

Research Article

Frequency Performance of the Extended State Observer for General Systems

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Abstract: The observing capability of the Active Disturbance Rejection Control (ADRC) for the general systems is discussed in this study. The frequency performance of the Linear Extended State Observer (LESO) is analyzed. And the theoretical result is verified by simulations. It is shown that ESO can estimate the required states at the designed speed, in spite of the different total uncertainties.

Keywords: Active disturbance rejection control, extended state observer, frequency performance

INTRODUCTION

Control design for the systems with uncertainties is a longstanding fundamental issue in automatic control. The uncertainties usually stem from 2 sources: the internal (parameter or structure) uncertainty and the external (disturbance) uncertainty. Hence, many disturbance estimating techniques appeared, such as Unknown Input Observer (UIO) (Basile and Marro, 1969), Perturbation Observer (POB) (Kwon and Chung, 2003), the Disturbance Observer (DOB) (Schrijver and Dijk, 2002). And the Active Disturbance Rejection Control (ADRC), which is based on the use of Extended State Observer (ESO), attracted many researchers' attention due to less dependence on the model information, strong capabilities for disturbance rejection and its simple control structure. The key of ADRC is to use ESO to estimate the total uncertainty, which lumps the internal nonlinear and uncertain dynamics and the external disturbance. Then the uncertainties of the system can be compensated actively.

The idea of ESO can be demonstrated in the following Single-Input and Single-Output (SISO) model:

$$\begin{cases} \dot{x}^{(n)} = f(x^{(n-1)}(t), x^{(n-2)}(t), \dots, x(t), w(t), t) + bu(t) \\ y = x(t) \end{cases} \quad (1)$$

where,

n : The order of the plant model

y : The output, u is the input

b : A constant

$w(t)$: The external disturbance

$f(\cdot)$: The total uncertainty (or total disturbance) of the system, containing both internal and external uncertainties

Introduce $e(t) = df/dt$. Then the total uncertainty $f(\cdot)$ can be taken as an extended state of the system (1) and the Eq. (1) can be written in the state form as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = x_{n+1} + bu \\ \dot{x}_{n+1} = e \end{cases}, \quad (2)$$

where, $X = [x_1 \dots x_{n+1}]^T \in R^{n+1}$ is the state vector of the system. The general form of ESO estimating both the states and the extended state for the uncertain system (1) is given in Gao *et al.* (2001) and Han (1995). As a special form of ESO, the following Linear ESO (LESO) is discussed in many studies, such as Zheng *et al.* (2007a, b):

$$\begin{cases} \dot{z}_1 = z_2 - \beta_1(z_1 - y) \\ \dots \\ \dot{z}_{n-1} = z_n - \beta_{n-1}(z_1 - y) \\ \dot{z}_n = z_{n+1} - \beta_n(z_1 - y) + bu \\ \dot{z}_{n+1} = -\beta_{n+1}(z_1 - y) \end{cases}, \quad (3)$$

where,

$Z = [z_1, \dots, z_{n+1}]^T \in R^{n+1}$ is the state vector of ESO

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$\beta_i, (i = 1, 2, \dots, n + 1)$ are the observer gains
And ESO (3) is designed to have the property that:

$$z_i(t) \rightarrow x_i(t), i = 1, 2, \dots, n + 1.$$

In Yang and Huang (2009), the capability of LESO (3) is analyzed for a wider scope of uncertain function $f(\cdot)$. It is shown that LESO can estimate the total uncertainty with bounded error if either the derivative of the total uncertainty is bounded or the total uncertainty itself is bounded. If the plant model is unknown, it will be difficult to discuss the error bound of ESO generally, since the total uncertainty may involve the high order derivatives of the system output. Sometimes, the orders of involved derivatives are even higher than the plant order. In this study, the capability of LESO is analyzed in frequency domain for the plants with unknown model. The main result is that the LESO can track the derivatives of the system output and the total uncertainty at the designed speed, while the stability of ESO is guaranteed.

In this study, the capabilities of ESO for estimating uncertainties are analyzed. The main result implies that ESO (6) can be used to observe the required states and the total uncertainty while the stability of ESO (6) is guaranteed. Moreover, the theoretical results are verified in detail by the simulations.

PROBLEM FORMULATION

In the following discussion, a symbol may represent both a time-domain variable and its Laplace transform when there is no confusion. Consider the following general system:

$$\begin{cases} \dot{\bar{x}} = h(\bar{x}, u, t) \\ y = \varphi(\bar{x}) \end{cases}, \quad (4)$$

where, \bar{x}, y and u represent the state vector, the output and the control input, respectively.

The design objective is to build an LESO (6) which can estimate the 1st to $(n-1)$ th order derivatives of y and the total uncertainty, guarantee $y(t)$ to be controlled to follow the command signal $v(t)$. And assume that the 1st to $(n-1)$ th order derivatives of $v(t)$ can be obtained in real time.

Due to the lack of the plant knowledge, the plant (4) may be modeled by the following dynamic system while designing the ADRC scheme:

$$y^{(n)} = f(\cdot) + b_e u, \quad (5)$$

where,

$n \geq 1$: An estimate of the order of the plant (4)

b_e : An estimate of the gain for the control input

$f(\cdot)$: The total uncertainty, which contains all the unknown dynamics: If n is greater than the plant order, $f(\cdot)$ will involve the derivative(s) of order(s) higher than the plant order

Remark 1: Since the gain for the control input of the system (4) may be unknown or even varying, b_e is different from b in (1) and (3). If the plant (4) is a linear system, b_e is an estimate of the plant's NZC, which is short for the numerator's zero-order coefficient (Zhao and Huang, 2011; Zhao, 2011).

The key in ADRC is to design the Extended State Observer (ESO) to estimate the total uncertainty $f(\cdot)$. Based on (5), the LESO (3) is modified as follows:

$$\begin{cases} \dot{z}_1 = z_2 - \beta_1(z_1 - y) \\ \dots \\ \dot{z}_{n-1} = z_n - \beta_{n-1}(z_{n-1} - y) \\ \dot{z}_n = z_{n+1} - \beta_n(z_n - y) + b_e u \\ \dot{z}_{n+1} = -\beta_{n+1}(z_{n+1} - y) \end{cases}, \quad (6)$$

where, $\beta_i (1 \leq i \leq n + 1)$ are positive parameters. And $\beta_i (1 \leq i \leq n + 1)$ should be designed such that $z_i (1 \leq i \leq n)$ tracks $y^{i-1} (1 \leq i \leq n)$ and z_{n+1} tracks $f(\cdot)$. Then the following control law is proposed for the tracking problem:

$$u = \frac{1}{b_e} (-z_{n+1} + \sum_{i=1}^n p_i (v^{(i-1)} - z_i)), \quad (7)$$

where, the parameters $p_i > 0$ for $1 \leq i \leq n$, $v^{(0)} = v$ is the reference signal and $v^{(i)} (1 \leq i \leq n)$ represents the i th order derivative of v .

Based on (6), there is:

$$z_i = F_{yi}(s) s^j y + F_{ui}(s) b_e u; (1 \leq i \leq n), \quad (8)$$

$$z_{n+1} = F_{y_{n+1}}(s) (s^n y - b_e u), \quad (9)$$

where,

$$F_{yi}(s) = \frac{\sum_{k=i}^{n+1} \beta_k s^{n+1-k}}{\sum_{k=0}^{n+1} \beta_k s^{n+1-k}}$$

$$F_{ui}(s) = \frac{\sum_{k=0}^{i-1} \beta_k s^{i-k}}{\sum_{k=0}^{n+1} \beta_k s^{n+1-k}}$$

$$\beta_0 = 1$$

Then the 2 properties follows:

Lemma 1: If $F_{yi}(s)$ ($1 \leq n \leq n + 1$) is stable, then its zero-frequency gain is $F_{yi}(0) = 1$.

Lemma 2: If $F_{ui}(s)$ ($1 \leq n \leq n$) is stable, then its zero-frequency gain is $F_{ui}(0) = 0$.

MAIN RESULTS

Theorem 1: There exist such parameters $b_e, \beta_1, \beta_2, \dots, \beta_{n+1}$ and P_1, P_2, \dots, P_n for the ESO (6) that $F_{y1}(s), F_{y2}(s), \dots, F_{yn+1}(s)$ and $F_{u1}(s), F_{u2}(s), \dots, F_{un}(s)$ are all stable transfer functions and:

$$\begin{aligned} |F_{yi}(j\omega) - 1| < \varepsilon_e, 0 \leq \omega \leq W_e, (1 \leq i \leq n + 1), \\ |F_{ui}(j\omega)| < \varepsilon_e, 0 \leq \omega \leq W_e, (1 \leq i \leq n) \end{aligned} \quad (10)$$

for any fixed $w_e > 0$ and $\varepsilon_e < 0$.

Before the proof of Theorem1, the following lemma shown in Zhao (2010) will be introduced:

Lemma 3: Assume $p(s)$ and $Q(s)$ be monic polynomials with real coefficients, $p(s)$ be Hurwitz, $degP(s) = m$ and $degQ(s) \leq m + 1$. Then there exists some positive k_0 such that $kP(s) + Q(s)$ is Hurwitz for any $k > k_0$.

Proof of theorem 1: Since $F_{y1}(s), F_{y2}(s), \dots, F_{yn+1}(s)$ and $F_{u1}(s), F_{u2}(s), \dots, F_{un}(s)$ all have the same denominator, their stability is equivalent to that:

$$B(s) \sum_{k=0}^{n+1} \beta_k s^{n+1-k} \quad (11)$$

is Hurwitz. Assume $\alpha_i = \beta_i / \beta_{i-1}, 1 \leq i \leq n + 1$ and $D_{n+1}(s) = 1$ Then define $D_i = s^{n+1-i} + \alpha_{i+1} D_{i+1}$ for $n \geq i \geq 0$ Thus $D_0(s) = B(s)$.

The following induction is made to show that $B(s) = D_0(s)$ is Hurwitz. First, considering $\alpha_{n+1} > 0$, $D_n(s) = s + \alpha_{n+1}$ is Hurwitz immediately. Then, $D_i(s) = s^{n+1-i} + \alpha_{i+1} D_{i+1}(s)$ is Hurwitz from Lemma 3, provided that $D_{i+1}(s)$ is Hurwitz and α_n is big enough. Thus, the induction is completed and the stability is proved.

Furthermore, (10) will be achieved by modifying the former induction. Assume $\varepsilon_e < 1$ and $w_e > 1$ without loss of generality. Let $\varepsilon_0 \in (0,1)$ be small enough such that:

$$\varepsilon_0 / (1 - \varepsilon_0) < W_e^{-n+1} \varepsilon_e \leq \varepsilon_e, \quad (12)$$

and set $\alpha_0 > n\varepsilon_0^{-1} w_e$. Then choose β_i satisfying:

$$\beta_{i+1} / \beta_i > \alpha_0, 0 \leq i \leq n. \quad (13)$$

Since $\omega \in [0, W]$ and $\beta_k > 0, 0 \leq k \leq n + 1$

$$\begin{aligned} \frac{1}{\beta_{n+1}} \left| \sum_{k=0}^{i-1} \beta_k (j\omega)^{n+1-k} \right| &\leq \sum_{k=0}^{i-1} \frac{\beta_k}{\beta_{n+1}} \omega^{n+1-k} \leq \sum_{k=0}^{i-1} \frac{\beta_k}{\beta_{n+1}} W_e^{n+1-k} \\ &\leq \sum_{k=0}^{i-1} (\alpha_0^{-1} W_e)^{n+1-k} \leq n\alpha_0^{-1} W_e < \varepsilon_0 \end{aligned}$$

holds for $1 \leq i \leq n + 1$. Especially, when $i = n + 1$, there is:

$$\frac{1}{\beta_{n+1}} \left| \sum_{k=0}^n \beta_k (j\omega)^{n+1-k} \right| < \varepsilon_0 < 1.$$

Then, for $1 \leq i \leq n + 1$. And $\omega \in [0, W]$, there is:

$$\begin{aligned} |F_{yi}(j\omega) - 1| &= \left| \frac{\sum_{k=0}^{i-1} \beta_k (j\omega)^{n+1-k}}{\sum_{k=0}^{n+1} \beta_k (j\omega)^{n+1-k}} \right| \\ &\leq \frac{\frac{1}{\beta_{n+1}} \left| \sum_{k=0}^{i-1} \beta_k (j\omega)^{n+1-k} \right|}{\left| 1 - \frac{1}{\beta_{n+1}} \sum_{k=0}^n \beta_k (j\omega)^{n+1-k} \right|} \leq \frac{\varepsilon_0}{1 - \varepsilon_0} \\ &< W_e^{-n+1} \varepsilon_e \leq \varepsilon_e. \end{aligned} \quad (14)$$

Moreover, for $1 \leq i \leq n$

$$\begin{aligned} |F_{ui}(j\omega)| &= \left| \frac{\sum_{k=0}^{i-1} \beta_k (j\omega)^{i-k}}{\sum_{k=0}^{n+1} \beta_k (j\omega)^{n+1-k}} \right| \\ &= |(j\omega)^{n-i} (F_{yi}(j\omega) - 1)| \leq W^{n-i} |F_{yi}(j\omega) - 1| < \varepsilon_e. \end{aligned}$$

Hence (10) holds for any $\omega \in [0, W]$.

According to Theorem1, each $F_{yi}(s)$ ($1 \leq i \leq n + 1$) is a low-pass transfer function with bandwidth no lower than w_e and zero-frequency gain 1. Meanwhile, $F_{ui}(1 \leq i \leq n)$ is a band-stop transfer function with stop band wider than $[0, W_e]$ and satisfying $F_{ui}(0) = 0$ and $\lim_{\omega \rightarrow \infty} |F_{ui}(j\omega)| = 0$. Hence, taking U as disturbance, ESO (6) can be used for the uncertain minimum-phase system $H(s)$ to observe the required states. Moreover, ESO (6) plus the controller (7) can guarantee the close-loop stability. Then, the practical performances will be shown by simulation in next section.

SIMULATIONS

Consider the following group of plants:

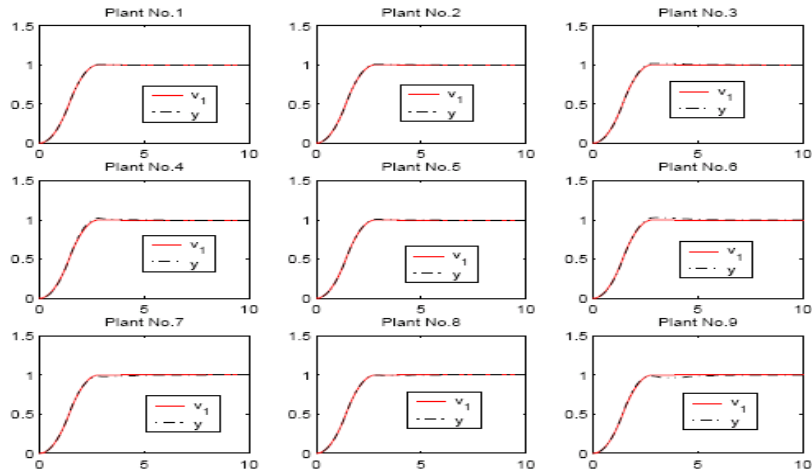


Fig. 1: Close-loop performances with ADRC

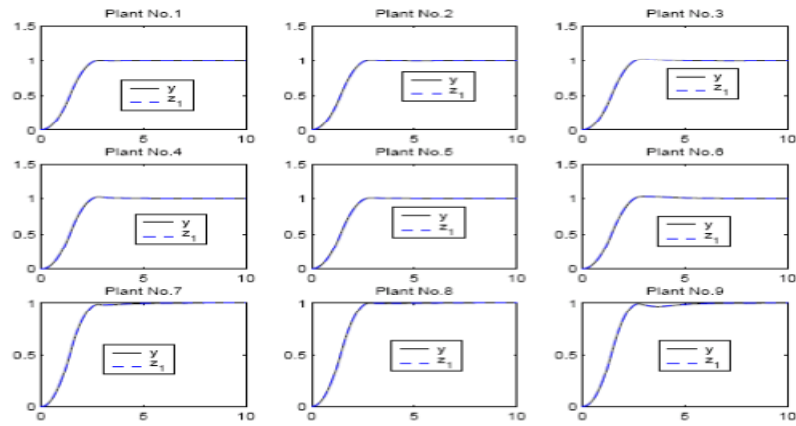


Fig. 2: Performances of ESO: $y(t)$ and z_1

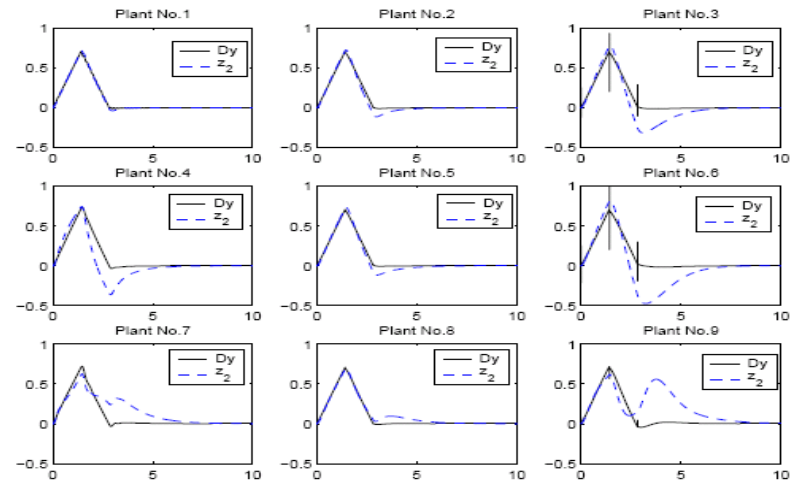


Fig. 3: Performances of ESO: $y(t)$ and z_2

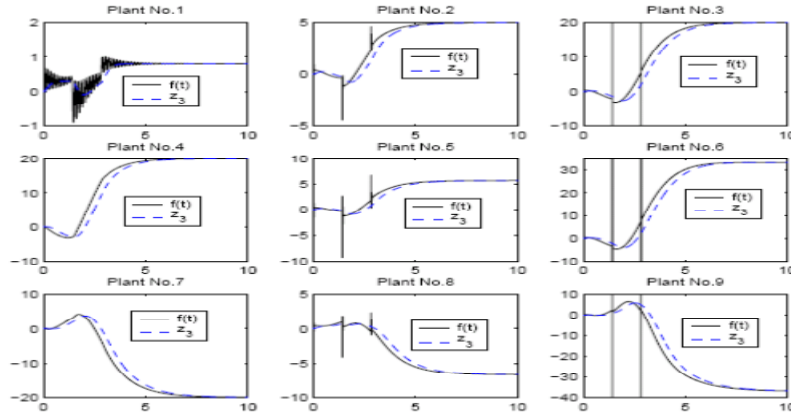


Fig. 4: Performances of ESO: $f(t) = y(t) - b_e u(t)$ and z_3

$$\text{No.1: } y(s) = \frac{500}{s^2 + 3s - 4} u(s),$$

$$\text{No.2: } y(s) = \frac{80(s+1)}{s^2 + 3s - 4} u(s),$$

$$\text{No.3: } y(s) = \frac{20(s^2 + 3s + 2)}{s^2 + 3s - 4} u(s),$$

$$\text{No.4: } y(s) = \frac{100(s+1)}{s^3 + 8s^2 + 11s - 20} u(s),$$

$$\text{No.5: } y(s) = \frac{175(s^2 + 3s + 2)}{s^3 + 8s^2 + 11s - 20} u(s),$$

$$\text{No.6: } y(s) = \frac{10(s^3 + 6s^2 + 11s + 6)}{s^3 + 8s^2 + 11s - 20} u(s),$$

$$\text{No.7: } y(s) = \frac{100(s^2 + 3s + 2)}{s^4 + 6s^3 - 5s^2 - 42s + 40} u(s),$$

$$\text{No.8: } y(s) = \frac{100(s^3 + 6s^2 + 11s + 6)}{s^4 + 6s^3 - 5s^2 - 42s + 40} u(s),$$

$$\text{No.9: } y(s) = \frac{3(s^4 + 12s^3 + 47s^2 + 72s + 36)}{s^4 + 6s^3 - 5s^2 - 42s + 40} u(s).$$

where,

$$\beta_1 = 200$$

$$\beta_2 = 2500$$

$$\beta_3 = 7000$$

$$b_e = 100$$

$$p_1 = 300$$

$$p_2 = 5$$

The close-loop performances for the nine systems are shown in Fig. 1. The outputs all track $v_1(t)$ with no static errors. The capability of ESO is shown in Fig. 2-4. Figure 2 shows that $z_1(t)$ follow the output $y(t)$ very quickly for all the nine plants. In Fig. 3, z_2 follows $y(t)$ in all cases. According to Section 2, z_3 is designed to follow the total uncertainty. Figure 4 demonstrates that ESO (15) has a good capability to follow the total uncertainties, despite the uncertainties are quite different in the nine plants. With the help of ESO (15), all the uncertainties can be estimated and compensated by the controller (16). Hence the stable tracking can be achieved with good transient quality.

CONCLUSION

In this study, the capabilities of ESO for estimating uncertainties are analyzed. The main result implies that ESO (6) can be used to observe the required states and the total uncertainty while the stability of ESO (6) is guaranteed. Moreover, the theoretical results are verified in detail by the simulations.

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The objective is to make their outputs track the reference $v_1(t)$ by designing a parameter-fixed ADRC. In detail, $v_1(t)$ satisfies the condition that v_1 maintains to be 1 after a finite period of time and $v_2(t) = v_1(t)$ is continuous and available. In this study, $v_1(t)$ is generated by the Tracking Differentiator (TD) in Huang and Zhang (2002). And the following ADRC scheme is designed for the nine plants:

$$\begin{cases} \dot{z}_1 = z_2 - \beta_1(z_1 - y), \\ \dot{z}_2 = z_3 - \beta_2(z_1 - y) + b_e u, \\ \dot{z}_3 = -\beta_3(z_1 - y), \end{cases} \quad (15)$$

$$u = b_e^{-1}(-z_3 + p_1(v_1 - z_1) + p_2(v_2 - z_2)), \quad (16)$$

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