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Research Article

A Sequential Energy Detection Based Spectrum Sensing Scheme in Cognitive Radio

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Abstract: Reliable and swift spectrum sensing is a crucial technical challenge of cognitive radio. This study proposes a Sequential Energy Detection (SED) scheme to reduce the average required sample number and sensing time for spectrum sensing in low signal-to-noise ratio regime. In the scheme, the data samples are first grouped into data blocks and the Sequential Probability Ratio Test (SPRT) use the energies of the data blocks as the statistic variables. The resulting detection rule exhibits simplicity in implementation and in analysis and retains the high sample-efficiency of sequential probability ratio test. The detection performance in terms of Average Sample Number (ASN) is evaluated theoretically. Simulation results are provided to verify the theoretical analysis.

Keywords: Cognitive radio, Spectrum sensing, Energy detector, Sequential probability ratio test

INTRODUCTION

Cognitive Radio (CR), which enables secondary users to opportunistic access to the unoccupied licensed spectrum bands, has recently emerged as a promising technology to improve the spectrum utilization efficiency and mitigate the spectrum scarcity problem (Haykin, 2005; Quan *et al*., 2008). Spectrum sensing, as an essential functionality in CR communications, is used to exploit spectrum access opportunities for the secondary users and avoid harmful interference to the potential primary users. Several signal detection techniques, such as Energy Detection (ED) (Atapattu *et al*., 2011; Nguyen-Thanh and Insoo, 2011), matched filter detection (Sahai and Cabric, 2005), cyclostationary feature detection (Han *et al*., 2006), can be used in spectrum sensing. Matched filter is known as the optimal detector when the transmitted signal is known, however its implementation complexity is impractically large since it requires perfect knowledge of the primary users. Cyclostationary feature detection is more robust than energy detector, but it is computationally complex and it still needs some prior knowledge of the primary user, such as modulation types and symbol rates. Since CR users usually have limited knowledge about the primary signals, the no coherent ED becomes the most frequently employed technique for spectrum sensing. However, ED requires a large number of the received signal samples to achieve a desired level of accuracy in the low SNR regime (Liang *et al*., 2008), which decreases the throughput of cognitive radio system. Furthermore, ED suffer from the SNR wall due to the uncertainty of the noise, which means the secondary user cannot detect the presence of primary user when it received power is lower than some threshold even if the detection time is infinite (Tandra and Sahai, 2008).

Sequential Probability Ratio Test (SPRT) requires the smallest Average Sample Number (ASN) to meet specified probabilities of false-alarm and miss-detection (Wald, 1945) and has drawn continued interest for decades. Motivated by this remarkable advantage, an autocorrelation based SPRT has been used in spectrum sensing to reduce the average required sample size (Chaudhari *et al.*, 2009), however it is only suitable for the Orthogonal Frequency Division Multiplexing (OFDM) based primary signals. In Zou *et al.* (2009), another SPRT based cooperative spectrum sensing scheme was presented and evaluated using Gaussian distributed primary signal. In Xin *et al.* (2009), a sub-optimal sequential detection scheme for spectrum sensing, namely Sequential Shifted Chi-Square Test (SSCT), was proved to be applicable to most of the primary signals. Nonetheless, when the primary signals are taken from a finite alphabet, for example the widely used Phase-Shift-Keying (PSK) signals and M-Ary Quadrature Amplitude Modulation (MQAM) signals, evaluating the probability ratio requires the deterministic statistical distribution of the primary signals. Acquiring such statistical distribution is practically difficult in general. For this reason, the existing SPRT based spectrum sensing schemes cannot be used in such primary signal types.

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To overcome this drawback, this study proposes a Sequential Energy Detection (SED) scheme. In the scheme, the data samples are first grouped into data blocks and the Sequential Probability Ratio Test (SPRT) use the energies of the data blocks as the test statistic variables in order to avoid the deterministic knowledge of the primary signals. The major advantages of the proposed scheme are as follows. Firstly, the test statistics can be approximated as Gaussian random variables by central limit theorem without consideration of the statistical distribution of primary signals, therefore the scheme is easy to implement. Secondly, the detection performance of SED described in terms of Average Sample Number (ASN) function is easily evaluated by theoretical analysis. Lastly, the SED can achieve higher sample-efficiency than SSCT, which can be explained by the conclusion that the sequential probability ratio test (SPRT) is optimum in the sense that the ASN function is less than or equal to the ASN function for any other sequential test.

SYSTEM MODEL FOR SPECTRUM SENSING

System Model: In local spectrum sensing problem, each secondary user conducts spectrum sensing independently to make a decision on the presence of primary user based on its own observations. Local spectrum sensing can be formulated as a binary hypothesis test problem as follows:

$$\begin{cases} H_0 : x(k) = n(k) \\ H_1 : x(k) = h(k)s(k) + n(k) \end{cases} \tag{1}$$

where, H_0 and H_1 are respectively correspondent to hypotheses of absence and presence of primary signals, $x(k)$ represents the signal received by secondary user, $s(k)$ denotes the primary user's transmitted signal, $n(k)$ is the additive white Gaussian noise (AWGN) and $h(k)$ is the amplitude gain of the channel between primary user and secondary user.

Energy Detection (ED) for spectrum sensing: In ED based spectrum sensing, the average energy of the received signal samples is first measured and then compared against a predetermined threshold λ to make a decision about the presence of a primary signal. The decision rule of ED can be described as:

$$Y = \frac{1}{N}\sum_{k=0}^{N-1} |x(k)|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{2}$$

where, Y denotes the test statistic of the energy detection and N denotes a fixed length of the observation samples. Given a SNR γ and a pair of target probabilities of false-alarm and miss-detection denoted by α and β, respectively, the required sample number is given by Liang *et al*. (2008):

$$N = \frac{1}{\gamma^2}\left(Q^{-1}(\alpha) - Q^{-1}(1-\beta)\sqrt{1+2\gamma}\right)^2 \tag{3}$$

where, $Q(\ .)$ is the complementary distribution function of the standard Gaussian.

Sequential probability ratio test for spectrum sensing: To reduce the number of required samples, SPRT conducts the Log-Likelihood ratio test for each received sample in a sequential manner instead of using a fixed sample size. That is, for K=1, 2,…, the test rules can be described as:

$$\begin{cases} l(\mathbf{X}_K) \le \lambda_0, & \text{Accept } H_0 \\ l(\mathbf{X}_K) \ge \lambda_1, & \text{Accept } H_1 \\ \lambda_0 < l(\mathbf{X}_K) < \lambda_1, & \text{Take Next Statistic} \end{cases} \tag{4}$$

where, $l(X_K)$ is the Log-Likelihood ratio (LLR) of the received sequence $X_K = \{x(0), x(1), \ldots, x(K)\}$. Assuming the received signals are independent of each other, the $l(X_K)$ can be computed as:

$$l(\mathbf{X}_K) = \ln\left(\prod_{i=1}^{K} \frac{f(x(i) \mid H_1)}{f(x(i) \mid H_0)}\right) \tag{5}$$

This SPRT based spectrum sensing scheme cannot be used in the MPSK and MQAM modulated signals, because for each received signal sample it is hard to determine which one is its exact statistical distribution in the M possible distributions.

SEQUENTIAL ENERGY DETECTION (SED) SCHEME FOR SPECTRUM SENSING

In this section, we present a SED based spectrum sensing scheme to avoid the deterministic knowledge of the primary signals for SPRT. The detection procedure of SED can be described as follows: At each stage of SED, take a package of the previous L samples, calculate a test statistic from these L observations and perform a 2 -threshold test based on the aggregation of all the test statistics. Stop sampling if one of the thresholds is crossed and make a decision between H_0 and H_1. Otherwise, receive the next L samples, calculate the additional test statistic and repeat the previous procedure.

We first group the received sequence $\{x(k)\}$ into data blocks of same size L, then use the average energies of the data blocks as the test statistics for SED, which are expressed as:

$$Y_i = \frac{1}{L}\sum_{j=0}^{L-1} |x(Li+j)|^2 \quad i = 0,1,2,\cdots. \tag{6}$$

We focus on the complex-valued phase-shift-keying (PSK) modulated primary signal and the circularly symmetric complex Gaussian noise with mean zero and variance σ_n^2. By central limit theorem, for a large number of samples N ($N \geq 20$ is often sufficient in practice), the test statistic can be approximated by Gaussian distribution given by:

$$Y_i \sim \begin{cases} N_r\left(\sigma_n^2, \frac{1}{L}\sigma_n^4\right) & H_0 \\ N_r\left((\gamma+1)\sigma_n^2, \frac{1}{L}(2\gamma+1)\sigma_n^4\right) & H_1 \end{cases}, \tag{7}$$

where, $N_r(\,.)$ denotes real Gaussian distribution and γ is the received SNR of the primary signal. According to the distributions of Y_i, the LLR of Y_i is obtained as:

$$\begin{aligned} l(Y_i) &= \ln \frac{f(Y_i \mid H_1)}{f(Y_i \mid H_0)} \\ &= \frac{L\gamma}{(2\gamma+1)\sigma_n^4}\left(Y_i - \frac{\sigma_n^2}{2}\right)^2 - \frac{L\gamma + 2\ln(2\gamma+1)}{4} \end{aligned} \tag{8}$$

We denote the collected K test statistics by a vector $Y_K=[Y_0,Y_1,\ldots Y_{K-1}]$. Assuming $Y_0,Y_1,\ldots$ and Y_{K-1} are independent of each other, the LLR of Y_K is obtained as:

$$\begin{aligned} l(\mathbf{Y}_K) &= \ln \frac{f(Y_0, Y_1, \cdots Y_{K-1} \mid H_1)}{f(Y_0, Y_1, \cdots Y_{K-1} \mid H_0)} \\ &= \ln \prod_{i=0}^{K-1} \frac{f(Y_i \mid H_1)}{f(Y_i \mid H_0)} = \sum_{i=0}^{K-1} l(Y_i) \end{aligned} \tag{9}$$

Now for K=1, 2,…, the decision rules can be given by:

$$\begin{cases} l(\mathbf{Y}_K) \leq \lambda_0, & \text{Accept } \mathrm{H}_0 \\ l(\mathbf{Y}_K) \geq \lambda_1, & \text{Accept } H_1 \\ \lambda_0 < l(\mathbf{Y}_K) < \lambda_1, & \text{Take Next Statistic} \end{cases}, \tag{10}$$

where, λ_0 and λ_1 are the 2 predetermined thresholds. Let α and β, respectively denote the target false-alarm probability and the target miss-detection probability. Ignoring the LLR threshold overshoots, which is a reasonable assumption if the number of the required test statistics is large, the thresholds satisfy:

$$\lambda_0 = \ln \frac{\beta}{1-\alpha}, \text{ and } \lambda_1 = \ln \frac{1-\beta}{\alpha} \tag{11}$$

AVERAGE SAMPLE NUMBER (ASN) FOR SED

Performance of sequential detection can be expressed in terms of the Average Sample Number (ASN). The ASN is defined as the number of samples required on average to reach a decision at a certain level of accuracy (Wald and Wolfowitz, 1948). Assuming the average test statistic numbers (ATSN) for SED under the hypotheses H_0 and H_1 are, respectively denoted by $E[K|H_0]$ and $E[K|H_1]$ and using Wald's equation, we obtain:

$$E[K \mid H_0] = \frac{\alpha\lambda_1 + (1-\alpha)\lambda_0}{E[l(Y_i) \mid H_0]} \tag{12}$$

and

$$E[K \mid H_1] = \frac{(1-\beta)\lambda_1 + \beta\lambda_0}{E[l(Y_i) \mid H_1]} \tag{13}$$

where, $E[l(Y_i)\ |H_0]$ and $E[l(Y_i)\ |H_1]$ denote the expectations of the LLR of Y_i under hypotheses H_0 and H_1, respectively. In order to obtain them, we first take the expectations of $(Y_i - \sigma_n^2/2)^2$ under the 2 hypotheses as follows:

$$\begin{aligned} E\left[(Y_i - \sigma_n^2/2)^2 \mid H_0\right] &= E\left[Y_i^2 \mid H_0\right] - \sigma_n^2 E\left[Y_i \mid H_0\right] + \sigma_n^4/4 \\ &= \left(\frac{1}{L} + \frac{1}{4}\right)\sigma_n^4, \end{aligned} \tag{14}$$

and

$$\begin{aligned} E\left[(Y_i - \sigma_n^2/2)^2 \mid H_1\right] &= E\left[Y_i^2 \mid H_1\right] - \sigma_n^2 E\left[Y_i \mid H_1\right] + \sigma_n^4/4 \\ &= \left(\frac{2\gamma+1}{L} + \frac{(2\gamma+1)^2}{4}\right)\sigma_n^4 \end{aligned} \tag{15}$$

By taking the expectations of both sides of (10) under hypotheses H_0 and H_1 respectively and using the results of (14) and (15), we obtain:

$$\begin{aligned} E[l(Y_i) \mid H_0] &= \frac{\gamma L}{(2\gamma+1)\sigma_n^4} E\left[(Y_i - \sigma_n^2/2)^2 \mid H_0\right] - \frac{\gamma L + 2\ln(2\gamma+1)}{4} \\ &= \frac{-\gamma^2(L+4)}{2(2\gamma+1)} \end{aligned} \tag{16}$$

and

$$\begin{aligned} E[l(Y_i) \mid H_1] &= \frac{\gamma L}{(2\gamma+1)\sigma_n^4} E\left[(Y_i - \sigma_n^2/2)^2 \mid H_1\right] - \frac{\gamma L + 2\ln(2\gamma+1)}{4} \\ &= \frac{\gamma^2 L}{2} \end{aligned} \tag{17}$$

Then, substituting (16) into (12) and (17) into (13), respectively we obtain:

$$E[K \mid H_0] = \frac{2(2\gamma+1)(\alpha\lambda_1 + (1-\alpha)\lambda_0)}{-\gamma^2(L+4)} \tag{18}$$

and

$$E[K \mid H_1] = \frac{2((1-\beta)\lambda_1 + \beta\lambda_0)}{\gamma^2 L} \tag{19}$$

Assuming the priori probabilities of H_0 and H_1 satisfy $P(H_0) = P(H_1) = 0.5$, the total ATSN can be written as:

$$\begin{aligned} E[K] &= P(H_0)E[K \mid H_0] + P(H_1)E[K \mid H_1] \\ &= \frac{(2\gamma+1)(\alpha\lambda_1 + (1-\alpha)\lambda_0)}{-\gamma^2(L+4)} + \frac{(1-\beta)\lambda_1 + \beta\lambda_0}{\gamma^2 L} \end{aligned} \tag{20}$$

According to (6), each test statistic is calculated from L samples therefore, the ASN is L times the length of ATSN, which is given by:

$$\begin{aligned} E[M] &= E[K] \cdot L \\ &\approx \frac{(2\gamma+1)(\alpha\lambda_1 + (1-\alpha)\lambda_0)}{-\gamma^2} + \frac{(1-\beta)\lambda_1 + \beta\lambda_0}{\gamma^2} \end{aligned} \tag{21}$$

where, the last step uses the approximation $L+4 \approx L$ when L is relatively large. It can be seen from (21) that, for given α, β and γ, the required ASN for SED is a constant that is not related to the group size L. Before the test procedure can be executed, we must select the group size L. In summary, L should satisfy the following 2 constraints:

- L should be relatively large in order to well approximate the test statistic Y_i as a Gaussian random variable
- L must be considerably smaller than the sample size N for ED in order to ensure that the ATSN is relatively large and the LLR threshold overshoots can be ignored, which in turn ensure the accuracy of the theoretical model

According to these constraints, generally speaking, it is appropriate to select L from the region [N/100, N/20] in the low SNR regime. It can be seen from (21) that, if L satisfy the abovementioned constraints, for given α, β and γ, the required ASN for SED is a constant that is not related to the group size L. In other words, to reach a given detection performance at a certain SNR level, a large group size L will result in a small ATSN and vice versa, but the product of L and ATSN does not change. Figure 1 shows a theoretical comparison of the required sample numbers for SED, SSCT and ED. It is shown that the sample numbers required by SED are consistently and significantly lower than those required by SSCT and ED under both the same constraints of performance parameters: $\alpha = \beta = 0.1$.

SIMULATION RESULTS

In this section, we evaluate the ASN of SED by computer simulations. In the simulations, we adopt QPSK modulated signal as the primary signal. In the first test example, we first obtain the miss-detection probabilities at different SNR levels for ED under the constraints that the sample number is 650 and the probability of false-alarm is 0.1. Then we use the just obtained miss-detection probabilities and $P_f = 0.1$ as the target probabilities to set the parameters of SED and SSCT. Figure 2 show that the practical false-alarm and miss-detection probabilities for SED and SSCT are close to the target ones.

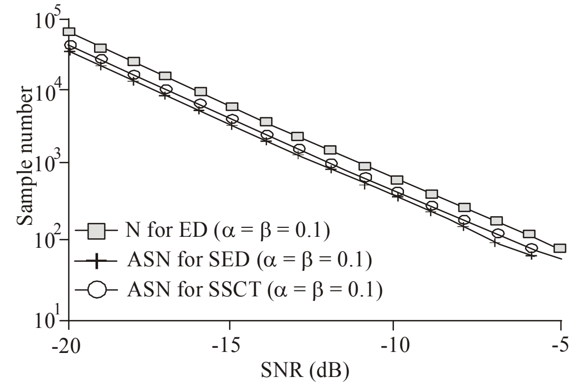

Fig. 1: Required sample numbers by SED, SSCT and ED under both the same constraints of performance parameters: $\alpha = \beta = 0.1$

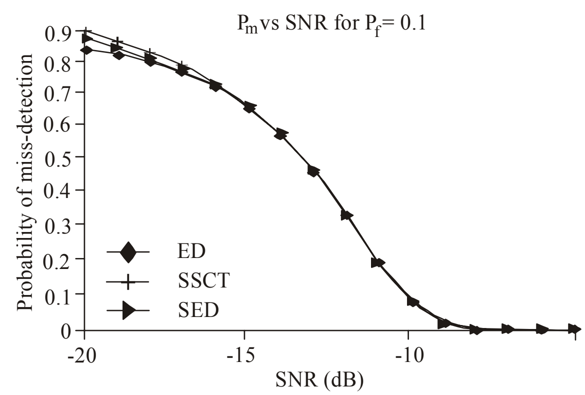

Fig. 2: Probability of the miss-detection as a function of SNR for ED, SSCT and SED

Figure 3 Shows that, compared with SSCT and ED, SED requires significantly reduced ASN under the same performance constraints. The inflexion points in Fig. 3 can be explained by that the requirement of decreasing miss-detection probability is a dominative reason for the increasing of the ASNs in the lower SNR region and the increasing of the SNR becomes a dominative reason for the decreasing of the ASNs in the higher SNR region.

In the second test example, we compare SED with SSCT and ED in terms of ASN and sample-efficiency and the false-alarm and miss-detection probabilities at several different SNR levels using computer simulations. The fixed sample size N for ED and all the

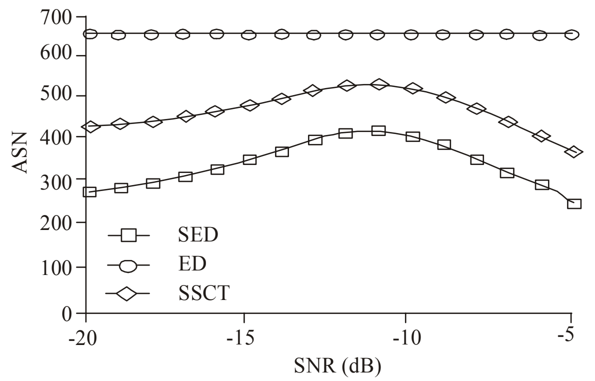

Fig. 3: Comparison of the required sample numbers for SED and SSCT and ED

Table 1: The SED versus SSCT and ED

SNR (dB)	-5	-10	-15	-20
α (target)	0.05	0.1	0.15	0.2
β (target)	0.05	0.1	0.15	0.2
L	10	20	90	300
α_{SED} (simu.)	0.056	0.103	0.152	0.25
α_{SSCT} (simu.)	0.055	0.103	0.153	0.26
α_{ED} (simu.)	0.055	0.102	0.151	0.21
β_{SED} (simu.)	0.048	0.102	0.148	0.23
β_{SSCT} (simu.)	0.046	0.099	0.154	0.25
β_{ED} (simu.)	0.047	0.096	0.149	0.198
ASN_{SED} (Theo.)	70	387	2505	16802
ASN_{SED} (simu.)	76	398	2568	16989
ASN_{SSCT} (simu.)	95	509	3154	20575
N (for ED)	140	721	4431	28616
η_{SED}	45.7%	44.8%	42.0%	40.6%
η_{SSCT}	32.1%	29.4%	28.8%	28.1%

values of the thresholds for ED, SSCT and SED are selected according to the target false-alarm probability and miss-detection probability pairs (α, β) = (0.05, 0.05), (0.1, 0.1), (0.15, 0.15) and (0.2, 0.2) corresponding to SNR = −5, −10, −15 and −20dB, respectively. The group sizes for SED are given in the second row of Table 1. Following the conventional terminology in squential detection, the sample-efficiency is defined as $\eta = 1 - ASN/N$. The third and fourth rows of Table 1 show that all the false-alarm probabilities and miss-detection probabilities obtained by Monte Carlo simulations for ED, SSCT and SED are close to the target ones at various SNR levels. All the 3 schemes achieve the expected detection performance at the similar degree of accuracy. The fifth row of Table 1 shows that the required ASNs of SED obtained by Monte Carlo simulations is less than those of SSCT and match well with the theoretical ASNs of SED at various SNR levels. The sixth row of Table 1 shows that, compared with energy detection, the SED can achieve about 40% ~ 45% sample-efficiency while the SSCT can only achieve about 28% ~ 32% sample-efficiency when satisfying the same desired performance levels. It is demonstrated that SED outperform SSCT in terms of sample-efficiency.

CONCLUSION

In this study, we have proposed a SED based spectrum sensing scheme by combining SPRT and ED. We have studied the ASN of SED by theory and simulation. It has been found that SED significantly reduces the ASN and sensing time while retaining a comparable detection performance compared with SSCT and ED. The proposed scheme exhibits implementation simplicity due to not relying on the deterministic knowledge of primary user..

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