Research Article

Instantaneous Gradient Based Dual Mode Feed-Forward Neural Network Blind Equalization Algorithm

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Abstract: To further improve the performance of feed-forward neural network blind equalization based on Constant Modulus Algorithm (CMA) cost function, an instantaneous gradient based dual mode between Modified Constant Modulus Algorithm (MCMA) and Decision Directed (DD) algorithm was proposed. The neural network weights change quantity of the adjacent iterative process is defined as instantaneous gradient. After the network converges, the weights of neural network to achieve a stable energy state and the instantaneous gradient would be zero. Therefore dual mode algorithm can be realized by criterion which set according to the instantaneous gradient. Computer simulation results show that the dual mode feed-forward neural network blind equalization algorithm proposed in this study improves the convergence rate and convergence precision effectively, at the same time, has good restart and tracking ability under channel burst interference condition.

Keywords: Blind equalization, constant modulus algorithm, dual mode algorithm, feed-forward neural network, instantaneous gradient

INTRODUCTION

In the digital communication systems, finite bandwidth and multi-path propagation characteristics of the communication channels can cause severe Inter-Symbol Interference (ISI) in high data rate communication systems (Yin-Bing et al., 2010), which may lead to high error rates in symbol detection. Adaptive equalization technology is an effective means of eliminating inter-symbol interference. Compare with traditional adaptive equalization, blind equalization technique does not require any training sequence to implement compensating and tracking of the channel, which can improve the quality of the communication, at the same time, save the communication bandwidth (Xiao and Yu-Hua, 2009). For interception of military information and multi-user broadcast communications, there is no training sequence can be exploited, under these conditions, blind equalization has important practical value. In all kinds of the blind equalization algorithms, neural network blind equalization is one of the nonlinear equalization algorithms, which can not only apply to the minimum phase channel, also applies to the maximum phase channel, including the nonlinear channel (Xiao et al., 2009). The channel is more or less has nonlinear characteristics in the actual communication system, so neural network blind equalization has important significance in the practical engineering. The feed-forward neural network has mathematical theory and simple realization process, so it is most widely used. Blind equalization by feed-forward neural network usually adopts the cost function of Constant Modulus Algorithm (CMA) (Yang et al., 2011), then the convergence rate is very slow and the steady state error is big. In contrast, Decision Directed (DD) algorithm has faster convergence rate and better convergence precision, but if the eye diagram of the received signal is not open, DD algorithm often divergence or convergence error. The basic idea of dual mode blind equalization is derived from the features of CMA and DD algorithm, set a reasonable threshold allows the algorithm to between two algorithms are timely switch, which a combination of two algorithms to their respective advantages and further improves the blind equalization performance. According to the threshold setting difference, different dual mode algorithm can be obtained. In this study, we use the instantaneous gradient change rate to set threshold and a new dual mode neural network blind equalization algorithm was proposed, at last by using computer simulations proved the validity of the algorithm compared with decision circle based dual mode algorithm and sign error based dual mode algorithm, meanwhile, the performance for restart and tracking Ability under the channel has burst interference was done to prove the practical engineering value.

Fig. 1: Block diagram of neural network blind equalization

DUAL MODE NEURAL NETWORK BLIND EQUALIZATION

Feed-forward neural network blind equalization: The principle of neural network blind equalization can be shown as Fig. 1 (LU, 2003). Neural network blind equalization use neural network to act as blind equalizer and the principle is same to transversal equalizer.

In Fig. 1, x(n) is the input sequence of unknown channel, h(n) is the channel impulse response sequence, the output sequence of channel is s(n), n(n) is a gauss white noise at to s(n), then y(n) as the neural network equalizer input sequence is received, the output sequence of equalizer $\tilde{x}(n)$ can be decided to obtain the recovery sequence $x(n)$. According to the transmission theory of communication system can know:

$$s(n) = x(n) * h(n) \quad (1)$$

$$y(n) = s(n) + n(n) \quad (2)$$

If we take w(n) as equivalent convolution weight coefficient of neural network filter:

$$\tilde{x}(n) = w(n) * y(n) \quad (3)$$

The purpose of blind equalization is to recover the sending sequence x(n), $\tilde{x}(n)$ can be obtained directly by observed sequence y(n) and satisfy:

$$\tilde{x}(n) = x(n - D)e^{i\phi} \quad (4)$$

where,

D : Constant delay
$\phi$ : Constant phase shift

The sending sequence recovery quality is not affected by D, phase shift $\phi$ can be get rid of by decision set. Combining (1) and (3) and ignoring convolution noise, $\tilde{x}(n)$ can be shown as:

$$\tilde{x}(n) = h(n) * w(n) * x(n) \quad (5)$$

It can be seen that the condition of availability of (4) is that associate impulse response of channel and equalizer must satisfy (6):

$$h(n) * w(n) = [0, \cdots, 0, e^{i\phi}, 0, \cdots, 0]^T \quad (6)$$

The realization of feed-forward neural network blind equalization can establish network learning objective function by use of observed sequence. The weight coefficient of neural network is updated via special algorithm, which can make objective function achieve minimum, that is, associate impulse response of equivalent convolution weight w(n) and channel satisfies (6). According to SW theorem, the input and output signal have the same variance and the absolute value of kurtosis are equal, which is the necessary and sufficient conditions to equalization. It indicates that the implement of blind equalization relies on signal statistical property. Feed-forward neural network blind equalization takes cost function of CMA algorithm as objective function to train network, while the cost function of CMA just make use of signal high-order statistical property indirectly. Blind equalization by FNN has slow convergence rate because of adopting error Back Propagation (BP) algorithms (Godard, 1980) and it is easy to fall into locally minimum point due to non-convexity of cost function.

Cybenko has proved that trilevel feed-forward neural network can approximate any continuous function with any precision. The topological structure of trilevel feed-forward neural network can be shown as Fig. 2, where, $w_{ij}(n) (i = 1, 2 \cdots m, j = 1, 2 \cdots n)$ is weight coefficient from input layer to hidden layer, $W_j(n)$ ($j = 1, 2 \cdots n$) is weight coefficient from hidden layer to output layer, $u_j(n)$, $v_j(n)$ and l(n) is input sequence of hidden layer, output sequence of hidden layer and input sequence of input layer respectively. Then the state equations can be shown as:
Fig. 2: Trilevel FNN structure

\[
y(n) \rightarrow w_{ji} \rightarrow y(n-1) \rightarrow \ldots \rightarrow y(n-m+1) \rightarrow \bar{x}(n) \rightarrow w_j \rightarrow y(n-m)
\]

\[
\begin{align*}
    u_j(n) &= \sum_{i=1}^{n} w_{ij} (n) \bar{x}(n-i) \\
    v_j(n) &= f[u_j(n)] \\
    I(n) &= \sum_{j=1}^{n} w_j(n) v_j(n) \\
    \bar{x}(n) &= f[I(n)]
\end{align*}
\]

where, \( f(.) \) is the transfer function of neural network and the transfer function can be chosen as:

\[
f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}
\]

For training network, the monotonicity of transfer function must be ensured, so its derived function must be greater than zero. This requires adjustment parameters \( \lambda \) be greater than zero. The value of \( \lambda \) is determined by the amplitude of signal, that is, \( \lambda \) should be greater if amplitude of signal is greater, vice versa.

Combined with CMA cost function, the objective function of blind equalization by neural network is:

\[
J_D = \frac{1}{2} \left[ \bar{x}(n)^2 - R_{CM} \right]
\]

\[
R_{CM} = \frac{E[\bar{x}(k)^4]}{E[\bar{x}(k)^2]^2}
\]

Using BP algorithm, the iterative formula is shown as:

\[
w(n+1) = w(n) - \mu \frac{\partial J_D}{\partial w(n)}
\]

\[
\begin{align*}
    \frac{\partial J_D}{\partial w(n)} &= 2[\bar{x}^2(n) - R_{CM}] \frac{\partial \bar{x}(n)}{\partial w(n)} \\
    \frac{\partial \bar{x}(n)}{\partial w_j(n)} &= f'[I(n)]v_j(n)
\end{align*}
\]

for output layer:

\[
\frac{\partial \bar{x}(n)}{\partial w_j(n)} = f'[I(n)]v_j(n)
\]

from (14),(15) and (16), iterative formula is:

\[
w_j(n+1) = w_j(n) + \mu H(n)v_j(n)
\]

where, \( \mu \) is step-size:

\[
H(n) = -2[\bar{x}^2(n) - R_{CM}]f''(I(n))
\]

for hidden layer:

\[
\frac{\partial \bar{x}(n)}{\partial w_j(n)} = f'[I(n)]v_j(n)
\]

from (19) and (20), (14) becomes:

\[
w_{ij}(n+1) = w_{ij}(n) + \mu H_j(n)v(n-i)
\]

where,

\[
H_j(n) = f'[u_j(n)]v_j(n)H(n)
\]

**Dual mode blind equalization algorithm:** CMA is a special case of Godard algorithm, also is the most widely used blind equalization algorithm belonging to Bussgang for it is simple computation and stable performance. But CMA blind equalization convergence rate is slow and has big state steady error after convergence; also it is less sensitive to the phase of input signal, then a mend CMA (MCMA) (Ye-Cai et al., 2012) was proposed based on CMA, MCMA can effectively correct the phase deflection. The cost function of MCMA is:

\[
J_D = \frac{1}{2} \left[ \bar{x}_R(n)^2 - R_R \right] + \frac{j}{2} \left[ \bar{x}_I(n)^2 - R_I \right]
\]

where, \( \bar{x}_R(n) \) and \( \bar{x}_I(n) \) denote the real part and imaginary part of \( \bar{x}(n) \) respectively, \( R_R \) and \( R_I \) is defined as:
The cost function of DD is:

\[ J_{DD} = \frac{1}{2} \left[ \tilde{x}(n) - \hat{x}(n) \right]^2 \]  

(25)

In this study, dual mode algorithm is set according to MCMA and DD algorithm. Define the instantaneous gradient change rate as:

\[ g(n) = \frac{\|w_{ij}(n) - w_{ij}(n-1)\|}{\|w_{ij}(n-1) - w_{ij}(n-2)\|} \]  

(26)

According to the limit theorem and L’Hospital Rule, if the algorithm stability convergence, then \( \lim_{n \to \infty} g(n) = 1 \). The instantaneous gradient change rate reflects the network steady state and it can be used as the threshold for dual mode algorithm switching criterion. From the iterative process of neural network blind equalization algorithm, dual mode blind equalization neural network algorithm implement can only change \( H(n) \) in the iterative formula. According to MCMA:

\[ H_1(n) = -2 \left[ \tilde{x}_j^2(n) - R_x \right] \tilde{x}_j(n) \\
+ j \left[ \tilde{x}_j^2(n) - R_x \right] \tilde{x}_j(n) \right] f'(I(n)) \]  

(27)

and according to DD algorithm:

\[ H_2(n) = -2[\tilde{x}(n) - \hat{x}(n)]f'(I(n)) \]  

(28)

cv Hidden layer weights of neural network are updated according to the error back propagation algorithm without modification. Here according to the instantaneous gradient change rate is given the dual mode neural network weights updating formula when \( |g(n)| \) large or less than \( \delta \):

\[
\begin{align*}
   w_i(n+1) &= w_i(n) + \mu H_1(n) v_i(n) \\
   w_j(n+1) &= w_j(n) + \mu H_2(n) v_j(n)
\end{align*}
\]  

(29)

**COMPUTER SIMULATIONS**

In the simulations, equivalent probability binary sequence is adopted to act as sending signal and QPSK modulation is utilized. Adding noise is band-limited gauss white noise with zero mean. The structure of wavelet neural network equalizer set to 25×20×1. The diagonal elements of weights between input and hidden layer and the center elements of weights between hidden and output layer initialize to 1, the other weights set to 0. Step size \( \mu = 0.001 \). The comparison is in terms of mean square error (MSE) (Li-Jun and Ze-Min, 2010), which is defined as:

\[ MSE \ (n) = \frac{1}{n} \sum_{k=1}^{n} \left[ \tilde{x}(k) - \hat{x}(k) \right]^2 \]  

(30)

The simulation communication channel adopt mix-phase channel which the impulse response is \( h = [0.3132, -0.1040, 0.8908 and 0.3143] \). The threshold of dual mode sets \( \delta = 0.05 \). In order to verify the performance of the dual mode neural network blind equalization (DUAL-IGCR) proposed in this study, dual mode based decision circle blind equalization (DUAL-DC) (Ye-Cai and Yan-Ping, 2007) and dual mode based sign error blind equalization (DUAL-SE) (Jia-Qi and Ning, 2009) is done in the simulation for comparison. Figure 3 shows the convergence curve with SNR = 18.6 dB after 500 times Monte Carlo simulation. From Fig. 3 can see that DUAL-IGCR has faster convergence rate and lowest MSE.
To further verify the performance of DUAL-IGCR algorithm for channel burst interference, when the iterative times $n = 5000$, let the refraction way phase reversal that is $h = [0.3132, -0.1040, 0.8908, -0.3143]$ at that time to simulation channel burst interference, the result of 500 times Monte Carlo simulation is shown as Fig. 4. From Fig. 4 can see that the dual mode method proposed in this study has good restart and tracking ability under channel burst interference condition.

**CONCLUSION**

In this study a new dual mode neural network blind equalization algorithm based on the instantaneous gradient change rate was proposed, the theory analysis shows that the instantaneous gradient change rate reflects the stability of the network and the threshold can set according to it. Simulation results prove that the proposed algorithm has good equalization performance and has good restart and tracking ability under channel burst interference condition, therefore this study has a certain practical value.

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