

Research Article

First-Order Reactant in Homogeneous Turbulence Prior to the Ultimate Phase of Decay for Four-Point Correlation in Presence of Dust Particle

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Abstract: In this study, following Deissler's approach the decay in homogeneous turbulence at times preceding to the ultimate phase for the concentration fluctuation of a dilute contaminant undergoing a first-order chemical reaction for the case of four-point correlation in presence of dust particle is studied. Two, three and four-point correlation equations have been obtained and the correlation equations are converted to spectral form by their Fourier-transforms. The terms containing quintuple correlations are neglected in comparison to the third and fourth order correlation terms. Finally, integrating the energy spectrum over all wave numbers, the energy decay of homogeneous dusty fluid turbulent flow for the concentration fluctuations ahead of the ultimate phase for four-point correlation has been obtained and is shown graphically.

Keywords: Deissler's method, dust particle, fourier-transform, navier-stokes equation, turbulent flow

INTRODUCTION

In recent year, the motion of dusty viscous fluids in a rotating system has developed rapidly. The motion of dusty fluid occurs in the movement of dust-laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. The behavior of dust particles in a turbulent flow depends on the concentrations of the particles and the size of the particles with respect to the scale of turbulent fluid.

Deissler (1958, 1960) developed a theory 'on the decay of homogeneous turbulence before the final period.' Using Deissler's theory, Kumar and Patel (1974) studied the first-order reactant in homogeneous turbulence before the final period of decay. Kumar and Patel (1975) extended their problem for the case of multi-point and multi-time concentration correlation. Loeffler and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Batchelor (1953) studied the theory of homogeneous turbulence. Chandrasekhar (1951) obtained the invariant theory of isotropic turbulence in magneto-hydrodynamics. Sarker and Kishore (1991) studied the decay of MHD turbulence before the final period. Sarker and Islam (2001) obtained the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker (2001) also obtained the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Aziz *et al.*

(2009) studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Aziz *et al.* (2010) also extended their previous problem in presence of dust particle. Corrsin (1951) obtained on the spectrum of isotropic temperature fluctuations in isotropic turbulence. Azad *et al.* (2011) studied the statistical theory of certain distribution functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. It is noted that in all above cases, the researcher had considered three-point correlations and through their study they obtained the energy decay law ahead of the ultimate phase.

By analyzing the above theories we have studied the first-order reactant in homogeneous fluid turbulence prior to the ultimate phase of decay for four-point correlation in presence of dust particle. Through this study we would like to find the decay law of first order reactant in homogeneous fluid turbulence in presence of dust particle. Presently we shall try to show in this study the chemical reaction rate $R = 0$ causes the concentration fluctuation of decay in presence of dust particle is more rapidly than for the reaction rate $R \neq 0$ in the absence of dust particle $f = 0$. The comparison between the effects of the chemical reaction in the homogeneous fluid turbulence and the dust particles are graphically discussed. It is observed that the chemical reaction rate increases than the concentration fluctuation to decay more decreases and vice versa.

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MATERIALS AND METHODS

Basic equation: The differential equation governing the concentration of a dilute contaminant undergoing a first-order chemical reaction in homogeneous dusty fluid turbulence could be written as:

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = D \frac{\partial^2 u_i}{\partial x_k \partial x_k} - Ru_i + f(u_i - v_i) \quad (1)$$

where,

- $u_i(\hat{x})$ = A random function of position and time at a point p
- $u_k(\hat{x}, t)$ = The turbulent velocity
- R = The constant reaction rate
- D = The diffusivity
- t = The time
- f = kN/p , dimension of frequency N , Constant number density of dust particle
- m_s = $(4/3)\pi R_s^3 \rho_s$ is the mass of single spherical dust particle of radius R_s
- ρ_s = The constant density of the material in dust particle
- u_i = The turbulent velocity component
- v_i = The dust particle velocity component
- x_k = The space coordinate and repeated subscript in a term indicates a summation of terms, with the subscripts successively taking on the values 1, 2 and 3.

Two-point correlation and spectral equations: Under the restrictions that the turbulence and the concentration fields are homogeneous, the local mass transfer has no effect on the velocity field; the reaction rate and the diffusivity are constant. The differential equation of a dilute contaminant undergoing a first-order chemical reaction in homogeneous system could be written as:

$$\frac{\partial \Gamma}{\partial t} + u_k \frac{\partial \Gamma}{\partial x_k} = D \frac{\partial^2 \Gamma}{\partial x_k \partial x_k} - R\Gamma \quad (2)$$

Subtracting the mean of (2) from Eq. (2), we obtain:

$$\frac{\partial X}{\partial t} + u_k \frac{\partial X}{\partial x_k} = D \frac{\partial^2 X}{\partial x_k \partial x_k} - RX \quad (3)$$

where, $X(\hat{x}, t)$ is the fluctuation of concentration about the mean at a point $p(\hat{x})$ and time t . The two-point correlation for the fluctuating concentration can be written, as:

$$\frac{\partial \langle XX' \rangle}{\partial t} + u_k \frac{\partial \langle XX' \rangle}{\partial x_k} = D \frac{\partial^2 \langle XX' \rangle}{\partial x_k \partial x_k} - R \langle XX' \rangle \quad (4)$$

where, X'_j is the fluctuating concentration at the point $p'(\hat{x})$ and the point p' is at point a distance r from the point p . The symbol $\langle \rangle$ is the ensemble average.

Putting the Fourier transforms:

$$\langle XX'(\tau) \rangle = \int_{-\infty}^{\infty} \theta(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (5)$$

$$\langle u_k XX'(\tau) \rangle = \int_{-\infty}^{\infty} \varphi_k(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (6)$$

$$\langle u'_k XX'(\tau) \rangle = \int_{-\infty}^{\infty} \varphi'_k(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (7)$$

into Eq. (4), one obtains:

$$\frac{\partial \theta}{\partial t} + (Dk^2 + 2R)\theta = ik_k [\varphi_k(\hat{K}, t) - \varphi_k(-\hat{K}, t)] \quad (8)$$

Three-point correlation and spectral equations:

Taking the Navier-Stokes equations for a first-order chemical reaction in homogeneous dusty fluid turbulence at p is:

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = D \frac{\partial^2 u_i}{\partial x_k \partial x_k} - Ru_i + f(u_i - v_i)$$

and the fluctuation equations at p' & p'' one can find the three-point correlation Equation as:

$$\begin{aligned} & \frac{\partial \langle u_i X X'' \rangle}{\partial t} + \frac{\partial \langle u_k u'_k X X'' \rangle}{\partial x_k} + \frac{\partial \langle u_i u'_k X X'' \rangle}{\partial x'_k} + \frac{\partial \langle u_i u''_k X X'' \rangle}{\partial x''_k} \\ & = -\frac{1}{\rho} \frac{\partial \langle p X X'' \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i X X'' \rangle}{\partial x_k \partial x_k} \\ & + D \left(\frac{\partial^2}{\partial x_k \partial x_k} + \frac{\partial^2}{\partial x'_k \partial x'_k} \right) \langle u_i X X'' \rangle - 2R \langle u_i X X'' \rangle \\ & + f(\langle u_i X X'' \rangle - \langle v_i X X'' \rangle) \end{aligned} \quad (9)$$

Using the transformations:

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r'_k} \right) \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial x_k}, \frac{\partial}{\partial x''_k} = \frac{\partial}{\partial r'_k}$$

In order to convert Eq. (9) to spectral form, using six-dimensional Fourier transforms (Kumar and Patel, 1974) and:

$$\langle v_i X'X'' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_i(\hat{K}, \hat{K}', t) \cdot \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}'$$

with the fact that $\langle u_i u_k'' X'X'' \rangle = \langle u_i u_k' X'X'' \rangle$
 one can write Eq. (9) in the form:

$$\begin{aligned} & \frac{\partial \beta_j}{\partial t} + [v(\hat{K} + \hat{K}')^2 + D(\hat{K} + \hat{K}') + 2R - Qf] \beta_j \\ &= -i(k_k + k_k') \psi_{jk} + i(k_k + k_k') \psi_{jk'} + \frac{1}{\rho} i(k_j + k_j') \gamma \end{aligned} \quad (10)$$

where, $\mu_i(\hat{K}, \hat{K}', t) = L \beta_j(\hat{K}, \hat{K}', t)$ and $1-L = Q$, with L and Q arbitrary constants. If the momentum Eq. (3) at p is multiplied by $X'X''$ and divergence of the time averages is taken, the resulting equation will be:

$$\frac{\partial^2 \langle u_i u_k X'X'' \rangle}{\partial x_i \partial x_k} = -\frac{1}{\rho} \frac{\partial^2 \langle p X'X'' \rangle}{\partial x_i \partial x_i} \quad (11)$$

In Fourier space it can be written as:

$$-(k_k + k_k') \psi_{ik'} = \frac{1}{\rho} (k_i + k_i') \gamma \quad (12)$$

Substituting this into Eq. (10), we obtain:

$$\begin{aligned} & \frac{\partial \beta_j}{\partial t} + [v(\hat{K} + \hat{K}')^2 + D(\hat{K} + \hat{K}') + 2R - Qf] \beta_j \\ &= -i(k_k + k_k') \psi'_{jk} \end{aligned} \quad (13)$$

Four-point correlation and spectral equations:

Again by taking the Navier-Stokes equations at p and p' and the concentration equations at p''p''' and following the same procedure as before, one can get the four-point correlations as:

$$\begin{aligned} & \frac{\partial \langle u_i u_j' X'X'' \rangle}{\partial t} + \frac{\partial \langle u_i u_k u_j' X'X'' \rangle}{\partial x_k} + \frac{\partial \langle u_i' u_k' u_j X'X'' \rangle}{\partial x_k'} \\ &+ \frac{\partial \langle u_i u_j' u_k' X'X'' \rangle}{\partial x_k''} + \frac{\partial \langle u_i u_j' u_k'' X'X'' \rangle}{\partial x_k'''} \\ &= -\frac{1}{\rho} \left[\frac{\partial \langle p u_j' X'X'' \rangle}{\partial x_i} + \frac{\partial \langle p' u_i X'X'' \rangle}{\partial x_j'} \right] \end{aligned}$$

$$\begin{aligned} & + v \left(\frac{\partial^2}{\partial x_k \partial x_k} + \frac{\partial^2}{\partial x_k' \partial x_k'} \right) \langle u_i u_j' X'X'' \rangle \\ &+ D \left(\frac{\partial^2}{\partial x_k'' \partial x_k''} + \frac{\partial^2}{\partial x_k''' \partial x_k'''} \right) \langle u_i u_j' X'X'' \rangle \\ &- 2R \langle u_i u_j' X'X'' \rangle \\ &+ f \left(\langle u_i u_j' X'X'' \rangle - \langle v_i u_j' X'X'' \rangle \right) \end{aligned} \quad (14)$$

In order to convert Eq. (14) to spectral form, using the transformations:

$$\begin{aligned} \frac{\partial}{\partial x_k} &= -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k''} \right) \\ \frac{\partial}{\partial x_k'} &= \frac{\partial}{\partial x_k}, \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}, \frac{\partial}{\partial x_k'''} = \frac{\partial}{\partial r_k''} \end{aligned}$$

and nine-dimensional Fourier transforms (Kumar and Patel, 1974) and:

$$\begin{aligned} & \langle v_i u_j' X'X'' \rangle(\hat{r}, \hat{r}', \hat{r}'') \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_{ij}(\hat{K}, \hat{K}', \hat{K}'', t) \\ & \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}' + \hat{K}'' \cdot \hat{r}'')] d\hat{K} d\hat{K}' d\hat{K}'' \end{aligned}$$

and with the fact that:

$$\begin{aligned} & \langle u_k'' u_i u_j' X'X'' \rangle(\hat{r}, \hat{r}', \hat{r}'') \\ &= \langle u_k'' u_i u_j' X'X'' \rangle(\hat{r}, \hat{r}', \hat{r}'') \end{aligned}$$

Eq. (14) can be written as:

$$\begin{aligned} & \frac{\partial g_{ij}}{\partial t} + [v(\hat{K} + \hat{K}' + \hat{K}'')^2 + v\hat{K}^2 \\ &+ D(\hat{K}' + \hat{K}'')^2 + 2R - Sf] g_{ij} \\ &= -i(k_k + k_k' + k_k'') [h_{ijk}(\hat{K}, \hat{K}', \hat{K}'')] \\ &- ik_k [h_{ijk}(-\hat{K} - \hat{K}' - \hat{K}'', \hat{K}', \hat{K}'')] \\ &- i(k_k' + k_k'') h_{ijk}(\hat{K}, \hat{K}', \hat{K}'') \\ &- \frac{1}{\rho} [-(k_i + k_i' + \bar{k}_i'') \Omega_j(\hat{K}, \hat{K}', \hat{K}'') \\ &+ k_j' \Omega_i(-\hat{K} - \hat{K}' - \hat{K}'', \hat{K}', \hat{K}'')] \end{aligned} \quad (15)$$

where, $\mu_{ij}(\widehat{R}, \widehat{K}', \widehat{K}'', t) = M g_{ij}(\widehat{R}, \widehat{K}', \widehat{K}'', t)$ and $1-M = S$, with M and S are an arbitrary constants. Following the same procedure as was used in obtaining (12), we get:

$$\Omega_j(\widehat{K}, \widehat{K}', \widehat{K}'') \quad (16)$$

$$= -\rho \frac{(k_k + k'_k + k''_k)}{(k_i + k'_i + k''_i)} h_{ijk}(\widehat{K}, \widehat{K}', \widehat{K}'')$$

Equation (15) and (16) are the spectral equations corresponding to the four-point correlations. To get a better picture of the of the first-order reactant of homogeneous dusty fluid turbulence decay system from its initial period to its final period, four-point correlations are to be considered. The same ideology could be applied to the concentration phenomenon. Here, we neglect the quintuple correlations since the decay faster than the lower-order correlations. As pointed out by Deissler (1958, 1960) when the quintuple correlations are neglected, the corresponding pressure-force terms which are related to them are also neglected. Under these assumptions, Eq. (15) and (16) give the solution as:

$$g_{ij}(\widehat{K}, \widehat{K}', \widehat{K}'', t)$$

$$= [g_{ij}(\widehat{K}, \widehat{K}', \widehat{K}'', t_0) \exp\{-D[2N_s k^2 + (1+N_s)(k'^2 + k''^2)$$

$$+ 2N_s(k_k k'_k + k'_k k''_k + k_k k''_k)] - 2R - Sf\}(t - t_0)] \quad (17)$$

where, $N_s = v/D$. A relation between $g_{ij}(\widehat{R}, \widehat{K}', \widehat{K}'', t_0)$ and $\varphi'_{ij}(\widehat{R}, \widehat{K}')$ can be found by taking $\widehat{r}'' = 0$ in the expression for $\langle u_i u'_j X'' X'''(\widehat{r}, \widehat{r}', \widehat{r}'') \rangle$ and comparing it with the expression for $\langle u_i u'_k X'' X'''(\widehat{r}, \widehat{r}') \rangle$:

$$\phi'_{ik}(\widehat{K}, \widehat{K}') = \int_{-\infty}^{\infty} g_{ij}(-\widehat{K} - \widehat{K}', \widehat{K}', \widehat{K}'', t) d\widehat{K}'' \quad (18)$$

Substituting this in to Eq. (13), we obtains:

$$\frac{\partial \beta_j}{\partial t} + D[(1 + N_s)(K^2 + K'^2) + 2N_s \widehat{K} \cdot \widehat{K}'] + \frac{2R}{D} - \frac{Sf}{D} \beta_j$$

$$= -i(k_k + k'_k) \int_{-\infty}^{\infty} g_{ij}(-\widehat{K} - \widehat{K}', \widehat{K}', \widehat{K}'', t) d\widehat{K}''$$

which with the help of Eq. (17), gives:

$$\frac{\partial \beta_j}{\partial t} + D[(1 + N_s)(K^2 + K'^2) + 2N_s \widehat{K} \cdot \widehat{K}'] + \frac{2R}{D} - \frac{Sf}{D} \beta_j$$

$$= -2i(k_k + k'_k) [g_{ij}(-\widehat{K} - \widehat{K}', \widehat{K}', \widehat{K}'', t_0)]_0$$

$$\times \frac{a\pi^{3/2}}{D^{3/2}(1 + N_s)^{3/2}(t - t_0)^{3/2}}$$

$$\times \exp \left\{ \begin{aligned} & -D \left[\frac{N_s(2 + N_s)k^2}{(1 + N_s)} + 2N_s k k' \right] \\ & + (1 + N_s)k'^2 + \frac{2R}{D} - \frac{Qf}{D} \end{aligned} \right\} (t - t_0) \quad (19)$$

Now, the solution of Eq. (19) is:

$$\beta_j(\widehat{K}, \widehat{K}', t_0) = [\beta_j(\widehat{K}, \widehat{K}', t_0)]_0$$

$$\times \exp \left\{ \begin{aligned} & -D[(1 + N_s)(K^2 + K'^2) + 2N_s \widehat{K} \cdot \widehat{K}'] \\ & + \left(\frac{2R}{D} - \frac{Qf}{D} \right) \end{aligned} \right\} (t - t_0)$$

$$- \frac{2i(k_k + k'_k) b_i \pi^{3/2}}{D^{3/2}(1 + N_s)^{3/2}}$$

$$\times \left[\frac{-2}{(t - t_0)^{1/2}} + \frac{4kD^{1/2} F(\omega)}{(1 + N_s)^{1/2}} \right]$$

$$\times \exp \left\{ -D \left[\frac{N_s(2 + N_s)k^2}{(1 + N_s)} + 2N_s k k' + (1 + N_s)k'^2 + (-Sf/D) \right] \right\} (t - t_0) \times \exp[-2R(t - t_0)] \quad (20)$$

where,

$$F(\omega) = \exp(-\omega^2) \int_0^{\infty} \exp(n^2) dn$$

$$\omega = k \left[\frac{(t - t_0)D}{91 + N_s} \right]^{1/2}$$

and

$$[g_{ij}(-\widehat{K} - \widehat{K}', \widehat{K}', \widehat{K}'', t_0)]_0 = b_i \quad (21)$$

Now, by taking $\widehat{r}' = 0$ in the expression for $\langle u' X'' X'''(\widehat{r}, \widehat{r}') \rangle$ is performed, we obtain and comparing the result with the expression $\langle X u'_k X'(\widehat{r}) \rangle$:

$$\varphi_k(\widehat{K}) = \int_{-\infty}^{\infty} \beta_k(\widehat{K} \cdot \widehat{K}') dk' \quad (22)$$

Substituting Eq. (21) into (8), we obtain:

$$\frac{\partial G}{\partial t} + (2Dk^2 + 2R)G = W \quad (23)$$

where, $G = 2\pi k^2 \theta$ and:

$$\begin{aligned} W &= 2\pi i k^2 \exp[-2R(t-t_0)] \\ &\times \int_{-\infty}^{\infty} k_k [\beta_k(\hat{K}, \hat{K}') - \beta_k(-\hat{K}, -\hat{K}')]_0 \\ &\times \exp\{-D[(1+N_s)(k^2+k'^2) \\ &+ 2N_s \hat{K} \cdot \hat{K}' + (-Qf/D)](t-t_0)\} \\ &+ \frac{2i\pi^{5/2}}{D^{3/2}(1+N_s)^{3/2}} \int_{-\infty}^{\infty} [b_i(\hat{K}, \hat{K}') - b_i(-\hat{K}, -\hat{K}')] \\ &\times \left[\frac{-2}{(t-t_0)^{1/2}} + \frac{4kF(w)D^{1/2}}{(1+N_s)^{1/2}} \right] \\ &\times \exp\left\{-D\left[\frac{N_s(2+N_s)k^2}{(1+N_s)} + 2N_s k k' + \right. \right. \\ &\left. \left. (1+N_s)k'^2 + (-Sf/D)\right](t-t_0)dk'\right\} \quad (24) \end{aligned}$$

In analogy with the turbulent energy spectrum function, the quantity G, in Eq. (23) can be called concentration energy spectrum function and W, the energy transfer function, is responsible for the transfer of concentration from one wave number to another. In order to find solution completely and following Deissler (1958, 1960) we assume that:

$$\begin{aligned} (2\pi)^2 i k_k [\beta_0(\hat{K}, \hat{K}') - \beta_0(-\hat{K}, -\hat{K}')]_0 \\ = -\xi_0(k^2 k'^4 - k^4 k'^2) \end{aligned}$$

And

$$\begin{aligned} \frac{8\pi^{7/2}}{D^{3/2}(1+N_s)^{3/2}} i [b(\hat{K}, \hat{K}') - b(-\hat{K}, -\hat{K}')]_0 \\ = -\xi_1(k^4 k'^6 - k^6 k'^4), \end{aligned}$$

where, ξ_0 and ξ_1 are arbitrary constants depending on the initial conditions. If these results are used in Eq. (24) and the integration is performed, we obtain:

$$W = -\frac{\xi_0 \pi^{1/2} N_s}{2(t-t_0)^{3/2} D^{3/2} (1+N_s)^{5/2}}$$

$$\begin{aligned} &\times \exp\left\{-D\left[\frac{1+2N_s}{1+N_s}\right]k^2 + (-Qf/D)\right\}(t-t_0)\} \\ &\times \left\{ \frac{15k^4}{4D^2 N_s^2 (t-t_0)^2} \left(\frac{N_s}{1+N_s}\right) + \left[5\left(\frac{N_s}{1+N_s}\right)^2 \right. \right. \\ &\left. \left. - \frac{3}{2}\right] \frac{k^6}{N_s D(t-t_0)} + \left[\left(\frac{N_s}{1+N_s}\right)^3 - \left(\frac{N_s}{1+N_s}\right)\right]k^8 \right\} \\ &+ \frac{\xi_1 \pi^{1/2} N_s}{(t-t_0)^{7/2} D^{3/2} (1+N_s)^{7/2}} \\ &\times \exp\left\{-2D\left[\frac{N_s}{1+N_s}\right]k^2 + \left(-\frac{Sf}{2D}\right)\right\}(t-t_0)\} \\ &\times \left\{ \frac{105k^6}{8D^3 N_s^2 (t-t_0)^3} \right. \\ &\left. \left(\frac{N_s}{1+N_s}\right) + \frac{15k^8}{4D^3 N_s (t-t_0)^2} \right. \\ &\times \left[7\left(\frac{N_s}{1+N_s}\right)^2 - 1\right] + \left(\frac{N_s k^{10}}{D(1+N_s)(t-t_0)}\right) \\ &\times \left[\frac{21}{2}\left(\frac{N_s}{1+N_s}\right)^2 - 5\right] + \frac{N_s^2 k^{12}}{(1+N_s)^2 (t-t_0)} \left[\left(\frac{N_s}{1+N_s}\right)^2 - 1\right] \right. \\ &\left. - \frac{2\xi_1 \pi^{1/2} N_s}{(t-t_0)^{3/2} D(1+N_s)^4} \right. \\ &\left. \times \exp\left\{-D\left[\frac{1+2N_s}{1+N_s}\right]k^2 + \left(-\frac{Sf}{D}\right)\right\}(t-t_0)\} \right. \\ &\left. \times \left\{ \frac{105k^7}{8D^3 N_s^2 (t-t_0)^3} \right. \right. \\ &\left. \left(\frac{N_s}{1+N_s}\right) + \frac{15k^9}{4D^3 N_s (t-t_0)^2} \right. \\ &\times \left[7\left(\frac{N_s}{1+N_s}\right)^2 - 1\right] + \left(\frac{N_s k^{11}}{D(1+N_s)(t-t_0)}\right) \\ &\times \left[\frac{21}{2}\left(\frac{N_s}{1+N_s}\right)^2 - 5\right] + \frac{N_s^3 k^{13}}{(1+N_s)^2} \left[\left(\frac{N_s}{1+N_s}\right)^2 - 1\right] \right. \\ &\left. \times \int_0^{\infty} \exp(-n^2) dn \right\} \exp[-2R(t-t_0)] \quad (25) \end{aligned}$$

It is very interesting that:

$$\int_{-\infty}^{\infty} W \, dk = 0 \tag{26}$$

This indicates that the expression for W satisfies the condition of continuity and homogeneity. Physically it was to be expected, since W is the measure of transfer of energy and the total energy transformed to all wave numbers is to be zero, which is what Eq. (26) gives. The linear Eq. (23) can be solved to give:

$$G = J(k) \exp \left[- (2 D k^2 + 2 R)(t - t_0) \right] + \int W \times \exp \left[- (2 D k^2 + 2 R)(t - t_0) \right] dt \tag{27}$$

where, $J(k) = \frac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as (Corrsin, 1951).

Thus, by integrating the right-hand side of equation, we obtain:

$$\begin{aligned} G = & \exp[-2R(t-t_0)] \left\{ \frac{N_0(1+N_s)\varepsilon^2}{D\pi(t-t_0)} \right. \\ & \times \exp\left[-2(1+N_s)\eta^2 + \left(\frac{Qf}{D}\right)\right] \\ & + \frac{\xi_0 \pi^{1/2} N_s}{2(t-t_0)^{9/2} D^{11/2} 2(1+N_s)^{3/2}} \\ & \times \exp\left[-(1+2N_s)\varepsilon^2 + \left(\frac{Qf}{D}\right)\right] \\ & \times \left[\frac{3\varepsilon^4}{2N_s} + \frac{(7N_s-6)\varepsilon^6}{3} - f_1\varepsilon^8 + 2f_1\varepsilon^9 F(\varepsilon) \right] \\ & - \frac{\xi_1 \pi^{1/2}}{2(t-t_0)^7 D^{15/2} (1+N_s)^{3/2}} \\ & \times \left[\frac{105\varepsilon^6}{32} + \frac{5}{16}(11-24N_s^2+8N_s)\varepsilon^8 + f_2\varepsilon^{10} \right. \\ & \left. + f_3\varepsilon^{12} - 2f_3\varepsilon^{14} \exp(-2\varepsilon^2) \right. \\ & \left. [E_i(2\varepsilon^2) - 0.5772] + (Sf/D) \right] \\ & - \frac{2\xi_1 \pi^{1/2} N_s}{2(t-t_1)^7 D^{15/2} 2(1+N_s)^{3/2}} \end{aligned}$$

$$\begin{aligned} & \times \exp \left\{ - [(1+2N_s)\varepsilon^2 + 2 + (Sf/D)] \right. \\ & \times \left\{ \frac{105}{8N_s} \left[-\frac{\varepsilon^8}{3} - \frac{2\varepsilon^{10}}{3} - \frac{23\varepsilon^{12}}{30} + \frac{498\varepsilon^{14}}{630} \right] \right. \\ & \times \exp[(-2\varepsilon^2) + (Sf/D)][E_i(2\varepsilon^2) - 0.5772] \\ & + 3.1808 \times 10^{-2} \varepsilon^{14} - 2.7178 \times 10^{-2} \varepsilon^{16} \\ & + 1.3968 \times 10^{-2} \varepsilon^{18} - 4.6204 \times 10^{-3} \varepsilon^{20} \\ & + 1.1584 \times 10^{-3} \varepsilon^{22} - 2.3139 \times 10^{-4} \varepsilon^{24} \\ & + 3.8589 \times 10^{-5} \varepsilon^{26} - 5.5108 \times 10^{-4} \varepsilon^{28} \\ & + 1.1584 \times 10^{-3} \varepsilon^{22} - 2.3139 \times 10^{-4} \varepsilon^{24} \\ & + 6.8898 \times 10^{-7} \varepsilon^{30} - \dots] \\ & - \frac{15(N_s^2 - 2N_s - 1)}{4N_s} \left[-\frac{\varepsilon^{10}}{2} - \frac{5\varepsilon^{12}}{6} + \frac{14\varepsilon^{14}}{15} \right] \times \exp \\ & [(-2\varepsilon^2) + (Sf/D)][E_i(2\varepsilon^2) - 0.5772] \\ & + 2.0261 \times 10^{-2} \varepsilon^{14} + 3.5487 \times 10^{-3} \varepsilon^{16} \\ & + 5.4047 \times 10^{-4} \varepsilon^{18} + 7.2369 \times 10^{-5} \varepsilon^{20} \\ & + 8.6172 \times 10^{-6} \varepsilon^{22} + 9.22057 \times 10^{-7} \varepsilon^{24} \\ & + 8.9477 \times 10^{-8} \varepsilon^{26} + 7.9374 \times 10^{-9} \varepsilon^{28} \\ & + 6.4814 \times 10^{-10} \varepsilon^{30} + \dots] \\ & + (11N_s^2 - 20N_s - 10)N_s \left[-\varepsilon^{12} + \frac{4}{3}\varepsilon^{14} \right] \times \exp \\ & [(-2\varepsilon^2) + (Sf/D)][E_i(2\varepsilon^2) - 0.5772] \\ & + 8.2505 \times 10^{-2} \varepsilon^{14} + 1.7495 \times 10^{-2} \varepsilon^{16} \\ & + 3.1572 \times 10^{-3} \varepsilon^{18} + 4.90805 \times 10^{-4} \varepsilon^{20} \\ & + 6.66926 \times 10^{-5} \varepsilon^{22} + 8.0290 \times 10^{-6} \varepsilon^{24} \\ & + 8.6642 \times 10^{-7} \varepsilon^{26} + 8.4643 \times 10^{-8} \varepsilon^{28} \\ & + 7.5495 \times 10^{-9} \varepsilon^{30} + \dots] \\ & - (1+2N_s)N_s^3 \left[\varepsilon^{14} \exp[(-\varepsilon^2) + (Sf/D)] \right] \times \end{aligned}$$

$$\begin{aligned}
 & [E_i(2\varepsilon^2) - 0.5772] \\
 & + 2.2654 \times 10^{-1} \varepsilon^{14} + 1.0679 \times 10^{-1} \varepsilon^{16} \\
 & - 3.3996 \times 10^{-3} \varepsilon^{18} + 9.0694 \times 10^{-3} \varepsilon^{20} \\
 & - 1.1099 \times 10^{-3} \varepsilon^{22} + 3.7350 \times 10^{-4} \varepsilon^{24} \\
 & - 4.444 \times 10^{-5} \varepsilon^{26} + 8.2388 \times 10^{-6} \varepsilon^{28} \\
 & - 8.47486 \times 10^{-7} \varepsilon^{30} + \dots] \}} \}} \quad (28)
 \end{aligned}$$

where,

$$\begin{aligned}
 \varepsilon^2 &= D \frac{(t-t_1)k^2}{(1+N_s)}, f_1 = \frac{4}{3}(3N_s^2 - 2N_s + 3) \\
 f_2 &= \left(\frac{15 - 40N_s^2(2N_s + 1) + 4N_s^2(11N_s^2 - 20N_s)}{16(1+N_s)^2 D} \right) \\
 f_3 &= \left(\frac{15 + 20(4N_s^2 - 2N_s) + 4N_s^2(9N_s^2 - 20N_s) - 16N_s^2}{8(1+N_s)^3} \right)
 \end{aligned}$$

In a turbulent phenomenon, such as turbulent kinetic energy, we associate the so-called concentration energy with the fluctuating concentration, defined by the relation:

$$\frac{1}{2} \langle XX' \rangle = \int_{-\infty}^{\infty} G(k, t) dk \quad (29)$$

The substitution of Eq. (28) and subsequent integration with respect to k leads to the result:

$$\begin{aligned}
 & \frac{1}{2} \langle X^2 \rangle = \exp[-2R(t-t_0)] \\
 & \times \left[\begin{aligned} & A(t-t_0)^{\frac{3}{2}} + B(t-t_0)^{-5} \exp[R_2(t-t_0)] \\ & \left[C(t-t_0)^{-\frac{15}{2}} - D_1(t-t_0)^{-\frac{15}{2}}(t-t_1)^{\frac{1}{2}} - E(t-t_0)^{-15} \right. \\ & \left. - F(t-t_0)^{-\frac{29}{2}} - G(t-t_0)^{-15}(t-t_1)^{\frac{1}{2}} \right] \exp[R_3(t-t_0)] \end{aligned} \right] \quad (30)
 \end{aligned}$$

This is the decay law of first order reactant in homogeneous turbulence in presence of dust particle, where,

$$A = \frac{N_0}{D^{\frac{3}{2}} 2^{\frac{7}{2}} \pi^{\frac{1}{2}}}, B = \frac{\xi_0 R_1}{D^6}, C = cd, D_1 = ce,$$

$$E = cf'(gh + gj_1 - k_1 l_1 - k_1 n + pq + ps - t'vw)$$

$$F = cf'(gi_1 - k_1 m - ht'), G = cf'pr_1$$

$$c = \frac{15 \pi \xi_1}{D^8 2^{15/2} N_s^{7/2} (1+N_s)}$$

$$d = \frac{105}{32} + \frac{35(11 - 24N_s^2 + 8N_s)}{64N_s} + \frac{63f_2}{8N_s^2} +$$

$$\frac{693f_3}{64N_s^3} - \frac{1808f_3}{256} + \frac{41.5910f_3 N_s^{\frac{7}{2}}}{(1+N_s)^{\frac{1}{2}}}$$

$$e = 2f_3 \frac{D^{\frac{1}{2}} I(-2N_s \varepsilon^2, \varepsilon)^2}{(1+N_s)^{\frac{1}{2}} \pi^{\frac{1}{2}}}$$

$$f' = \frac{37.12311 \xi_1 N_s \pi^{\frac{1}{2}}}{D^8 \varepsilon^2 (1+N_s)}, g = \frac{105}{2N_s}$$

$$h = -\frac{1}{3} - \frac{6}{\varepsilon_1} - \frac{75.90}{\varepsilon_1^2} - \frac{2.52052 \times 10^{-3} \varepsilon_1^{\frac{9}{2}}}{(1+N_s)^{\frac{15}{2}}}$$

$$i_1 = 1.118 \varepsilon_1^{\frac{9}{2}} \frac{(1+N_s)^{\frac{1}{2}}}{D} \times I[(1+2N_s)\varepsilon, \varepsilon^2]$$

$$j_1 = \frac{163}{\varepsilon_1^3} (3.1808 \times 10^{-2} - 4.0767 \times 10^{-1} \varepsilon_1^{-1} + 3.561804 \varepsilon_1^{-2}$$

$$- 2.2386 \times 10 \varepsilon_1^{-3} + 1.1786 \times 10^2 \varepsilon_1^{-4} - 5.4148 \times 10^2 \varepsilon_1^{-5}$$

$$+ 2.02576 \times 10^3 \varepsilon_1^{-6} - 8.7048 \times \varepsilon_1^{-7}$$

$$+ 3.1561 \times 10^4 \varepsilon_1^{-8} - \dots)$$

$$k_1 = \frac{135(6N_s^2 - 2N_s - 1)}{N_s \varepsilon_1}, l_1 = -\frac{1}{2} - \frac{55}{6\varepsilon_1}$$

$$- \frac{2.3510 \times 10^{-3} \times 163 \varepsilon_1^{\frac{11}{2}}}{2^{15} (1+N_s)^{\frac{15}{2}}}$$

$$m = 7.4469 \varepsilon_1^{\frac{11}{2}}$$

$$\begin{aligned} & \times \frac{(1+N_s)^{\frac{1}{2}}}{D} I[(1+2N_s)\varepsilon^2, \varepsilon^2] \\ n &= \frac{163}{\varepsilon_1^2} (2.0261 + 5.3230 \times 10^{-2} \varepsilon_1^{-1} \\ & + 1.3782 \times 10^{-1} \varepsilon_1^{-2} + 3.5063 \times 10^{-1} \varepsilon_1^{-3} \\ & + 8.7676 \times 10^{-1} \varepsilon_1^{-4} + 2.57710^{-1} \varepsilon_1^{-5} \\ & + 5.2347 \varepsilon_1^{-6} + 1.2538 \varepsilon_1^{-7} + 2.9690 \times 10 \varepsilon_1^{-8} + \dots) \\ p &= 396(11N_s^2 - 20N_s - 10)N_s \varepsilon_1^{-2} \\ q &= -1 - 3.0532 \times 10^{-4} \varepsilon_1^{\frac{13}{2}} (1+N_s)^{\frac{-15}{2}} \\ r_1 &= 1.0638 \varepsilon_1^{\frac{13}{2}} (t-t_1)^{\frac{1}{2}} \frac{(1+N_s)^{\frac{-1}{2}}}{D} \\ & \times I[(1+2N_s)\varepsilon^2, \varepsilon^2] \\ S &= 13 \varepsilon_1^{-1} (8.2504 \\ & + 2.6243 \times 10^{-1} \varepsilon^{-1} + 8.0509 \times 10^{-1} \varepsilon^{-2} \\ & + 2.3780 \times \varepsilon_1^{-3} + 6.7856 \varepsilon_1^{-4} + 1.8789 \times 10 \varepsilon_1^{-5} \\ & + 5.0688 \times 10^1 \varepsilon_1^{-6} + 1.3370 \times 10^2 \varepsilon_1^{-7} \\ & + 3.4583 \times 10^2 \varepsilon_1^{-8} + \dots) \\ t' &= -1.4219 N_s^2 \varepsilon_1^{\frac{11}{2}} (1+N_s)^{\frac{1}{2}} \\ u &= \left\{ \left(-3.1886 \times 10^{-3} + 1.4443 \times 10^2 (t-t_0)^{\frac{1}{2}} \right. \right. \\ & \left. \left. \times D^{\frac{1}{2}} (1+N_s)^{\frac{-1}{2}} I[(1+2N_s)\varepsilon^2, \varepsilon^2] \right\} \\ v &= 1.8102 \times 10^2 (1+2N_s)^{\frac{1}{2}} \varepsilon_1^{\frac{-15}{2}} \\ w &= (2.2653 \times 10^{-1} + 1.60195 \varepsilon_1^{-1} \\ & - 8.6664 \times 10^{-1} \varepsilon^{-2} + 4.3941 \times 10 \varepsilon_1^{-3} \end{aligned}$$

$$\begin{aligned} & -1.1293 \times 10^2 \varepsilon_1^{-4} + 8.7405 \times 10^2 \varepsilon_1^{-5} \\ & - 2.6001 \times 10^3 \varepsilon_1^{-6} + 1.3013 \times 10^4 \varepsilon_1^{-7} \\ & - 3.8822 \times 10^4 \varepsilon_1^{-8} + \dots) \\ R_1 &= \frac{\pi}{2(1+N_s)(1+2N_s)^{5/2}} \left[\frac{9}{16} + \frac{5N_s(7N_s-6)}{16(1+2N_s)} \right. \\ & \left. - \frac{105f_1N_s}{32(1+2N_s)} + \frac{f_1N_s(1+2N_s)^{5/2}}{2^{21/2}(1+N_s)^{11/2}} \right. \\ & \left. \times \sum_{n=0}^{\infty} \frac{1.3.5...(2n+9)}{n!(2N_s+1)2^{2n}(1+N_s)^n} \right] \varepsilon_1 = 2(1+2N_s) \\ R_2 &= (Qf/D) \quad R_3 = (Sf/D) \end{aligned}$$

f = constant number density dust particle and

$$I[-\alpha\varepsilon^2, \beta\varepsilon^2] = \int_0^{\infty} \frac{\exp(-\alpha\varepsilon^2)}{[\varepsilon^{14} \exp(\beta\varepsilon^2) E_i(\beta\varepsilon^2)]} dk$$

RESULTS AND DISCUSSION

The first term of Eq. (30) corresponds to the concentration energy for two-point concentration; the second term represents first order reactant in homogeneous fluid turbulence in presence of dust particle for three-point correlation. The expression $\exp[R_2(t-t_0)]$ represents the dusty fluid turbulence for three-point correlation; the expression $\exp[R_3(t-t_0)]$ and the remainder are due to fluid turbulence in presence of dust particle for four-point correlation. In Eq. (30) we obtained the decay law of first order reactant in homogeneous dusty fluid turbulence for four-point correlation after neglecting quintuple correlation terms. The equation contains the terms $(t-t_0)^{-3/2}, (t-t_0)^{-5}, (t-t_0)^{-15/2}$. Thus, the terms associated with the higher-order correlation die out faster than those associated with the lower-order ones. Therefore, the assumption that the higher-order correlations can be neglected in comparison with lower-order correlations seems to be valid in our case. If the fluid is clean (*i.e.*, $f = 0$) the Eq. (30) becomes:

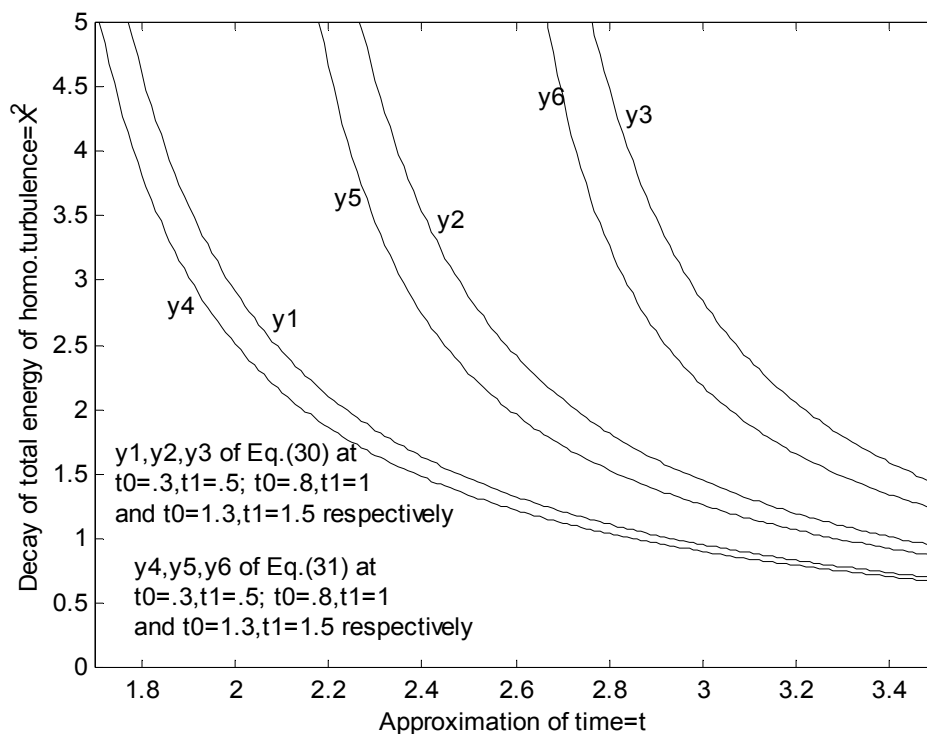


Fig. 1: Comparison between Eq. (30) and (31) if $R = 0.25$

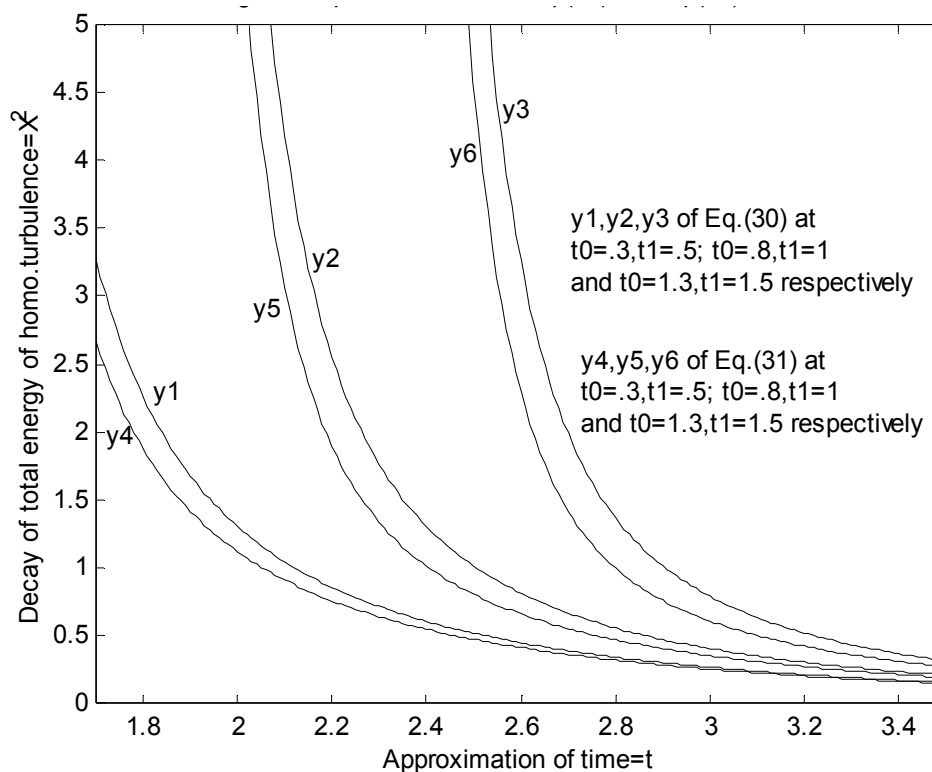


Fig. 2: Comparison between Eq. (30) and (31) if $R = 0.5$

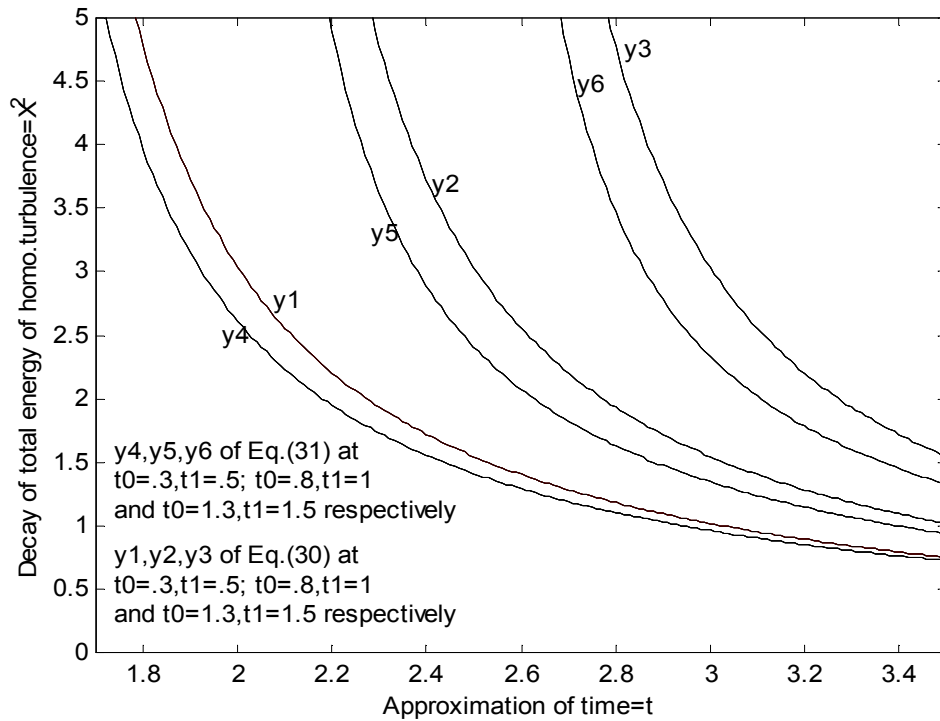


Fig. 3: Comparison between Eq. (30) and (31) if $R = 0$

$$\frac{1}{2} \langle X^2 \rangle = \exp[-2R(t-t_0)] \times [A(t-t_0)^{-\frac{3}{2}} + B(t-t_0)^{-5} - \left\{ C(t-t_0)^{-\frac{15}{2}} - D_1(t-t_0)^{\frac{15}{2}}(t-t_1)^{\frac{1}{2}} - \left[E(t-t_0)^{-15} - F(t-t_0)^{-\frac{29}{2}} - G(t-t_0)^{-15}(t-t_1)^{\frac{1}{2}} \right] \right\}] \quad (31)$$

this is obtained by Kumar and Patel (1974). Here A, B, C, D_1, E, F, G are constants which can be determined. With $R = 0$ and the contaminant replaced by the temperature, the results show complete argument with the result obtained by Loeffler and Deissler (1961) for the decay of temperature fluctuation in homogeneous turbulence before the final period up to three-point correlations. For large times, the last terms become negligible and give the-3/2 power decay law for the final period. In figures, Eq. (30) represented by the curves y_1, y_2, y_3 and (31) by y_4, y_5 and y_6 , respectively. For $R_2 = 0.25$ and $R_3 = 0.25$, the comparison between the Eq. (30) and (31) are shown in Fig. 1, 2 and 3 corresponding to the values $R = 0.025, 0.5$ and 0 , respectively, the energy decays rapidly in presence of dust particle of homogeneous fluid

turbulence than clean fluid which indicates in the figures clearly.

CONCLUSION

From the figures and discussions, this study shows that the terms associated with the higher-order correlations die out faster than those associated with the lower-order ones and if the chemical reaction rate increases than the concentration fluctuation to decay more decreases and vice versa. At the chemical reaction rate $R = 0$ of homogeneous fluid turbulence in presence of dust particle causes the concentration fluctuation of decay more rapidly than they would for the chemical reaction rate $R \neq 0$ and $f = 0$. Also we conclude that due to the effect of homogeneous turbulence in the flow field of the first order chemical reaction for four-point correlation in presence of dust particle prior to the ultimate phase of decay, the turbulent energy decays more rapidly than the energy decay for the first order reactant in homogeneous turbulence before the final period.

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