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Research Article

Improving Forecasts of Generalized Autoregressive Conditional Heteroskedasticity with Wavelet Transform

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Abstract: In the study, we discussed the generalized autoregressive conditional heteroskedasticity model and enhanced it with wavelet transform to evaluate the daily returns for 1/4/2002-30/12/2011 period in Brent oil market. We proposed discrete wavelet transform generalized autoregressive conditional heteroskedasticity model to increase the forecasting performance of the generalized autoregressive conditional heteroskedasticity model. Our new approach can overcome the defect of generalized autoregressive conditional heteroskedasticity family models which can't describe the detail and partial features of times series and retain the advantages of them at the same time. Comparing with the generalized autoregressive conditional heteroskedasticity model, the new approach significantly improved forecast results and greatly reduces conditional variances.

Keywords: Brent oil, daily returns, DWT-GARCH, GARCH, volatility, wavelet transform

INTRODUCTION

The GARCH (abbr. Generalized Autoregressive Conditional Heteroskedasticity) family models found many important and significant characteristics of returns in financial assets. These characteristics could be stated as the following: heteroscedasticity and volatility clustering property (Engle, 1982; Bollerslev, 1986); asymmetric relation property (Nelson, 1991) and nonlinearity property (Higgens and Bera, 1992; Klaassen, 2002). Due to the advancement, GARCH family models have rapidly expanded. However, via literature review, we found that the GARCH family models could better simulate the nonlinear characteristics of time series, but it could not describe the detail and partial features of time series. Thus they lose a lot of information when modeling. In order to overcome this shortcoming of GARCH family models, we developed DWT-GARCH (abbr. Discrete Wavelet Transform GARCH) model by combining discrete wavelet transform with GARCH model to predict daily returns in Brent oil market.

Engle designed an ARCH (Abbr. Autoregressive Conditional Heteroscedasticity) model to estimate conditional variance of inflation sequence of the UK in 1982. The model assumed that the variance of random errors associated with the pre-random error and the model included the mean equation and the conditional variance equation. Via improving the ARCH model, there are a number of improved models, such as GARCH (Bollerslev, 1986), ARCH-in-mean (Engle *et al*., 1987), exponential GARCH (Nelson, 1991), nonlinear ARCH (Higgens and Bera, 1992), threshold ARCH (Glosten *et al*., 1993), dual-threshold GARCH (Yang and Chang, 2008). PARCH (Ding *et al*., 1993), long memory GARCH (Ding and Granger, 1996) and FIGARCH (abbr. fractional integrated GARCH) (Bailie *et al*., 1996). And vector GARCH are also developed.

In the applications, there are also lots of researches. Korkmaz *et al*. (2009) used FIGARCH model to test long-term memory of stock returns of Istanbul Stock Exchange. Their research shows that the market does not have weak validity. Liu and Bruce (2009) used the family of GARCH models, including ACGARCH model (abbr. asymmetric component GARCH), to forecast volatility of Hang Seng index. Their research shows that the family of GARCH models also shows "robustness" when using abnormal distribution data to do volatility forecast. Different news has different impact on markets. Impact of general news on markets is gentle, but some great news will cause sharp jump of prices. Das and Sundaram (1999) thought the models' setting will have deviation if not consider characteristic of volatility jumping. Chan and Maheu (2002) developed GARCH model which can capture the jump intensity with time-varying. The model can well simulate the price jump. Time series often have characteristics of periodicity and seasonality. Kyrtsou and Terraza (2003) proposed a GARCH model related to a seasonal variable, established a more general

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Seasonal Mackey-Glass-GARCH model (Kyrtsou, 2005) and used this model to explain the long-term, short-term memories and nonlinear structure of return series (Kyrtsou and Terraza, 2010). Racicot *et al.* (2008) used ultra high frequency GARCH model to predict data volatility of irregular time intervals. And they compared it with the forecast effect of realized volatility model (Bollerslev and Wright, 2001). The results show that forecast effect has not been improved. So they thought that combine artificial neural network with family of GARCH model or use spline function before modeling to carry on the pretreatment for the data may improve the forecast effect. Wang (2009) established a hybrid model which has used grey theory, non-linear neural network and Glosten-Jagannathan-Runkle GARCH and forecasted the stock index options returns in Taiwan Futures Exchange. Bildirici and Ersin (2009) combined artificial neural networks with GARCH family models to establish ANN-GARCH (abbr. artificial neural networks GARCH) models and separately use GARCH family models and ANN-GARCH models to study volatility characteristics of daily return rate in Istanbul Stock Exchange. Their results show that only partial models' forecast accuracy of GARCH family model has been improved by network learning. Zhao *et al.* (2010) established Multiscale-IGARCH model by combining multi-scale analysis with IGARCH model and forecasted the China's soya futures margin.

MODEL AND DATA

DWT-GARCH model: We recorded the daily returns as r_t ($r_t = ln(P_t)-ln(P_{t-1})$) and chose GARCH models to predict r_t. General GARCH models can only describe the overall volatility features of the series, but cannot describe partial volatility features of the series and multi-scale information. Ignoring partial volatility features and multi-scale information of the time series will affect the precision of the model. To take advantage of prices' partial volatility features and multi-scale information of variables when forecasting, we established DWT-GARCH model by combining wavelet analysis theory with the GARCH. In accordance with the principle of DWT, we decompose r_t into different time scales, as model (1) shows:

$$r_t = r_{A^j,t} + \sum_1^J r_{D^j,t} \tag{1}$$

where,

$r_{A^j,t}$ = The approximate part of r_t

$r_{D^j,t}$ = The detail part

J = The layers of decomposition

According to the principle of multi-scale analysis, series in each time scale after decomposing are orthogonal ($\bullet$ denotes point multiplication between vectors), that is:

$$r_{A^j} \bullet r_{D^i} = 0 \, , r_{D^j} \bullet r_{D^i} = 0 \tag{2}$$

Discrete wavelet expression of model (1) is as follows:

$$r_t = \sum_1^k a_{J,k}\Phi_{J,k}(t) + \sum_1^k d_{J,k}\Psi_{J,k}(t) + \sum_1^k d_{J-1,k}\Psi_{J-1,k}(t) + \ldots \sum_1^k d_{1,k}\Psi_{1,k}(t) \tag{3}$$

where,

J≤2logN

k = $N/2^j$

N = The number of sample observations of r_t

Φ(t) = Scaling function

Ψ(t) = Wavelet function

'a' = Scaling coefficients to capture trend of series r_t, which can be obtained by the following formula:

$$a_{j,k} = \int \Phi_{j,k}(t) r_t dt \tag{4}$$

'd' denotes wavelet coefficients, used to capture the partial features of series r_t, which can be obtained by the following formula:

$$d_{j,k} = \int \Psi_{j,k}(t) r_t dt \tag{5}$$

Here, $r_{A^j} = \sum_1^k a_{J,k}\Phi_{J,k}(t)$ and

$$\sum_1^J r_{D^j} = \sum_1^k d_{J,k}\Psi_{J,k}(t) + \sum_1^k d_{J-1,k}\Psi_{J-1,k}(t) + \ldots + \sum_1^k d_{1,k}\Psi_{1,k}(t).$$

Let $r_{A^j,t} = \mu_{A^j,t} + \varepsilon_{A^j,t}$, $r_{D^j,t} = \mu_{D^j,t} + \varepsilon_{D^j,t}$ (6)

In model (6), suppose the residuals $\varepsilon_{Aj,t}$ and $\varepsilon_{Dj,t}$ are independent and:

$$\varepsilon_{A^j,t} = e_{A^j,t}\sqrt{h_{A^j,t}} \, , \; \varepsilon_{D^j,t} = e_{D^j,t}\sqrt{h_{D^j,t}}$$

Put model (6) into model (1), we get the model (7) which is a mean equation of DWT-GARCH (p, q). Corresponding variance equations are showed in model (8) and model (9):

$$r_t = \mu_{A^j,t} + \sum_1^J \mu_{D^j,t} + \varepsilon_{A^j,t} + \sum_1^J \varepsilon_{D^j,t} \tag{7}$$

$$h_{A^j,t} = \kappa_{A^j} + \sum_1^p \alpha_{A^j,i}\varepsilon^2_{A^j,t-i} + \sum_1^q \beta_{A^j,l} h_{A^j,t-l} \tag{8}$$

$$h_{D^j,t} = \kappa_{D^j} + \sum_1^p \alpha_{D^j,i}\varepsilon^2_{D^j,t-i} + \sum_1^q \beta_{D^j,l} h_{D^j,t-l} \tag{9}$$

DWT-GARCH can dig up self-correlation characteristics existing in multi-scale, thereby improve accuracy for forecasting. Record conditional variances obtained via DWT-GARCH as h_DWT. DWT can enhances the prediction accuracy of GARCH models and makes predictive value of r_t approach the actual value.

In order to compare prediction accuracy of DWT-GARCH with GARCH, these 2 kinds of models are used to conduct empirical research. When modeling with approximate series and detail series, we found that assumption of t distribution cannot pass the test in most cases. Therefore, we supposed the data follow the GED distribution. After model debugging, AR-GARCH (1, 1) can be written as follows:

The mean equation is as model (10):

$$r_t = \sum_{i=1}^{n} \theta_i AR(i) + \varepsilon_t \tag{10}$$

The conditional variance model is as model (11):

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{11}$$

Data: In dealing with high-frequency data, as density of singular point is very large and vanishing moments cannot be too high, therefore we chose db5 function to do wavelet decomposition. Via repeated test on high-frequency data about stock prices and returns, Ma *et al.* (2007) suggests the number of decomposition layer be no greater than 3 for they have many high-frequency components. We set the number of decomposition layer as 2. The 3 series obtained by decomposing were recorded as a2, d2 and d1. Here, a2 is the approximate series; d2 and d1 are detail series. Figure 1 showed the results of wavelet decomposition and daily returns for 1/4/2002-30/12/2011 period in Brent oil market. Multi-scale analysis excavates partial self-correlation feature of the series, but this feature cannot be directly observed from the original series. As mentioned, the original series is equal to the superposition of the 3 series obtained from wavelet lifting and there are no correlations between the 3 series. Because series after transform have significant self-correlation, we need to add AR item in the mean equation. Serial self-correlation is beneficial to improve forecast precision of time series.

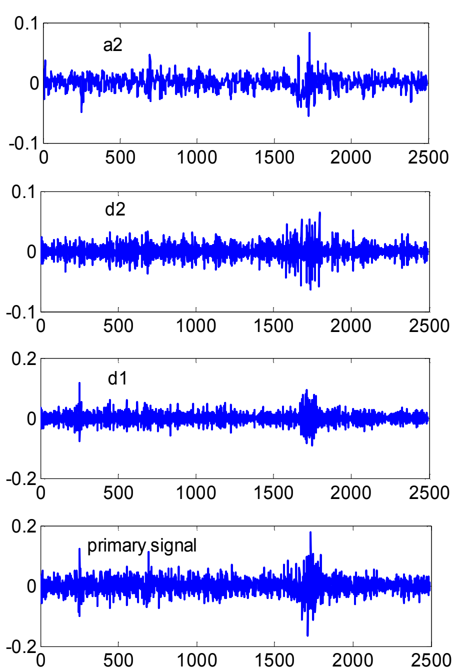

Fig. 1: Wavelet decomposition and primary signal of daily returns of brent oil

RESULTS

Table 1 and 2 give the estimation results of AR-GARCH and AR-DWT-GARCH. In the process of AR-GARCH (1, 1) parameter estimation of original series, R^2 = 0.0001, Adj-R^2 = 0.0001, S.E. = 0.0233, SSR = 1.3527, Log likelihood = 6036, AIC = -4.8459 and SC = -4.8342 and HQC = -4.8416. In the process of AR-DWT-GARCH parameter estimation of a2, R^2 = 0.9509, Adj-R^2 = 0.9508, S.E. = 0.0026, SSR = 0.0170, Log likelihood = 12056, AIC = -9.6819, SC = -9.6655 and HQC = -9.6759. In the process of AR-DWT-GARCH parameter estimation of d2, R^2 = 0.8292, Adj-R^2 = 0.8290, S.E. = 0.0045, SSR = 0.0513, Log likelihood = 10335, AIC = -8.3018, SC = -8.2831 and HQC = -8.2951. In the process of AR-DWT-GARCH parameter estimation of d1, R^2 = 0.7641, Adj-R^2 = 0.7638, S.E. = 0.0082, SSR = 0.16667,

Table 1: Parametric estimation of the AR-GARCH

Parameter	Coefficient	Z-value	Prob.
AR(3)	0.0339	1.7261	0.0843
ω	7.29×10^{-6}	2.6523	0.0080
α	0.0424	5.2099	0.0000
β	0.9426	81.9112	0.0000
GED_V	1.4681	25.6727	0.0000

Table 2: Parametric estimation of the AR-DWT-GARCH

Eqs.	Parameter	Coefficient	Z-value
a2	a2_AR (1)	2.4350	118.1642
	a2_AR (2)	-2.1358	-56.4578
	a2_AR (3)	0.6578	30.4777
	a2_ω	2.09×10^{-7}	8.3272
	a2_α	0.3761	14.0923
	a2_β	0.5992	30.3991
	a2_GED_V	3.3520	19.1114
d2	d2_AR (1)	0.9091	81.8700
	d2_AR (2)	-1.4265	-106.9146
	d2_AR (3)	0.7101	53.4988
	d2_AR (4)	-0.5691	-54.1029
	d2_ω	7.86×10^{-7}	3.5797
	d2_α	0.1753	6.0050
	d2_β	0.8101	29.4602
	d2_GED_V	0.9141	23.6545
d1	d1_AR (1)	-1.4466	-70.9852
	d1_AR (2)	-1.4453	-44.6780
	d1_AR (3)	-1.0152	-35.8427
	d1_AR (4)	-0.4216	-26.9593
	d1_ω	1.52×10^{-5}	8.9293
	d1_α	0.4013	7.4232
	d1_β	0.3973	8.8436
	d1_GED_V	1.4694	28.3377

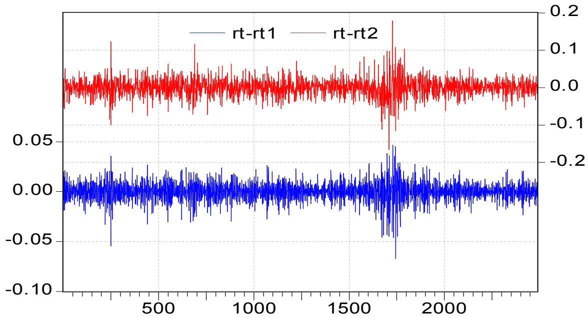

Fig. 2: Prediction residual of daily returns of brent oil

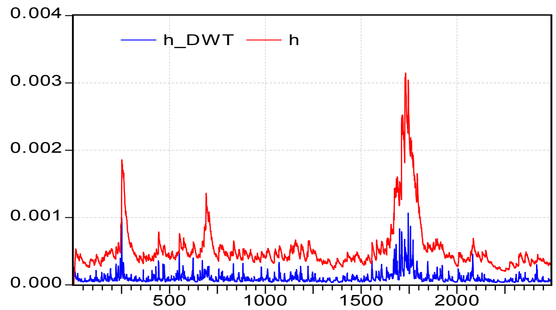

Fig. 3: Conditional variance of daily returns of brent oil

Log likelihood = 8714, AIC = -6.9985, SC = -6.9798 and HQC = -6.9917. Comparing various statistical values, we know, fitting precision of AR-DWT-GARCH model is higher than AR-GARCH and it uses more information in the time series.

Comparing R^2, Adj-R^2, Log likelihood, SE and SSR of AR-DWT-GARCH (1, 1) with those of AR-GARCH (1, 1), we found that fitting accuracy of AR-DWT-GARCH (1, 1) model is much better than AR-GARCH (1, 1) model. Values of AIC, SC and HQC showed that AR-DWT-GARCH (1, 1) model could excavate richer information from primary signal of daily returns than AR-GARCH (1, 1) model. Record r_t obtains from the forecast which are based on DWT-GARCH and the GARCH as r_{t1} and r_{t2} respectively. Record conditional variances obtain from DWT-GARCH and GARCH as h_DWT and h. Figure 2 showed the differences of original value r_t and predictive value r_{t1} and r_{t2} respectively, which were recorded as r_t- r_{t1} and r_t- r_{t2}. Prediction results in Fig. 2 showed that forecasting result of DWT-GARCH model was more precise. The smaller the conditional variance is, the better the forecast is. Figure 3 showed the values of conditional variance estimated by GARCH model and DWT-GARCH model. DWT-GARCH model had obvious advantages in forecasting the daily returns as shown.

CONCLUSION

In the study, we analyzed and combined GARCH with wavelet transform models. The main aim of the study is to augment the forecasting power of DWT-GARCH. Our results show that the GARCH family models combining with wavelet transform theory may be more powerful when forecasting the daily returns. It can not only describe the detail and partial features of volatility series, but also describe the heteroscedasticity, volatility clustering property, asymmetric relation property and nonlinearity property of volatility series. The DWT-GARCH model maintains approximate parts of the original series and captures the detail parts of volatility series at the same time. Via wavelet lifting, DWT-GARCH can dig up self-correlation characteristics existing in multi-scale, thereby improve forecast accuracy. These advantages make forecast accuracy of DWT-GARCH model higher than GARCH.

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