

## Research Article

### An Imprecise Probability Model for Structural Reliability Based on Evidence and Gray Theory

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**Abstract:** To avoid the shortages and limitations of probabilistic and non-probabilistic reliability model for structural reliability analysis in the case of limited samples for basic variables, a new imprecise probability model is proposed. Confidence interval with a given confidence is calculated on the basis of small samples by gray theory, which is not depending on the distribution pattern of variable. Then basic probability assignments and focal elements are constructed and approximation methods of structural reliability based on belief and plausibility functions are proposed in the situation that structure limit state function is monotonic and non-monotonic, respectively. The numerical examples show that the new reliability model utilizes all the information included in small samples and considers both aleatory and epistemic uncertainties in them, thus it can rationally measure the safety of the structure and the measurement can be more and more accurate with the increasing of sample size.

**Keywords:** Epistemic uncertainty, evidence theory, gray theory, imprecise probability model, structural reliability

## INTRODUCTION

In reliability assessment of a structure, some factors, such as the complexity of internal mechanism, the limitation of people's knowledge and errors in manufacturing, will produce a variety of uncertainties. How to handling these uncertainties is a serious problem associating with the confidence of reliability analysis results. There are three types of reliability model at present, which are probabilistic, fuzzy and non-probabilistic reliability model, respectively. Probabilistic reliability model is constructed on the basis of probability theory and has been widely used in reliability assessment for various structures (Park *et al.*, 2004; Christopher and Masoud, 2009). But in this model, the inherent stochastic characteristics of structure are considered as the only source of uncertainties. However, recent researches show that despite inherent stochastic characteristics, limitation of people's knowledge is another source to produce uncertainties and this type of uncertainty is so called "epistemic uncertainty" (Oberkampf *et al.*, 1998, 2001). Probability theory is effective on handling aleatory uncertainty, but is not applicable for epistemic uncertainty (Helton, 1997; George and Richard, 2001).

On the other hand, the precondition of using probabilistic reliability model is there having sufficient information to represent the probability distribution of uncertain quantity. But it is hard to meet this condition in many occasions. Above two shortages limit the application of probabilistic reliability model. Fuzzy reliability model is constructed on the basis of probabilistic reliability model and avoids the insufficiency of hypothesis of discrete finite state (Utkin and Gurov, 1996; Giasi *et al.*, 2003; Biondini *et al.*, 2004). But fuzzy reliability model is also having the aid of statistical method to obtain membership function, which is similar with probabilistic reliability model. The only difference between them is that the former uses subjective probability and the later uses objective probability (Lu and Feng, 2000). However, in general cases, obtaining the bounds of an uncertain quantity are easier than the probability distribution. In this view, Ben-Harm, Elishakoff and other researchers proposed the non-probabilistic reliability model based on interval analysis and convex model (Ben-Haim, 1994; Ben-Haim *et al.*, 1996; Elishakoff, 1994; Luo *et al.*, 2009). Although the model is simple in calculation, the results are always too conservative. Furthermore, the model only uses the lower and upper

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bounds of uncertain quantities and wastes the other valuable information, so it was difficult to be applied in engineering practice.

In engineering practices, as the limitation of experimental expenses and time, experimental samples and data are always inadequate. This makes the obtained information both involving aleatory and epistemic uncertainties. The existing data is insufficient to construct the probability distribution of uncertain quantity, but can get additional information more than merely an interval range. Evidence theory is the generalization of probability theory (Dempster, 1967; Shafer, 1976). Researches show that evidence theory has strong ability on handling both aleatory and epistemic uncertainties simultaneously (Helton *et al.*, 2004). So the motivation of this study is constructing a new imprecise probabilistic reliability model to evaluate structure reliability with insufficient experimental data using evidence theory and gray theory, in which gray theory is used to estimate the confidence interval of experimental data. The new model will consider both aleatory and epistemic uncertainties, so it is expected to get more reasonable results than the other reliability models.

**PROBABILISTIC RELIABILITY MODEL**

In reliability analyzing for an engineering structure, limit state equation can be constructed according to the designed functions and requirements. In probabilistic reliability model, if  $\mathbf{X} = \{x_1, x_2, \dots, x_n\}$  is the vector of  $n$  random variables for stochastic factors, then the structure limit state equation is:

$$Z = g(\mathbf{X}) = g(x_1, x_2, \dots, x_n) = 0 \tag{1}$$

According to the two states hypothesis in probabilistic reliability theory, failure surface  $\{\mathbf{X}|g(\mathbf{X}) = 0\}$  divides the basic variable space into two areas. If  $g(\mathbf{X}) > 0$ , then the structure is in reliable state and safe domain is:

$$\Omega_s = \{\mathbf{X} | g(\mathbf{X}) > 0\} \tag{2}$$

Otherwise the structure is in failure state and its failure domain is:

$$\Omega_f = \{\mathbf{X} | g(\mathbf{X}) < 0\} \tag{3}$$

If  $f(\mathbf{X})$  is the joint probability density function of  $\mathbf{X}$ , then the structure failure probability is:

$$P_f = P(F) = \int_{\Omega_f} f(\mathbf{X})d\mathbf{X} \tag{4}$$

where,  $F$  is the proposition  $F = \{\mathbf{X} \in \Omega_f\}$ . Correspondingly, the reliability is:

$$R_s = 1 - P_f \tag{5}$$

Generally, probability distribution of variable  $x_i$  can be obtained by statistic method with experimental data. However, if the size of experimental data is small, it is hard to determine the distribution pattern and distributed parameters for  $x_i$ . And more, for a same set of data, sometimes different distribution patterns may all be fitted (Ben-Haim, 1994). But these different distribution patterns have different header and tail information, which are very important for the accuracy of structure reliability calculation. Choosing an ill-suited distribution pattern will result in great error in reliability calculation (Ben-Haim, 1993; Elishakoff, 1995). So in the situation of small sample data, conventional probabilistic reliability model faces enormous challenges.

**IMPRECIS PROBABILITY MODEL FOR STRUCTURAL RELIABILITY UNDER SMALL SAMPLES**

The essence of small sample data is aleatory and epistemic uncertainties are coexisting. To a variable  $x_i$ , aleatory uncertainty is its inherent characteristics and this type of uncertainty cannot be eliminated. However, epistemic uncertainty is caused by inadequate data and knowledge and can be removed when there are enough data or knowledge to construct the precise probability distribution of  $x_i$ . The root cause of the defect of conventional probability reliability model is that it ignores the epistemic uncertainties in variables when there are not enough samples.

In this section, gray distance measure is used to estimate the gray confidence interval of small sample data. Then the Basic Probability Assignments (BPAs) are constructed and structural reliability is calculated by evidence reasoning.

**Gray confidence interval estimation:** If the sample set of  $x_i$  is  $A = \{a_1, a_2, \dots, a_m\}$ , then the interval of  $x_i$  should be estimated with  $A$  first. Traditional methods are working on the basis of large sample size and the prior distribution of the variable  $x_i$  (Chien, 1997; David, 1997; Law, 2006). But to small sample data, the prior distribution is generally unknown, so the traditional interval estimation methods are not available. Confidence interval of variable  $x_i$  can be estimated using gray theory in the following procedures:

- Calculating the gray distance measure of each sample  $a_i$  to each other sample  $a_j$  with (Ke *et al.*, 2007):

$$dr(a_i, a_j) = \frac{\xi \left\| d(A, a_j) \right\|_{\infty}}{|a_i - a_j| + \xi \left\| d(A, a_j) \right\|_{\infty}} \tag{6}$$

where,  $\xi$  is the resolution ratio. Generally  $\xi = 0.5$  (Deng, 1990) and:

$$\|d(A, a_j)\|_{\infty} = \max_k \{|A(k) - x_j|, k \in [1, m]\} \quad (7)$$

Take the mean value of these gray distance measures as the distance measure of  $a_i$  ( $i \in [1, m]$ ) to the whole sample space:

$$J_i = \left( \sum_{j=1}^m dr(a_i, a_j) \right) / m \quad (8)$$

Finally, normalize  $J_i$  ( $i \in [1, m]$ ) as:

$$w_i = J_i / \sum_{j=1}^m J_j \quad (9)$$

where,  $w_i$  is the weight of sample  $a_i$  to gray estimate  $\hat{X}_i$ .

- Calculate the gray estimate  $\hat{X}_i$  with weighted accumulation method as:

$$\hat{x}_i = \sum_{i=1}^m w_i a_i \quad (10)$$

- Calculate the gray confidence interval of  $x_i$ . From Eq. (6),  $dr(a_0, \hat{X}_i)$  is the gray confidence. For a given confidence  $\alpha$  ( $\alpha \in [1, 1/3]$ ), by the Inequality:

$$dr(a_0, \hat{x}_i) \geq \alpha \quad (11)$$

the confidence interval of  $\hat{X}_i$  at confidence  $\alpha$  can be obtained as  $[\inf(\alpha_0), \sup(\alpha_0)]$ .

**Calculation of focal elements and BPAs:** After get the interval estimation of variable  $x_i$ , i.e.,  $[\inf(\alpha_0), \sup(\alpha_0)]$ , the data set  $A = \{a_1, a_2, \dots, a_m\}$ ,  $\inf(\alpha_0)$  and  $\sup(\alpha_0)$  are arranged from small to large order as:

$$B = \{a_{(1)}, a_{(2)}, \dots, a_{(k)}\} \quad (12)$$

where,  $k = m+2$ . The mean of above set is:

$$\bar{a} = (a_{(1)} + a_{(2)} + \dots + a_{(k)}) / k \quad (13)$$

Constructing  $N$  minimum intervals  $A_{ij} = [\bar{a} - \Delta_j, \bar{a} + \Delta_j]$  ( $1 \leq j \leq N$ ) with their mid-values at  $\bar{a}$ , where  $\Delta_1 < \Delta_2 < \dots < \Delta_N$ . These intervals are the focal elements of variable  $x_i$ . The probability of samples being included in each interval is  $P_j$  ( $1 \leq j \leq N$ ). If

there are  $M$  samples in the interval  $A_{ij}$  ( $1 \leq j \leq N$ ), then:

$$p_j = M / N \quad (14)$$

And

$$Bel(A_{ij}) = p_j (1 \leq j \leq N) \quad (15)$$

Then the BPA of focal element  $A_{ij}$  is:

$$m(A_{ij}) = \begin{cases} Bel(A_{j_1}) & (j=1) \\ Bel(A_{ij}) - Bel(A_{i(j-1)}) & (2 \leq j \leq N) \end{cases} \quad (16)$$

Denotes the frame of discernment of  $X = \{x_1, x_2, \dots, x_n\}$  to be  $\Theta_X$  and its focal elements to be  $C_p$  ( $1 \leq p \leq N^n$ ), then  $c_p$  is a Cartesian product:

$$c_p = A_{1j_1} \times A_{2j_2} \times \dots \times A_{nj_n} \quad (17)$$

where,  $j_1, j_2, \dots, j_n \in [1, N]$ . By Dempster's rule, BPA of  $C_p$  is:

$$m(c_p) = m(A_{1j_1}) \times m(A_{2j_2}) \times \dots \times m(A_{nj_n}) \quad (18)$$

**Structural reliability calculation with evidence theory:** Let  $\Theta_z$  be the frame of discernment in failure domain and  $Bel_z(F)$  and  $Pl_z(F)$  are belief and plausibility function of structure failure respectively, then:

$$Bel_z(F) \leq P_z(F) \leq Pl_z(F) \quad (19)$$

$Bel_z(F)$  is the lower bound of  $P_f$ ,  $Pl_z(F)$  is the upper bound and  $[Bel_z(F), Pl_z(F)]$  is the interval estimation of  $P_f$ . If information content about  $F$  is great enough, then  $Bel_z(F)$  will be close to  $Pl_z(F)$  indefinitely and  $|Bel_z(F) - Pl_z(F)|$  will tend to be an infinitesimal. So the mean of the two functions is the approximation of  $P_f$ :

$$\tilde{P}_f = (Bel_z(F) + Pl_z(F)) / 2 \quad (20)$$

Denote  $Z_0 = \{z: z < 0, z \in \Theta_z\}$ , then according to evidence theory,  $Bel_z(F)$  and  $Pl_z(F)$  can be expressed as:

$$Bel_z(F) = Bel_x[f^{-1}(Z_0)] = \sum_{c_i \in f^{-1}(Z_0)} m(c_i) \quad (21)$$

$$Pl_z(F) = Pl_x[f^{-1}(Z_0)] = \sum_{c_i \cap f^{-1}(Z_0) \neq \emptyset} m(c_i) \quad (22)$$

Then structure failure probability is obtained by Eq. (20). If it is necessary, structure reliability can also be calculated with Eq. (5).

If  $Z = g(X)$  is monotonic, then vertex method (Dong and Shah, 1987) can be used. Denote the mapping of  $c_i$  in  $Z$  as  $d_i$ . From the monotonicity of  $g(X)$ , the upper and lower bounds of  $d_i$  are only possible to obtain when  $X$  is the hypercube vertexes of  $c_i$ . Denote:

$$d_i = g(c_i) = [l_i, u_i] \tag{23}$$

Then:

$$d_i = g(c_i) = [\min_j \{g(v_j) : j = 1, \dots, n\}, \max_j \{g(v_j) : j = 1, \dots, n\}] \tag{24}$$

where,  $v_j$  ( $j = 1, 2, \dots, n$ ) are the vertexes of hypercube  $c_i$ . By Eq. (24), all  $d_i$  ( $i \in m^n$ ) in  $\Theta_Z$  can be calculated. Then

$$Bel_Z(F) = \sum_{d_i \subseteq Z_0} m(d_i) = \sum_{\sup(d_i) < 0} m(d_i) \tag{25}$$

$$Pl_Z(F) = \sum_{d_i \cap Z_0 \neq \emptyset} m(d_i) = \sum_{\inf(d_i) < 0} m(d_i) \tag{26}$$

If  $Z = g(X)$  is non-monotonic, then vertex method is unusable because of large computational errors. In this case, sampling method can be used to calculate  $Bel_Z(F)$  and  $Pl_Z(F)$ . Denote:

$$N\_Bel_Z = \{i : c_i \subset g^{-1}(Z_0)\} \tag{27}$$

$$N\_Pl_Z = \{i : c_i \cap g^{-1}(Z_0) \neq \emptyset\} \tag{28}$$

Then Eq. (21) and (22) can be expressed as:

$$Bel_Z(F) = \sum_{i \in N\_Bel_Z} m(c_i) \tag{29}$$

$$Pl_Z(F) = \sum_{i \in N\_Pl_Z} m(c_i) \tag{30}$$

The procedure to estimate  $N\_Bel_Z(F)$  and  $N\_Pl_Z(F)$  using sampling method is as follows:

**Step 1:** Randomly generating  $N_s$  ( $N_s \rightarrow \infty$ )  $X$  in  $c_k$  ( $c_k \subset \Theta_X$  and calculate  $Z$  with Eq. (1). If  $Z < 0$  hold for any  $Z$ , then  $i \in N\_Bel_Z$  and  $i \in N\_Pl_Z$ . If  $Z < 0$  hold for some  $Z$  and  $Z \geq 0$  hold for the others, then  $i \in N\_Pl_Z$ .

**Step 2:** For  $1 \leq i \leq N^n$ , repeat the step 1.

Then  $Bel_Z(F)$  and  $Pl_Z(F)$  can be calculated with Eq. (29) and (30).

After calculating  $Bel_Z(F)$  and  $Pl_Z(F)$ , approximation of  $P_f$  can be obtained with Eq. (20).

## NUMERICAL EXAMPLES

**Example 1:** The limit state equation of a structure is:

$$Z = g(X) = x_1^3 + x_2^3 - 18 = 0 \tag{31}$$

where,  $x_1$  and  $x_2$  obey normal distribution  $N(1, 0.2)$  and  $N(3, 0.09)$ , respectively. Randomly generate 10 samples with the probability distribution function of each variable, as shown in Table 1.

Set gray confidence  $\alpha = 0.8$  and gray confidence intervals of  $x_1$  and  $x_2$  are obtained as (0.89, 23.47) and (-4.25, 22.24). Construct 11 subintervals with their mid-values at  $\bar{x}_1$  and  $\bar{x}_2$ , respectively and  $P_j = j/11$  ( $1 \leq j \leq 11$ ).

Then focal elements and BPAs of  $x_1$  and  $x_2$  are calculated, the results are shown in Table 2.

As Eq. (31) is monotonic, vertex method is used in this example to calculate the belief and plausibility functions of the structure failure probability. The results are  $Bel_Z(F) = 0$  and  $Pl_Z(F) = 0.0165$ . So from Eq. (20) structure failure probability is  $\tilde{P}_f = 0.0083$  and from Eq. (5) structure reliability is  $\tilde{R}_s = 0.9917$ .

As the sample size is small, the results are imprecise. Increases the sample size, the results will be more accurate. Let  $m$  be 20, 30, 40 and 50, respectively and calculate the structure reliability with proposed method. The results are compared with that of Monte Carlo method with  $10^6$  samples and shown in Table 3. If the results of Monte Carlo method,  $R_s$ , can be taken as the exact value, then it can be seen from Table 3 that the results of proposed method is very close to the exact value when  $m$  increases to 50.

Table 1: Experimental data of  $x_1$  and  $x_2$

$x_1$					$\bar{x}_1$
1.36	3.380	6.760	8.850	9.370	9.86
9.78	10.24	12.57	14.67	21.75	
$x_2$					$\bar{x}_2$
-2.85	2.980	6.720	8.390	9.570	9.50
10.93	11.25	12.29	15.64	20.08	

Table 2: Focal elements and BPAs of  $x_1$  and  $x_2$

BPA	Focal elements of $x_1$	Focal elements of $x_2$
1/11	9.78, 10.24	8.39, 9.570
1/11	9.37, 10.24	8.39, 10.93
1/11	8.85, 10.24	8.39, 11.25
1/11	8.85, 12.57	6.72, 11.25
1/11	6.67, 12.57	6.72, 12.29
1/11	6.67, 14.67	6.72, 15.64
1/11	3.38, 14.67	2.98, 15.64
1/11	1.36, 14.67	2.98, 20.08
1/11	0.89, 14.67	-2.85, 20.08
1/11	0.89, 21.75	-2.85, 22.24
1/11	0.89, 23.47	-4.25, 22.24

Table 3: Results of reliability with different sample size in example 1

$m$	$\inf(R_s)$	$\sup(R_s)$	$\tilde{R}_s$	$R_s$
10	0.9835	1.0000	0.9918	0.9943
20	0.9865	0.9977	0.9921	0.9943
30	0.9900	0.9979	0.9939	0.9943
40	0.9911	0.9964	0.9937	0.9943
50	0.9931	0.9954	0.9942	0.9943

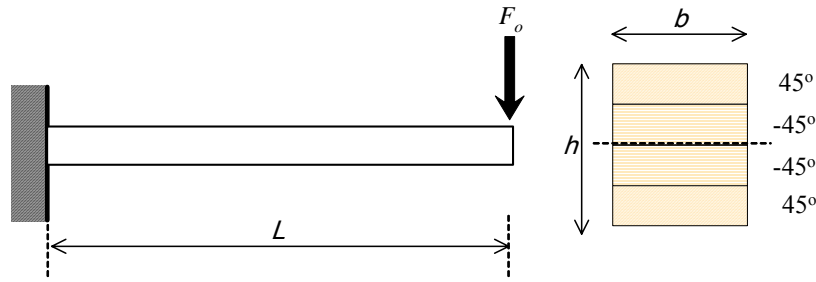


Fig. 1: Composite cantilever beam structure model

**Example 2:** A composite cantilever beam with a point load is shown in Fig. 1 (Bae *et al.*, 2004). To simplify the calculation of tip displacement of the composite beam, a symmetric laminated beam is used with one composite material and  $(\pm 45)$  s angle plies. The tip displacement is obtained by the classical laminated plate theory (Reddy, 1997) as:

$$\delta_{Tip} = \frac{F_0 L^3}{2h^3} \cdot \left( \frac{E_L^2 - 4G_{LT}E_T\nu_{LT}^2 + E_L(E_T + 4G_{LT} + 2E_T\nu_{LT})}{E_L G_{LT} (E_L + E_T + 2E_T\nu_{LT})} \right) \quad (32)$$

where,

$h$  : The height (3.83 cm) of the beam

$L$  : The length (50.7 cm) of the beam

$F_0$  : The applied load per width (360 kN)

$G_{LT}$  : The shear modulus (9.38 GPa)

$\nu_{LT}$  : The Poisson's ratio (0.03)

The Young's moduli  $E_L$  : Considered as uncertain variables and follow normal distribution  $N(143, 11.8)$

The Young's moduli  $E_T$  : Considered as uncertain variables and follow normal distribution  $N(33, 6.2)$

If the tip displacement should not exceed the limit state value of 5.59 cm, otherwise, the beam will fail and then the limit state equation of the structure is:

$$Z = \delta_{Tip} - Limit = 0 \quad (33)$$

and safe domain is:

$$\Omega_s = \{\delta_{Tip} : \delta_{Tip} - Limit < 0\} \quad (34)$$

Randomly generate 20 samples by the distribution function of  $E_L$  and  $E_T$ , respectively as shown in Table 4.

Set gray confidence  $\alpha = 0.8$  and gray confidence intervals of  $E_L$  and  $E_T$  are obtained as (115.26, 177.65) and (16.14, 48.26), respectively. Construct 22 subintervals with their mid-values at  $\bar{E}_L$  and  $\bar{E}_T$ , respectively and  $P_j = j/22$  ( $1 \leq j \leq 22$ ). Then focal

Table 4: Experimental data of  $E_L$  and  $E_T$

$E_L$					$\bar{E}_L$
118.08	129.27	131.23	132.56	135.72	145.32
138.71	139.74	140.45	140.98	143.06	
143.99	145.25	146.61	149.92	153.09	
156.12	159.29	162.54	167.37	172.47	
$E_T$					$\bar{E}_T$
17.40	18.87	21.18	23.24	26.72	31.12
28.25	28.64	29.55	30.63	31.63	
31.87	32.34	34.07	34.86	35.04	
36.41	37.11	38.85	39.69	46.14	

Table 5: Results of reliability with different sample size in example 2

$m$	Inf ( $R_s$ )	Sup ( $R_s$ )	$\bar{R}_s$	$R_s$
20	0.9876	1.0000	0.9928	0.9977
30	0.9883	0.9990	0.9937	0.9977
40	0.9932	0.9983	0.9958	0.9977
50	0.9963	0.9985	0.9974	0.9977

elements and BPAs of  $E_L$  and  $E_T$  are obtained. As Eq. (33) is non-monotonic, sampling method is used to calculate the structure reliability and the results are shown in Table 5. Let  $m$  be 30, 40 and 50, respectively and calculate the structure reliability with proposed method. The results are compared with that of Monte Carlo method with  $10^6$  samples, which is also shown in Table 5. It can be seen from Table 5 that with the increasing of  $m$ ,  $\bar{R}_s$  becomes more and more accurate too.

## CONCLUSION

When sample size is too small to construct the probability distribution of the variable, conventional probabilistic reliability model is not applicable. Although non-probabilistic reliability model based on interval analysis and convex model can calculate structure reliability under above situation, but in this model only bounds of data can be used. As many valuable data is discarded, the results of non-probabilistic reliability model are always too conservative to be applied in reality engineering structures. In this study, a new imprecise reliability model is proposed based on evidence theory and gray theory, which not only avoids the problem of constructing the distribution functions of variables, but also utilizes all of the experimental data. As the model

considers aleatory and epistemic uncertainties together, the results of reliability calculation can reflect the actual situation of structure. The numerical examples show the model has high precision and its precision will improve when sample size increases. On the other hand, the new model does not depend on the distribution information about variables. So the new model is very effective under small samples and is a beneficial supplement to conventional probabilistic and non-probabilistic reliability model.

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