Research Article
Solving Optimal Pricing Model for Perishable Commodities with Imperialist Competitive Algorithm

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Abstract: The pricing problem for perishable commodities is important in manufacturing enterprise. In this study, a new model based on the profit maximization principle and a discrete demand function which is a negative binomial demand distribution is proposed. This model is used to find out the best combination for price and discount price. The computational results show that the optimal discount price equals the cost of the product. Because the demand functions which involves several different distributions is so complex that the model is hard to solve with normal numerical method. Thus we combine the model with exterior penalty function and applied a novel evolution algorithm-Imperialist Competitive Algorithm (ICA) to solve the problem. Particle Swarm algorithm (PSO) is also applied to solve the problem for comparison. The result shows that ICA has higher convergence rate and execution speed.

Keywords: Imperialist competitive algorithm, particle swarm algorithm, perishable commodities, pricing model

INTRODUCTION

Perishable commodities are also called perishable product. With the development of the technology, intensification of market competition, shrinkage of the life cycle and upgrading, more and more products have features of the perishable commodity (such as MP3 players, personal computers and video games). In Weatherford and Bodily (1992) extended the concept of perishable asset (Lawrence and Samuel, 1992). He defined perishable commodities as goods that must be sold within a fixed period of time and its salvage value will shrink significantly or become negative after that period. Besides perishable commodities have many different features compare to common products. It has a feature of timeliness and the demand of the perishable commodities can fluctuate widely. Thus how to set or adjust the price is of great importance for the retailers to reach the maximum profit.

Research on pricing policies has been pursued by economists, marketing scientists and operations researchers from a wide range of perspectives. Because of the limitation of the sales horizon, many research focus on dynamic pricing police. Gallego and Ryzin study the optimal dynamic inventory pricing problems systematically using intensity control theory, bounds and heuristics (Guillermo and Garret Van, 1994). Feng and Gallego proposed a 2-price revenue management model of a single perishable product in (3). Feng and Xiao also present a continuous-time yield management model with multiple prices and reversible changes in price (Youyi and Baichun, 2000).

In the problem of optimal pricing for perishable commodities, stochastic demand is more suitable in studying problems in real world life since uncertainty widely exists. Stochastic demand, in fact, is a probabilistic demand and its probability distributions can be estimated from historical data. Young (2003) proposed a stochastic demand function based on customers' perceived value. Only those whose perceived value of the product is higher than the list price are willing to buy one unit of the product. This demand function is valid in many situations and can describe the effectual demand successfully.

However the demand function in his article is complicated because of the existence of some different stochastic distributions (Young, 2003). Hence pricing models based on discrete demand are usually difficult to solve with normal numerical methods. Heuristic algorithm is an effective method to cope with this bottle-neck. Heuristic algorithms have already been applied widely to optimization. Many optimization problems can successfully be solved with heuristic algorithms although some of the problems are NP-hard. Andrew and John proposed a hybrid tabu search/branch and bound approach to solve the Generalized Assignment Problem (GAP). Andrew and John (2010) Georgria Alexouda applied the Genetic Algorithm (GA) to design a product line using the seller's return criterion (Georgia and Konstantions, 2001) Komarudin develop an ant

colony system for solving unequal area facility layout problems (Komarudin, 2010).

In Esmaeil and Caro (2007) Lucas proposed a novel algorithm for optimization inspired by imperialistic competitive-Imperialist Competitive Algorithm (ICA) (Esmaeil and Caro, 2007) Compared to traditional evolutionary algorithms, ICA has higher convergence rate, better global optimal achievement and capacity of coping with difficult constraints. It has already been applied widely to various different fields such as computer science, telecommunication and control engineering. ICA is used for reverse analysis of an artificial neural network in order to characterize the properties of materials from sharp indentation test Biabangard-Oskouyi *et al* . (2008). ICA is also proved to be a reliable tool in beam-forming, Arash *et al.* (2007).

In this study, we will try to apply ICA to solve a pricing model for perishable goods, taking advantage of the algorithm's good performance.

PROBLEM STATEMENTS

Here we study one of the perishable commodities sellers in the market. The demand of perishable commodities is strong during the sales period after which the demand decreases significantly. Any product not sold by the end of the sales period will be disposed at cost price or a lower price. The reasons for this are:

- Some products have a short guarantee period
- Rapid changes in fashion and high inventory costs
- The value of the products will decrease with the upgrade of products

Thus in order to clarify our problems, we have 2 assumptions:

- The total number of customers visiting the store during the sales period is an exogenous variable, independent of other variables, for example not affected by the price. This assumption is valid in many situations. For one thing whether the customers visit the shop or not is largely determined by their buying habit. For another, sellers usually promote discount ranges for a family of products without specifying individual prices.
- The list price and the discount price of the product are to be determined at the beginning of the sales period. Any price negotiation between the seller and buyers is not allowed. This assumption conforms to the practical situation.

Thus the problem is described as follow: a certain units of perishable product have been retained in stock. The sellers have to determine the list price at the beginning of the sales period during which the list price is fixed. The list price must be higher than the cost. When the sales period is over, remain will be sold at the discount price which is also determined by the seller. The discount price is not allowed to exceed the cost.

In this section, we formulate the problem as a nonlinear programming. We have these notations: t: a single sales period time which is independent to the other variables in the model; s: units of a perishable (or seasonal) product have been retained in stock to satisfy the demand; ω: the list price for one unit of the product and is fixed during the sales period; λ: the arrival rate of customers, in our model λ is a random number with gamma uniform; n: the total number of customers visiting the store during the sales period t; φ: the discount price (or the salvage value) for one unit of the product after the sales period; c: the unit cost of the perishable (or seasonable) product. Here we do not consider any inventory carrying, ordering, disposal and shortage costs; $revenue_s(\omega)$: the expected revenue when the unit price is ω for s items; $profit_s(\omega)$: the total expected profit obtained from s items in stock $F(\omega)$: the cumulative distribution function used to estimate customers maximum perceived value at price ω; m: the total demand during the sales period t or equivalently, the number of potential buyers; $P_\omega [m]$: the demand distribution at price ω.

Demand distribution: As we mentioned in section 1, demand function is of vital importance in modeling. Gallego and Van Ryzin (1994) describe the expected demand as $E [m|\omega] = a \times exp (-b\omega)$, for $a,b>0$. Karlin and Carr considered several forms of demand function as follows:

- $E [m|\omega] = \max \{a-b\omega, 0\}$, for $a, b>0$
- $E [m|\omega] = \omega^{-a} \times exp (-b\omega)$, for $a, b>0$
- $E [m|\omega] = 1/ (1-\omega)^a$, for $a>0$

(Guillermo and Garret Van, 1994). In this study, we used the negative binomial demand distribution which is derived from Young's work. For each sales period, the total demand m at price ω during the sales period t follows a certain demand distribution $P_\omega[m]$. However $P_\omega[m]$ must satisfy the following 2 conditions:

- With the increase of the list price, the demand of the product will decrease according to the economic principle. This is because of the existence of the substitution effect and fierce competition in the market. The list price ω must be above the discount price φ. Thus the range of the list price ω should be (φ, ∞). Hence, it is reasonable to assume that:

$$\sum_{m=0}^{k} P_\omega[m] \leq \sum_{m=0}^{k} P_{\omega+\Delta}[m]$$

for any arbitrary $k = 0,1,2\ldots\infty$ and for any positive increments $\Delta>0$. The equality holds only for $k = \infty$.

- The probability of having any potential buyers becomes 0 when the list price ω increases towards infinity. Thus we can assume that $\lim_{\omega \to 0} \omega E[m$

$|\omega] = 0$ where $E[m|\omega]$ represents the expected number of potential buyers for a given list price ω. A demand distribution that satisfies all of the conditions is called a regular demand distribution as Guillermo and Garret Van (1994). Young proposed a regular demand distribution (Young, 2003). The total number of customers visiting the store during the sales period is denoted by a discrete random variable n, where $n = 0, 1, 2…\infty$. n is independent of all other parameters of the system including the price.

Among n customers, any one whose perceived value of the product is higher than the list price ω would like to buy one unit of the product at the price ω. The maximum that the i th customer is willing to pay for the product is denoted by a continuous random variable X_i, $0 \le X_i \le \infty$. $X_1, X_2, X_3, X_4, …, X_n$ are assumed to be independent and identically distributed random variables from a distribution having a probability density function, which is known or can be estimated from the history data.

The perceived value of customers at price ω is determined by the cumulative distribution $F(\omega)$, $0 \le F(\omega) \le 1$, where, $F(\omega)$ is a non-decreasing function of ω. $F(\omega)$ has the following feature: $\lim_{\omega \to 0} F(\omega) = 1$.

The potential buyers can be defined as those who would like to pay the price ω for one unit of the product with $X_i \ge \omega$. Among n customers arriving to the store during the sales period, the number of potential buyers' m when the price is ω can be expressed as a binomial random variable. Since the total number of customers' n is independent of the product price ω, the total number of potential buyer's m is a function of ω. That is:

$$P_\omega[m \mid n] = \binom{n}{m} [F(\omega)]^{n-m} [1 - F(\omega)]^m, m = 0, 1, 2, ..., n \quad (1)$$

In most situations, the total number of customers' n visiting the store during sales period t follows a Poisson distribution that is:

$$P[n \mid \lambda] = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0, 1, 2, ..., n \quad (2)$$

where, λ is the arrival rate of customers and is a random number with gamma distribution. Then for λ we have:

$$g(\lambda) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \lambda^{\alpha-1} e^{-\lambda/\beta}, 0 \le \lambda < \infty \quad (3)$$

With (1), (2) and (3), the total demand m in the sales period t at price ω can be deduced as follow:

$$P_\omega[m] = \int_{\lambda=0}^{\infty} \sum_{n=0}^{\infty} P_\omega[m \mid n] P[n \mid \lambda] g(\lambda) d\lambda$$

$$= \binom{m+\alpha-1}{m} \left[\frac{\beta t \{1 - F(\omega)\}}{1 + \beta t \{1 - F(\omega)\}}\right]^m \left[\frac{1}{1 + \beta t \{1 - F(\omega)\}}\right]^\alpha, m = 0, 1, 2, ..., \infty$$

In fact, this demand distribution is a negative binomial one. It has already been proved to satisfy the 2 conditions in Young's work.

Model: We formulate the problem as a non-linear programming. When the demand exceeds the supply, that is $s<m$, $revenue_s(\omega) = s\omega$.

When the supply exceeds the demand, which means a part of the products have to be sold at discount price, $revenue_s(\omega) = s\omega - (s-m)\varphi$.

Thus when the inventory level is s with a list price of ω, sellers expected revenue can be expressed as follow:

$$revenue_s(\omega) = \sum_{m=0}^{\infty} s\omega P_\omega[m] + \sum_{m=0}^{s-1} s\omega - (s-m)\phi P_\omega[m]$$

$$= s\omega - s\omega \sum_{m=0}^{s-1} P_\omega[m] + \sum_{m=0}^{s-1} [m\omega P_\omega[m] + sP_\omega[m] - msP_\omega[m]]$$

$$= s\omega - (\omega - \phi) \sum_{m=0}^{s-1} (s-m) P_\omega[m]$$

Thus the profit is:

$$profit_s(\omega) = revenue_s(\omega) - sc$$

$$= s(\omega - c) - (\omega - \phi) \sum_{m=0}^{s-1} (s-m) P_\omega[m]$$

Finally the problem can be formulated as a non-linear programming as followed:

$$MAX \quad profit_s(\omega) = s(\omega - c) - (\omega - \phi) \sum_{m=0}^{s-1} (s-m) P_\omega[m]$$

$$s.t. \quad \omega \ge c, \phi \le c$$

$$P_\omega[m] = \binom{m+\alpha-1}{m} \left[\frac{\beta t \{1 - F(\omega)\}}{1 + \beta t \{1 - F(\omega)\}}\right]^m \left[\frac{1}{1 + \beta t \{1 - F(\omega)\}}\right]^\alpha$$

$$m = 0, 1, 2…\infty$$

Optimizing the model with penalty function: Generally, an optimization problem with constraints can be formulated as a nonlinear programming as below:

$$\min f(x) \quad x \subset S \subset R$$
$$\begin{cases} u_i(x) = 0 & i = 1, 2, ..., p \\ v_j(x) \le 0 & j = 1, 2, ..., q \end{cases}$$

where,
$f(x)$: An objective function
S : Set of the feasible solutions in the feasible region
R : The search space
$v(x)$: The inequality constraint
$u(x)$: The equality constraint

In fact the constraints above are not strictly limited. For example, $v(x) \ge 0$ can also be replaced by $-v(x) \le 0$.

The constraints $u(x) = 0$ can also be expressed with 2 inequality constraints that is $u(x) \geq 0$ and $u(x) \leq 0$. Thus the constraints can be uniformed in the follow formulation: $Z_w(x), w = 1, 2,..., p+q$

One of the effective methods in solving constrained optimization problem is exterior penalty function method. It is a kind of Sequential Unconstrained Minimization Technique (SUMT), the principle of which is to find the global optima by solving a series of unconstrained optimization problem.

Usually, a penalty function is structured as follow:

$$F(x, \psi) = f(x) + \psi \sum_{w=1}^{p+q} [\max\{0, -z_w(x)\}]^2$$

where, ψ is a penalty factor. Then the nonlinear programming is equivalent to an unconstrained optimization, min $F(x, \psi)$

With this method, the model becomes:

$$\min F(\omega, \phi, \psi)$$
$$= -[revenue_s(\omega) - s \times c] + \psi[(\max\{0, c - \omega\})^2 + (\max\{0, \phi - c\})^2]$$
$$= -[s(\omega - c) - (\omega - \phi) \sum_{m=0}^{s-1} (s - m)P_\omega(m)] +$$
$$\psi[(\max\{0, c - \omega\})^2 + (\max\{0, \phi - c\})^2]$$

Although the programming is simplified to a minimum problem with the use of penalty function method, it is still hard to find the optimal combination of the list price and the discount price with the normal numerical methods. Thus we are going to find the global optimal solution with heuristic algorithms.

IMPERIALIST COMPETITIVE ALGORITHM

Imperialist competitive algorithm is a novel evolutionary algorithm for optimization which is a simulation of socio-political process of imperialistic competition. In optimization problem, the algorithm generates several certain amounts of countries at the beginning. Then the countries are divided into 2 types: imperialist states and colonies. To form the initial empires, each imperialist gets a certain numbers of colonies based on their power (the lower the cost the more powerful the country).

In order to simulate the assimilation policy, the colonies of an empire are moved toward the imperialist. This process is shown in Fig. 1. In Fig. 1, the colony moves toward the imperialist by x unit. The direction of the movement is the vector from colony to imperialist. Here x is a random variable with uniform (or any proper) distribution. That is $x \sim U(0, \beta \times d)$ where β is a number greater than 1 and d is the distance between colony and imperialist. β is set to be larger than 1 so that the colonies can get closer to the imperialist state from both sides. In order to search the points around the imperialist, a random amount of deviation is added to
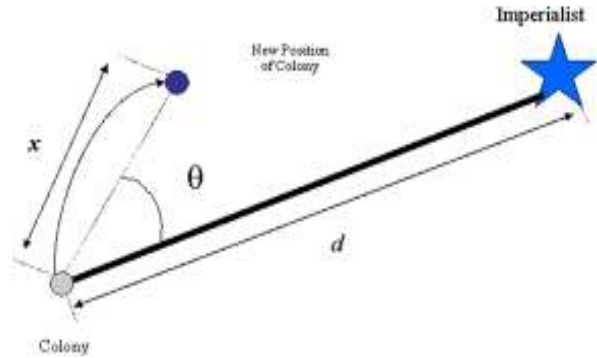

Fig. 1: Movement of colonies toward their relevant imperialist

the direction of movement. Also, θ is a random number with uniform (or any proper distribution). That is $\theta \sim U(-\gamma, \gamma)$ where γ is a parameter that adjusts the deviation from the original direction.

The values of β and γ are arbitrary and can be adjusted according to specified problem. In most situation, $\beta = 2$ and $\gamma = \pi/4$ have resulted in good convergence of countries to the global minimum.

Revolution is an important operator in the algorithm. Revolution causes a country to suddenly change its socio-political characteristics. The colony randomly changes its position in the socio-political axis instead of being assimilated by an imperialist. The revolution rate in the algorithm indicates the percentage of colonies in each colony which will randomly change their position. A very high revolution rate reduces the convergence rate of the algorithm.

A colony may reach to a position with lower cost than that of imperialist. In such a situation, the algorithm will exchange the position of the imperialist and a colony. Then all the colonies start moving toward the new position of the imperialist. This process can be also considered as the imperialist is replaced by one of its colony.

The tendency to occupy more colonies leads to the imperialistic competition. In the algorithm, this process is modeled by picking some of the weakest colonies of the weakest empires and making a competition among all empires to possess these colonies. It is shown in Fig. 2.

However the weakest colonies of the weakest empires are not certainly possessed by the most powerful empire. In other words, each of empires will have a likelihood of taking possession of the mentioned colonies.

After a certain amount of iterations, all the colonies will be under the control of a unique empire and the world achieves unification. In this situation, all the colonies and imperialists are in the same position and have the same cost. Then the algorithm stops.

However, different criteria can be used to stop the algorithm. One idea is to use maximum decades, which is a number of maximum iteration of the algorithm, to stop the algorithm. The algorithm can also be stopped when it has reached the best solution in different
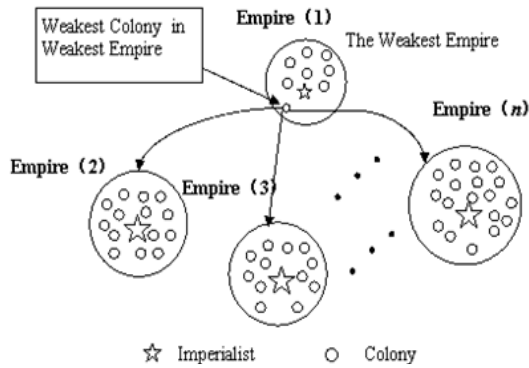
Fig. 2: Imperialistic competition

decades but the solution cannot be improved for some consecutive decades.

COMPUTATIONAL RESULTS

In this section, ICA is applied to a pricing problem for perishable commodities and the results will be discussed. In order to study the capabilities of ICA, the problem will also be solved with Particle Swarm Algorithm (PSO) and a comparison of the 2 algorithms is presented.

A sample pricing problem: Suppose that the seller initially has s items of a fashion coat (perishable product) to be sold during the period $t = 1$. During the sales period, the coats are sold at a price of ω. When the sales period is over (for example the winter is over), the coats are sold at a discount price φ. According to the history data, the arrival rate of the customers' λ is obtained by a gamma distribution. That is $\lambda \sim (3, 2)$; the perceived value of each customer X_i follow a (150, 20) normal distribution.

We are going to find the optimal combination of the ω and φ on a fixed inventory level. Let $s=5, 10, 15, ..., 50$.

According to the model above, we have the nonlinear programming as below:

$$profit_s(\omega) = revenue_s(\omega) - 100s$$
$$= s(\omega - 100) - (\omega - \phi)\sum_{m=0}^{s-1}(s - m)P_\omega[m]$$

$$s.t. \ \omega \geq 100, \ \phi \leq 100$$
$$s = 5,10,15,...,50$$

$$P_\omega[m] = \binom{m+2}{m}[\frac{2\{1-F(\omega)\}}{1+2\{1-F(\omega)\}}]^m[\frac{1}{1+2\{1-F(\omega)\}}]^3$$

The cumulative distribution function used to estimate customers max perceived value at price ω is:

Table 1: Parameters of ICA

Number of country	200
Number of initial empire	8
Number of iteration	250
Assimilation coefficient β	2
Assimilation angle γ	0.5

Table 2: Parameters of PSO

Particle size	200
Number of iteration	250
Accelerated factor 1	1.5
Accelerated factor 2	2.5

Table 3: The optimum list price ω and optimum discount price Ø

Inventory level/s	Optimum ω by ICA	Optimum Φ by ICA	Optimum ω by PSO	Optimum Φ by PSO
5	145.7464	100	145.7211	100
10	141.2469	100	141.1563	100
15	140.1892	100	140.0135	100
20	140.0593	100	139.7964	100
25	140.1129	100	139.7645	100
30	140.1938	100	139.7606	100
35	140.2784	100	139.7602	100
40	140.3637	100	139.7601	100
45	140.4494	100	139.7601	100
50	140.5354	100	139.7601	100

Table 4: Maximum profit

Inventory levels	Maximum profit by ICA	Maximum profit by PSO
5	134.1270	134.1276
10	161.1107	161.1070
15	165.3665	165.3714
20	165.8876	165.8986
25	165.9405	165.9542
30	165.9298	165.9596
35	165.9175	165.9601
40	165.9024	165.9601
45	165.8794	165.9601
50	165.8649	165.9601

$$F(\omega)\frac{1}{\sqrt{40\pi}}\int_{-\infty}^{\omega}e^{-\frac{(x-150)^2}{40}}dx$$

Thus, we have:

$$\min F(\omega, \phi, 10)$$
$$= -[revenue_s(\omega) - s \times 100]$$
$$+ \psi[(\max\{0, 100 - \omega\})^2 + (\max\{0, \phi - 100\})^2]$$
$$= -[s(\omega - 100) - (\omega - \phi)\sum_{m=0}^{s-1}(s - m)P_\omega(m)]$$
$$+ \psi[(\max\{0, 100 - \omega\})^2 + (\max\{0, \phi - 100\})^2]$$

Here the penalty factor $\psi = 10$.

Parameters of the algorithm and results: The minimum problems will be solved with ICA and PSO in this section. The parameters of the ICA and PSO are summarized in Table 1 and 2, respectively.

It is necessary to point out that maximum number of iteration should increase when the unit of the products s increases otherwise the algorithm will fail to find the optimal list price and discount price although reaching the maximum profit successfully. The results are shown in Table 3 and 4.
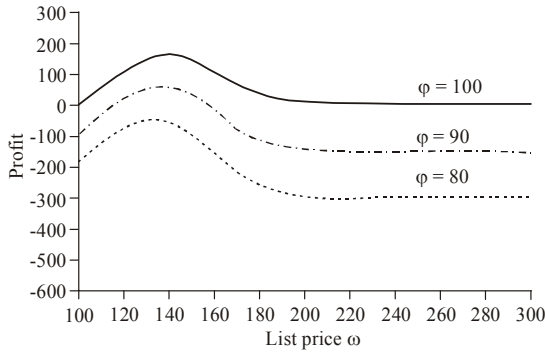
Fig.3: The seller's profit for 15 units of the product at discount price $\varphi = 100, 90\ 80$
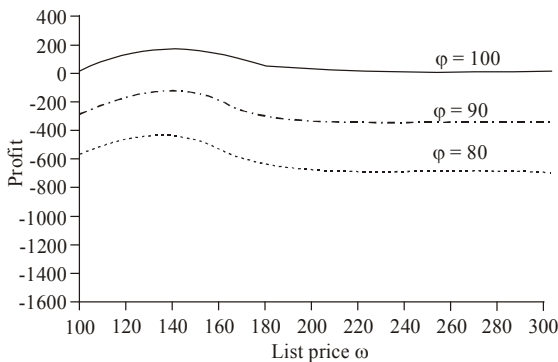

Fig.4: The seller's profit for 35 units of the product at discount price $\varphi = 100, 90\ 80$
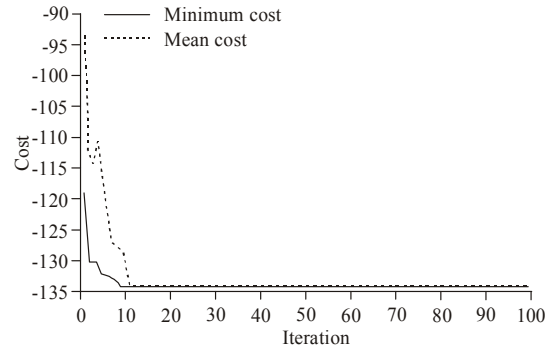

Fig. 5: Mean and minimum cost of the pricing problem when $s = 50$ for ICA
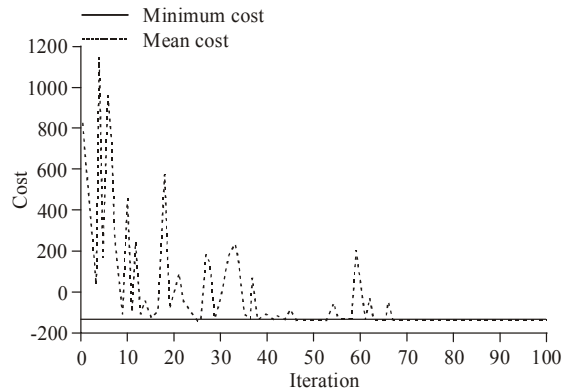
Fig. 6: Mean and minimum cost of the pricing problem when $s = 50$ for PSO

DISCUSSION

From the data, we can find out that with the increase of the inventory level, the optimal list price decreases and the max profit rises. Results by ICA show that when the inventory level is higher than 25, the max profit goes down slightly. Results by PSO indicate that the max profit is reached at the inventory level larger than 30. The difference of the profit is negligible. Both results conform to the economic principle which means the profit does not continuous increase with the inventory level and has a maximum value when the market reaches its equilibrium. Both the results show that the proposed model is workable.

Also from the data, we find out that the optimal discount price should be the cost. Figure 3 and 4 depicts the seller's total profit as a function of the product price ω. We consider 2 cases where the inventory level s is 15 and 35. Profit decreases as shown in Fig. 3 and 4 when the discount price $\varphi = 100, 90$ and 80. Thus 100 is the optimum discount price.

Advantages and perspective of ICA approach: From Table 4, we can find that the maximum profit reached by ICA and PSO are nearly the same and the difference is negligible. The optimum combination of the list price and discount price got by 2 algorithms are also identical. However ICA has a higher convergence rate. As shown in Fig. 5, ICA reach the optimal solution in 10 iterations while it takes almost 70 iterations for the PSO as shown in Fig. 6. Furthermore, as we mentioned in below, the maximum number of the iteration of the algorithms have to increase in order to find the optimal combination as the inventory level rises. However this will take the algorithm longer to solve the model. So the algorithm is required to have high execution speed. ICA excels PSO in execution speed. When $s = 50$, it takes ICA 53.687s to have 800 iterations while PSO has executed for more than a minute at the 35 iterations. It is obvious that ICA will be a more efficient tool in solving real-time pricing problems.

CONCLUSION

In this study, we formulated a multi-criterion pricing model for perishable commodities to find out the best combination of the list price and the discount price. Then, we applied a novel heuristic algorithm called Imperialist Competitive Algorithm (ICA) to solve the problem. Then we examined ICA against another heuristic algorithm PSO. Computational results shows that ICA has high executive speed and find the global optimum with less iteration compared to PSO.

Also computational results indicate that the optimal discount price should be the cost of the commodities.

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