, Table 2 and Fig. 2 are gained.

It can be drawn that in order to coordinate the supply chain, the wholesale price w should decrease with the risk aversion k_r. Especially, for a risk neutral retailer, the wholesale price w reaches the maximum.

If w = 0.06, by substituting the parameters into the model of partial buyback contract, Table 3 and Fig. 3 are gained.

It can be drawn that in order to coordinate the supply chain, the buyback proportion λ should increase with the risk aversion k_r. Especially, for a risk neutral retailer, the buyback proportion λ reaches the minimum.

If λ = 0.56, by substituting the parameters into the model of partial buyback contract, Table 4 and Fig. 4 are gained.

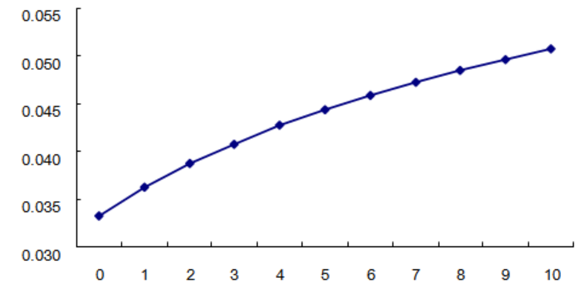

Fig. 1: The relation between the risk aversion k_r and the buyback price b in full buyback contract

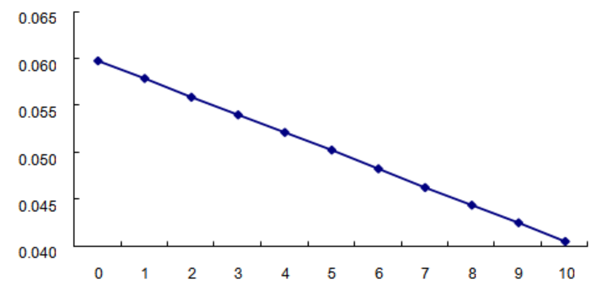

Fig. 2: The relation between the risk aversion k_r and the wholesale price w in full buyback contract

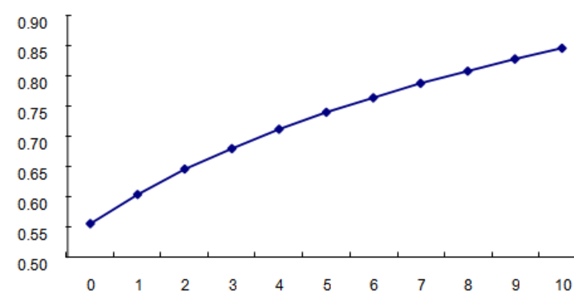

Fig. 3: The relation between the risk aversion k_r and the buyback proportion λ in the partial buyback contract

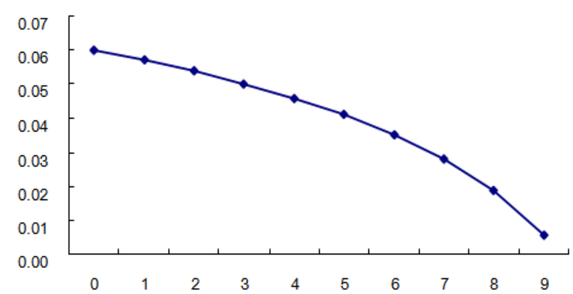

Fig. 4: The relation between the risk aversion k_r and the wholesale price w in the partial buyback contract

It can be drawn that in order to coordinate the supply chain, the wholesale price w should decrease with the risk aversion k_r. Especially, for a risk neutral retailer, the wholesale price w reaches the maximum.

CONCLUSION

This research studies the buyback contract of a supply chain system composed of a risk-neutral supplier and a risk-averse retailer. The buyback contract was divided into two cases, the credit for all unsold goods and the credit for a partial return of goods, which are theoretically analyzed and simulated numerically respectively. The conclusion is drawn that when the retailer prefers risk aversion, the supply chain system can still be achieved coordination. In the full buyback contract, the buyback price should be higher than the price of the risk neutral case, while the wholesale prices should be less than the price of the risk neutral case. The buyback price is the increasing function of the risk-aversion, while the wholesale price is the decreasing function of the risk-aversion. In the partial buyback contract, the buyback proportion should higher than the proportion of the risk neutral case, while the wholesale price should be less than the price of the risk neutral case. The buyback proportion is an increasing function of the risk-aversion, while the wholesale price is a decreasing function of the risk-aversion.

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