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Research Article

A Power Penalty Approach to a Horizontal Linear Complementarity Problem

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Abstract: A power penalty approach has been proposed to linear complementarity problem but not to Horizontal Linear Complementarity Problem (HLCP) because the coefficient matrix is not positive definite. It is skillfully proved that HLCP is equivalent to a variational inequality problem and a mixed linear complementarity problem for the first time. A power penalty approach is proposed to the mixed linear complementarity problem based on approximating the HLP by a nonlinear equation. It has been proved that the solution to this equation is feasible and converges to that of the HLP at a rate of at least $O(\lambda)^{-k/2}$ in the Euclidean norm.

Keywords: Convergence rate, horizontal linear complementarity problem, power penalty methods, strong monotone, variational inequality

INTRODUCTION

Consider a horizontal linear complementarity problem as follows:

Problem 1: Given A ε R^{nxn}, B ε R^{nxn} are matrixes and q ε R^n is a vector, find a pair x, y ε R^n such that:

$$Ax\text{-}By = q \tag{1}$$

$$x \leq 0 \tag{2}$$

$$y \leq 0 \tag{3}$$

$$x^T y = 0 \tag{4}$$

It is worth noting that HLP becomes LCP if B = I and if B is nonsingular, HLCP can reduce to LCP. So HLCP is the generalization of LCP. HLCP also includes the Linear Optimization (LO) and Convex Quadratic Optimization (CQO) (Zhang, 1994). LCP arise in many mathematical models from economy and technology. And various approaches to LCP are developed in recent years which are based commonly on Newton's method and Interior-Point Methods (IPMs). Some alternative the weighted path-following interior-point methods were proposed. Ding and Li (1998) studied the weighted path-following methods for LCP. Jansen *et al*. (1996) presented the primal-dual target-following algorithms for LO. Some IPMs for LO, CQO and LCP have been extended to HLCP, Such as Bonnans and Gonzaga (1996) studied the HLCP, (Huang, 2000) proposed an high-order feasible interior point method for HLCP with O(√n log $\varepsilon_0/\varepsilon$) iterations. Sturm (1999) derived the superlinear convergence properties for monotone linear complemntarity problem under no strictly complementary solution exists. Zhang and Zhang (1995) presented a class of infeasible IPMs for HLCP.

However, there was a limited study of penalty methods for LCP. Recently, (Wang and Xiaoqi, 2008) first presented a power penalty method for LCP in R^n based on approximating the LCP by a nonlinear equation and (Huang and Wang, 2010) developed it to NLCP and shown that the solution to the penalty equation converges to that of the NCP in the Euclidean norm at a rate of at least $O(\lambda^{-k/\xi})$. Inspired by their study, we develop a power penalty method for solving HLCP, based on the idea in Wang and Xiaoqi (2008) and Huang and Wang (2010). We first approximate the HLP by a nonlinear system of equations in which a power penalty term with a penalty constant λ>1and a power parameter k>0 are contained. If coefficient matrixes A, B obey the Assumption A1, we show that the solution to the penalty equation converges to that of the HLP in the Euclidean norm at a rate of at least $O(\lambda^{-k/2})$.

In this study, we use $\| \ \|_p$ to denote the usual l_p-norm on R^n for any p>1. When p = 2, it becomes the Euclidean norm. $[u]_+ = \max\{u, 0\}$, $[u] = \min\{-u, 0\}$, and $y_\sigma = (y^\sigma_1, y^\sigma_2, \ldots, y^\sigma_n)^T$ for any $y = (y_1, y_2, \ldots, y_n)^T$ and constant σ>0.The outline of the study is as follows. In next section, we briefly introduce the problem, prove skillfully that HLCP is equivalent to a variation

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inequality problem and a mixed linear complementarity problem and give its penalty formulation.

THE PROBLEM AND ITS PENALTY FORMULATION

Let the cone:

$$K = \{(x^T, y^T)^T \mid x \in R^n, y \in R^n, y \leq 0\} \quad (5)$$

which is a closed, convex and self- dual in R^{2n}.
If we let:

$$F(x, y) = \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} \quad (6)$$

and β is a constant and define the following variational inequality problem corresponding to Problem 1.

Problem 2: Find $(x^T, y^T)^T$ such that for all $(u^T, v^T)^T \in K$:

$$\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} \geq 0 \quad (7)$$

It can easily be shown that the variational inequality problem is equivalent to a mixed linear complementarity as follows:
Find a pair x, y ε Rn such that:

$$\begin{aligned} &G(x, y) = Ax - By - q = 0 \\ &\quad y \leq 0 \\ &H(x, y) = (I - \beta A)x + \beta By - \beta q \leq 0 \\ &\quad y^T H(x, y) = 0 \end{aligned} \quad (8)$$

We can prove Problem 1 and 2 is equivalent, namely, the Theorem 2 is true:

Proposition 1: $(x^T, y^T)^T$ is a solution for Problem 1 if and only if $(x^T, y^T)^T$ is a solution of Problem 2 at one time.

Proof: If $(x^T, y^T)^T$ is a solution for Pronlem 1, it is obvious $(x^T, y^T)^T \in K$ and for any $(u^T, v^T)^T \in K$, we have:

$$\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} = v^T x \geq 0$$

since v≤0 and x≤0. Therefore, $(x^T, y^T)^T$ is a solution of Problem 2.
Conversely, if $(x^T, y^T)^T$ is a solution of Problem 2, we have y≤0. We first prove:

$$(I - \beta A)x - \beta By - \beta q \leq 0$$

If it were not true, then there would exist at least an index i, such that the i[th] component of:

$$(I - \beta A)x - \beta By - \beta q$$

Satisfies:

$$\left[(I - \beta A)x - \beta By - \beta q \right]_i > 0$$

Since $(u^T, v^T)^T \in K$ is arbitrary, we choose u = x and:

$$v = \begin{cases} y_j, & j \neq i \\ y_i - \varepsilon, & j = i \end{cases}$$

For j = 1, 2,…, n and an arbitrary constant ε>0. Substituting this into (2) gives:

$$\begin{aligned} &\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} \\ &= (v_i - y_i)\left[(I - \beta A)x + \beta By + \beta q \right]_i \\ &= -\varepsilon \left[(I - \beta A)x + \beta By + \beta q \right]_i < 0 \end{aligned}$$

This contradicts the fact that $(x^T, y^T)^T$ is a solution to Problem 2. Thus, we have:

$$(I - \beta A)x - \beta By - \beta q \leq 0$$

Next, we show that Ax-By-q = 0. If it is not true, there must exist at least one indexi, such that $(Ax\text{-}By\text{-}q)_i \neq$. Now we choose u and v satisfying v = y and:

$$u = \begin{cases} x_j, & j \neq i \\ x_j - \varepsilon \operatorname{sgn}\left[(Ax - By - q)_i \right], & j = i \end{cases}$$

For j 1, 2,…, n, where, sign denotes the sign function and an arbitrary constant ε>0. Substituting this into (2) yields:

$$\begin{aligned} &\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} \\ &= (u_i - x_i)(Ax - By - q)_i \\ &= -\varepsilon \operatorname{sgn}\left[(Ax - By - q)_i \right](Ax - By - q)_i < 0 \end{aligned}$$

Which is impossible as $(x^T, y^T)^T$ is a solution to Problem 2. Therefore, we have Ax-By-q = 0.
So we have x≤0 from the facts:

$$(I - \beta A)x - \beta By - \beta q \leq 0$$

And,

$$Ax - By - q = 0$$

Finally, let us show that $x^T y = 0$. Since $(u^T, v^T)^T \varepsilon K$ is arbitrary, we choose $(u^T, v^T)^T$, respectively as follows:

$$\left(u^T, v^T\right)^T = \left(x^T, 2y^T\right)^T$$

And,

$$\left(u^T, v^T\right)^T = \left(x^T, \frac{1}{2}y^T\right)^T$$

Substituting these into (7) respectively and noticing $Ax-By-q = 0$ yields:

$$\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} = y^T x \geq 0$$

$$\left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)^T \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix} = -\frac{1}{2}y^T x \geq 0$$

So we can deduce $x^T y = 0$. This completes the proof of the proposition.

Next, we present a power penalty method for Problem 1. Consider the following penalty problem:

Problem 3: Finding $(x^T_\lambda, y^T_\lambda)^T \varepsilon R^{2n}$, such that:

$$\varphi_\lambda(x, y) = F(x_\lambda, y_\lambda) + \lambda \begin{pmatrix} 0 \\ [y_\lambda]_+^{\frac{1}{k}} \end{pmatrix} = 0 \tag{9}$$

where, $F(x, y) = \begin{bmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{bmatrix}$ which is equivalent to:

$$\begin{pmatrix} Ax_\lambda - By_\lambda - q \\ x_\lambda \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ [y_\lambda]_+^{\frac{1}{k}} \end{pmatrix} = 0 \tag{10}$$

And $\lambda>1$ and $k>0$ are parameters.

This is penalized equations corresponding to Problem 1, where the penalty term penalizes the positive part of y_λ. When $\lambda\to\infty$, we expect that the solution $(x^T_\lambda, y^T_\lambda)^T$ of the problem 3 converges to that of Problem 1. From (10), it is obvious that $x_\lambda \leq 0$, $y_\lambda \leq 0$ in this way we can get a feasible solution which differs obviously from Wang and Xiaoqi (2008) and Huang and Wang (2010) and the rate of convergence depend distinctly on both of the parameters in the penalty term.

CONVERGENCE ANALYSIS

In this section we first give a assumption, and based on this assumption, Problem 3 has unique solution and establish some upper bounds for the distance between the solution the Problem 2 and 3, respectively which are based on the following assumption on the coefficient matrix H:

A1: There exist constant β such that coefficient matrix:

$$H = \begin{bmatrix} A & -B \\ I - \beta A & \beta B \end{bmatrix}$$

is positive definite, i.e., there exist constants β and $\alpha > 0$ such that $z^T Hz \geq \alpha \|z\|_2^2$ for any $z \varepsilon R^{2n}$.

Remark 1: Under the Assumption A1, we have no difficulty to proof that:

$$F(x, y) = \begin{pmatrix} Ax - By - q \\ (I - \beta A)x + \beta By + \beta q \end{pmatrix}$$

is strong monotone and can proof $ø_\lambda$ (x, y) is also strong monotone. So we can draw conclusion based on theory of variational inequality that the problem 2 has unique solution and we can affirm that the problem 3 has unique solution.

Remark 2: when B = I and A positive definite, H is positive definite if and only if $\beta[2I - \beta A(A+A^T)^{-1} A^T]$ is positive definite. If we choose β satisfying $0 < \beta < \frac{2}{\lambda_{max}}$, where, λ_{max} is the maximal eigenvalue of $A(A + A^T)^{-1}$, H is positive definite Which weaken the conditions in Wang and Xiaoqi (2008).

Theorem 1: Under the Assumption A1:

$$\varphi_\lambda(x, y) = F(x_\lambda, y_\lambda) + \lambda \begin{pmatrix} 0 \\ [y_\lambda]_+^{\frac{1}{k}} \end{pmatrix}$$

is strong monotone.

Proof: In fact: for any:

$$\left(x^T, y^T\right)^T, \left(u^T, v^T\right)^T \in R^{2n}$$

$$\left(\left(x^T, y^T\right) - \left(u^T, v^T\right)\right)\left(\varphi_\lambda(x, y) - \varphi_\lambda(u, v)\right)$$

$$= \left(\left(x^T, y^T\right) - \left(u^T, v^T\right)\right)[\left(F(x, y) - F(u, v)\right) + \lambda \begin{pmatrix} 0 \\ [y]_+^{\frac{1}{k}} \end{pmatrix} - \lambda \begin{pmatrix} 0 \\ [v]_+^{\frac{1}{k}} \end{pmatrix}]$$

$$=\left(\left(x^T,y^T\right)-\left(u^T,v^T\right)\right)[(F(x,y)-F(u,v))]+$$

$$\lambda\left(\left(x^T,y^T\right)-\left(u^T,v^T\right)\right)\left(\begin{pmatrix}0\\ [y]_+^{\frac{1}{k}}\end{pmatrix}-\begin{pmatrix}0\\ [v]_+^{\frac{1}{k}}\end{pmatrix}\right)$$

$$=\left(\left(x^T,y^T\right)-\left(u^T,v^T\right)\right)[(F(x,y)-F(u,v))]$$
$$+\lambda(y-v)^T([y]_+^{\frac{1}{k}}-[v]_+^{\frac{1}{k}})$$
$$\geq\left(\left(x^T,y^T\right)-\left(u^T,v^T\right)\right)[(F(x,y)-F(u,v))]$$
$$\geq\alpha\left\|\left(\left(x^T,y^T\right)-\left(u^T,v^T\right)\right)\right\|_2^2$$

So Problem 3 has unique solution based the Theorem 2.3.3 of Facchinei and Pang (2003).

In the rest of our discussion, we start our convergence analysis with the lemma as follow:

Lemma 1: Let $(x^T_\lambda, y^T_\lambda)^T$ be the solution to Problem 3 for any $\lambda\geq 0$, then there exists a positive constant M which is independent of $(x^T_\lambda, y^T_\lambda)^T$, λ and K, such that:

$$\left\|\left(x_\lambda^T,y_\lambda^T\right)\right\|_2\leq M \tag{11}$$

Proof: let $(x^T_\lambda, y^T_\lambda)^T$ be the solution to Problem 3 for any $\lambda\geq 0$, Left-multiplying both sides of (9) by $(x^T_\lambda, y^T_\lambda)^T$ gives:

$$\left(x_\lambda^T,y_\lambda^T\right)\phi(x_\lambda,y_\lambda)+\lambda\left(x_\lambda^T,y_\lambda^T\right)\begin{pmatrix}0\\ [y_\lambda]_+^{\frac{1}{k}}\end{pmatrix}=0$$

Or

$$\left(x_\lambda^T,y_\lambda^T\right)\phi(x_\lambda,y_\lambda)+\lambda y_\lambda^T\left[y_\lambda\right]_+^{\frac{1}{k}}=0 \tag{12}$$

Since

$$y_\lambda^T\left[y_\lambda\right]_+^{\frac{1}{k}}=\left[y_\lambda\right]_+^T\left[y_\lambda\right]_+^{\frac{1}{k}}\geq 0$$

We have $(x^T_\lambda, y^T_\lambda)$ $F(x_\lambda, y_\lambda)\leq 0$ So $(x^T_\lambda, y^T_\lambda)$ $F(x_\lambda, y_\lambda)$ $-F(0, 0))\leq -(x^T_\lambda, y^T_\lambda)$ $F(0, 0)$ Using Cauchy-Schwarz inequality and F(x, y) is strong monotone, we have:

$$\alpha\left\|\begin{pmatrix}x_\lambda\\ y_\lambda\end{pmatrix}\right\|_2^2\leq\left\|\begin{pmatrix}x_\lambda\\ y_\lambda\end{pmatrix}\right\|_2\left\|F(0,0)\right\|_2=\sqrt{2}\left\|\begin{pmatrix}x_\lambda\\ y_\lambda\end{pmatrix}\right\|_2\left\|q\right\|_2$$

Therefore, $\|(x_\lambda^T,y_\lambda^T)^T\|_2\leq M$, where, M is obviously independent of $(x^T_\lambda, y^T_\lambda)^T$, λ and k.

Remark 3: F (x, y) is continuous obviously, Lemma 1 shows that for any $\lambda\geq 0$, there exists a positive constant L which is independent of $(x^T_\lambda, y^T_\lambda)^T$ λ and k, such that:

$$\left\|F(x_\lambda,y_\lambda)\right\|_2\leq L \tag{13}$$

Lemma 2: let $(x^T_\lambda, y^T_\lambda)^T$ be the solution to Problem 3 for any $\lambda\geq 0$, then there exists a positive constant C which is independent of $(x^T_\lambda, y^T_\lambda)^T$, λ and k, such that:

$$\left\|\begin{pmatrix}[x_\lambda]_+\\ [y_\lambda]_+\end{pmatrix}\right\|_2\leq\frac{C}{\lambda^k} \tag{14}$$

Proof: Let $(x^T_\lambda, y^T_\lambda)^T$ be the solution to Problem 3 for any $\lambda\geq 0$,we have $x_\lambda\leq 0$,which indicate $[x_\lambda]_+ = 0$, so we only proof $\|[y_\lambda]_+\|_2\leq C/\lambda^k$ under the conditions of the Lemma 2.

Let C be a generic positive constant which is independent of $(x^T_\lambda, y^T_\lambda)^T$, λ and k. Left -multiplying both sides of (7) by, $([x_\lambda]^T_+, [y_\lambda]^T_+)^T$, i.e., $(0, [y_\lambda]^T_+)^T$ gives:

$$[y_\lambda]_+^Tx_\lambda+\lambda[y_\lambda]_+^T\left[y_\lambda\right]_+^{\frac{1}{k}}=0 \tag{15}$$

Adding both sides of (15) by $-[y_\lambda]^T_+ y_\lambda$ gives:

$$[y_\lambda]_+^Tx_\lambda-[y_\lambda]_+^Ty_\lambda+\lambda[y_\lambda]_+^T\left[y_\lambda\right]_+^{\frac{1}{k}}=-[y_\lambda]_+^Ty_\lambda \tag{16}$$

Since $-[y_\lambda]^T_+ y_\lambda = -[y_\lambda]^T_+ [y_\lambda]_+\leq 0$ we have from (16):

$$\lambda[y_\lambda]_+^T\left[y_\lambda\right]_+^{\frac{1}{k}}=\leq-\left([y_\lambda]_+^Tx_\lambda-[y_\lambda]_+^Ty_\lambda\right)$$
$$=\left([y_\lambda]_+^T,[y_\lambda]_+^T\right)\begin{pmatrix}-x_\lambda\\ y_\lambda\end{pmatrix}$$

Using Cauchy-Schwarz inequality, we have from the above equation:

$$\lambda\left\|[y_\lambda]_+\right\|_p^p\leq\left([y_\lambda]_+^T,[y_\lambda]_+^T\right)\begin{pmatrix}-x_\lambda\\ y_\lambda\end{pmatrix}$$

$$\leq\left\|\left([y_\lambda]_+^T,[y_\lambda]_+^T\right)\right\|_p\left\|\begin{pmatrix}x_\lambda\\ y_\lambda\end{pmatrix}\right\|_q \tag{17}$$

where, p = 1 + 1/k, q = 1 + k and 1/p + 1/q = 1. By the fact that all norms in R^n are equivalent and Lemma 1, we see that (17) implies:

$$\left\|[y_\lambda]_+\right\|_2\leq\frac{C}{\lambda^k}$$

where, positive constant C is independent of $(x^T_\lambda, y^T_\lambda)$ λ and k. Finally, (13) follows from the above estimate.

As follows, we present and prove our main convergence result.

Theorem 2: Let $(x^T, y^T)^T$ and $(x_\lambda^T, y_\lambda^T)^T$ be the solutions to Problem 2 and 3, respectively. There exists positive constant C that is independent of $(x_\lambda^T, y_\lambda^T)^T$, λ and k such that:

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_\lambda \\ y_\lambda \end{pmatrix} \right\|_2 \le \frac{C}{\lambda^{k/2}} \tag{18}$$

Proof: Since $(x^T, y^T)^T$ is the solutions to Problem 2, it satisfies (7):

Notice $\left[\begin{bmatrix} x \\ y \end{bmatrix}\right] - \left[\begin{bmatrix} x_\lambda \\ y_\lambda \end{bmatrix} = \begin{bmatrix} x + \\ y + \end{bmatrix} \begin{bmatrix} x_\lambda \end{bmatrix} - [x_\lambda]_+ \\ [y_\lambda] - [y_\lambda]_+ \right]$

where, $\eta_\lambda = x + [x_\lambda]_-$ and $r_\lambda = y + [y_\lambda]_-$. Since:

$$x - \eta_\lambda = -[x_\lambda]_- \le 0,\ y - r_\lambda = -[y_\lambda]_- \le 0$$

We have $\begin{pmatrix} x - & \eta\lambda \\ y - & r\lambda \end{pmatrix} \epsilon\, K$

Let $\begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} x - & \eta\lambda \\ y - & r\lambda \end{bmatrix}$, in (7), it follows:

$$\left(\begin{pmatrix} x - \eta_\lambda \\ y - r_\lambda \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)^T F(x, y) \ge 0$$

Or

$$-\begin{pmatrix} \eta_\lambda \\ r_\lambda \end{pmatrix}^T F(x, y) \ge 0 \tag{19}$$

We notice that $(x^T_\lambda, y^T_\lambda)^T$ is the solution to the Problem 3, Therefore, left multiplying both sides of (9) by $(\eta^T_\lambda, r^T_\lambda)^T$, we have:

$$\begin{pmatrix} \eta_\lambda \\ r_\lambda \end{pmatrix}^T F(x_\lambda, y_\lambda) + \lambda \begin{pmatrix} \eta_\lambda \\ r_\lambda \end{pmatrix}^T \begin{pmatrix} 0 \\ [y_\lambda]_+^{\frac{1}{k}} \end{pmatrix} = 0$$

Adding up (17) and (18) gives:

$$\left(\eta_\lambda^T, r_\lambda^T\right)^T (F(x_\lambda, y_\lambda) - F(x, y)) + \lambda r_\lambda^T [y_\lambda]_+^{\frac{1}{k}} \ge 0 \tag{20}$$

Note that:

$$r_\lambda^T [y_\lambda]_+^{\frac{1}{k}} = (y + [y_\lambda]_-)^T [y_\lambda]_+^{\frac{1}{k}} = y^T [y_\lambda]_+^{\frac{1}{k}} \le 0$$

Since $y \le 0$ and $[y_\lambda]^{1/k}_+ \ge 0$, Thus (20) leads to:

$$\left(\eta_\lambda^T, r_\lambda^T\right)^T (F(x_\lambda, y_\lambda) - F(x, y)) \ge 0$$

Or

$$\left(\eta_\lambda^T, r_\lambda^T\right)^T (F(x, y) - F(x_\lambda, y_\lambda)) \le 0$$

So we have:

$$\left(\begin{pmatrix} x - x_\lambda \\ y - y_\lambda \end{pmatrix} + \begin{pmatrix} [x_\lambda]_+ \\ [y_\lambda]_+ \end{pmatrix} \right)^T (F(x, y) - F(x_\lambda, y_\lambda)) \le 0$$

It follows that:

$$\begin{pmatrix} x - x_\lambda \\ y - y_\lambda \end{pmatrix}^T (F(x, y) - F(x_\lambda, y_\lambda))$$

$$\le -\begin{pmatrix} [x_\lambda]_+ \\ [y_\lambda]_+ \end{pmatrix}^T (F(x, y) - F(x_\lambda, y_\lambda)) \tag{21}$$

From the Theorem 1 and the well-known Cauchy-Schwarz inequality, it follows that:

$$\alpha \left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_\lambda \\ y_\lambda \end{pmatrix} \right\|_2^2 \le \left\| \begin{pmatrix} [x_\lambda]_+ \\ [y_\lambda]_+ \end{pmatrix} \right\|_2 \| F(x, y) - F(x_\lambda, y_\lambda) \|_2 \le \frac{C_1}{\lambda^k}$$

So we have (18).

CONCLUSION

Although we do not make numerical experiments, from theoretical results and numerical experiments in (8-9), the power penalty approach should be effective for obtaining the solution of linear complementarily problem.

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