

Research Article

Heat and Mass Transfer Over a Vertical Plate with Periodic Suction and Heat Sink

K.K. Asogwa, I.J. Uwanta, A.A. Momoh and E. Omokhuale

Department of Mathematics, Usmanu Danfodiyo University, PM B 2346, Sokoto, Nigeria

Abstract: The aim of this study is to determine heat and mass transfer over a vertical plate in the presence of periodic suction and heat sink. The dimensionless governing equations are solved using perturbation technique. The velocity, temperature and concentration profiles are studied for different physical parameters like Suction parameter s , Heat sink F , thermal Grashof number Gr , mass Grashof number Gc , chemical reaction parameter K , Prandtl number Pr and Schmidt number Sc . It is observed that the velocity increases with increase in F and s . It is also observed that temperature increases with increasing F , Pr but s decreases with rise in temperature. While concentration increases with increasing Sc and K . the aim of the study is to determine the rate of heat and mass transfer of the system.

Keywords: Heat and mass transfer, vertical plate, periodic suction, heat sink

INTRODUCTION

The study of heat and mass transfer has been the object of extensive research due to its possible applications in many branches of Science and technology like geophysics, aeronautics, engineering and human medicine etc. In view of these applications, a series of investigations were made to study heat and mass transfer. Gersten and Gross (1974) analysed the flow and heat transfer along a plane wall with periodic suction.

Singh and Rana (1992) studied the effect of periodic suction velocity on the heat transfer and the flow through highly porous medium bounded by a flat surface at constant temperature. The study of heat transfer in mercury and electrolytic solution past an infinite porous plate with constant solution in presence of transverse magnetic field and heat sink were presented by Sahoo *et al.* (2003). Singh and Takhar (2007) have investigated the effectsof heat and mass transfer on the three dimensional flow of viscous fluid along an infinite porous vertical plate with periodic suction velocity. Hayat *et al.* (1998) have analysed the periodic unsteady flow of a non-newtonian fluid. Ogulu and Prakash (2006) discussed the heat transfer of unsteady magnetohydrodynamic flow past an infinite vertical moving plate with variable suction. Uwanta (2009) analysed mass transfer of free convective flow over a vertical plate with heat sink and jumped wall temperature. Ericson *et al.* (1965) examined heat and mass transfer on a moving continuous flat plate with suction or injection. Jha (2001) investigated transient free-convective flow in a vertical channel with heat

sinks. Recently, Ahmed *et al.* (2010) analysed the free convective oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity and a constant free stream velocity. Das and Mishra (2009) studied unsteady mixed convective MHD flow and mass transfer past an accelerated infinite vertical porous plate with suction. Effects of variable suction and thermophoresis on steady MHD combine free-forced convective semi-infinite permeable inclined plate in the presence of thermal radiation is studied by Alam *et al.* (2008). Govindarajulu and Thangaraj (1992) investigated the effect of variable suction on free convection over a vertical plate in a porous medium. Jaiswaf and Soundalgekar (2001) presented the oscillatory plate temperature effects on a flow past an infinite vertical porous plate with constant suction and embedded in a porous medium. Kim (2000) discussed unsteady Magneto Hydrodynamic (MHD) convective heat transfer past a semi infinite vertical porous moving plate with variable suction. Chen (2004) studied heat and Mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and convection. Eswara and Bommaiah (2004) analysed the effect of variable viscosity on laminar flowdue to a point sink. Eswara *et al.* (2004) studied unsteady MHD forced flow due to a point sink. Ibrahim *et al.* (2008) worked on the effects of the chemical reactions and radiations absorption on the unsteady MHD free convection flowpast a semi-infinite vertical permeable moving plate with heat source and suction. Mahanti and Gaur (2009) considered effects of varying viscosity and thermal conductivity on steady

Corresponding Author: K.K. Asogwa, Department of Mathematics, Usmanu Danfodiyo University, P.M.B 2346, Sokoto State of Nigeria

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free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink.

In this study, we extend the study of Muthucumaraswamy *et al.* (2009) to include suction parameter, heat sink and chemical reaction and the boundary conditions for velocity, concentration and temperature differs. It is assume that the effect of viscous dissipation is negligible in the energy equation and there is a chemical reaction between the diffusing species and the fluid.

FORMULATION OF THE PROBLEM

Consider heat and mass transfer over a vertical plate with periodic suction and heat sink. The plate is subjected to a periodic suction and heat sink moving in its own plane with a velocity $u = 0$ the x-axis is taken along the plate vertically upwards and the y- axis is normal to it.

The equations for the fluid flow are momentum; energy and mass concentration are as follow given by:

$$\frac{\partial u^*}{\partial t'} - v_0^* (1 + \varepsilon e^{i\omega t'}) \frac{\partial u^*}{\partial y'} = g\beta(T - T_\infty) + g\beta^*(C^* - C_\infty) + \nu \frac{\partial^2 u^*}{\partial y'^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} - v_0^* (1 + \varepsilon e^{i\omega t'}) \frac{\partial T}{\partial y'} = k \frac{\partial^2 T}{\partial y'^2} - Q_0 (T - T_\infty) \quad (2)$$

$$\frac{\partial C^*}{\partial t'} - v_0^* (1 + \varepsilon e^{i\omega t'}) \frac{\partial C^*}{\partial y'} = D \frac{\partial^2 C^*}{\partial y'^2} - K^* C^* \quad (3)$$

where,

- u^* = The velocity of the fluid
- v_0^* = The suction parameter
- t = Time
- K^* = Chemical reaction term
- ν = The kinematics viscosity
- g = The gravitational constant
- β = The thermal conductivity
- β^* = Modified thermal conductivity
- T = The temperature of the fluid
- k = Thermal conductivity
- ρ = Density
- C_p = Heat capacity at constant pressure
- D = Diffusion term
- C^* = The mass concentration
- Q_0 = The heat sink term
- y' = Distance
- ω = A frequency parameter
- ε = Perturbation parameter

With the following initial and boundary conditions:

$$\left. \begin{aligned} u = 0, T = T_\infty + \varepsilon A(T_\omega - T_\infty)e^{i\omega t}, C^* = C_\omega^* + \varepsilon B(C_\omega^* - C_\infty)e^{i\omega t}, y = 0 \\ u \rightarrow 0, T \rightarrow 0, C \rightarrow 0, y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

where, A and B are constants.

Introducing the following dimensionless variables:

$$\left. \begin{aligned} U = \frac{u}{(v u_0)^{\frac{1}{3}}}, t = t' \left(\frac{u_0^2}{\nu} \right)^{\frac{1}{3}}, Y = y \left(\frac{u_0}{\nu} \right)^{\frac{1}{3}} \\ \theta = \frac{T - T_\infty}{A(T_\omega - T_\infty)}, Gr = \frac{g\beta(T_\omega - T_\infty)}{u_0}, C = \frac{C^* - C_\infty^*}{B(C_\omega^* - C_\infty^*)} \\ Gc = \frac{g\beta^*(C_\omega^* - C_\infty^*)}{u_0}, Pr = \frac{\mu C_p}{\kappa}, Sc = \frac{\nu}{D} \\ s = \frac{v_0^*}{(v u_0)^{\frac{1}{3}}}, b = \frac{1}{\rho C_p}, F = \frac{Q_0 \nu^{\frac{1}{3}}}{\rho C_p u_0^{\frac{2}{3}}}, K = K^* \left(\frac{\nu}{u_0^2} \right)^{\frac{1}{3}} \times \frac{1}{B(C_\omega^* - C_\infty^*)} \end{aligned} \right\} \quad (5)$$

Substituting the dimensionless variables of Eq. (5) into (1) to (4), lead to:

$$\frac{\partial u}{\partial t} - s(1 + \varepsilon e^{i\omega t}) \frac{\partial u}{\partial Y} = GrA\theta + GcBC + \frac{\partial^2 u}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} - bs(1 + \varepsilon e^{i\omega t}) \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - F\theta \quad (7)$$

$$\frac{\partial C}{\partial t} - s(1 + \varepsilon e^{i\omega t}) \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \quad (8)$$

where,

- s = The suction parameter
- F = Heat sink
- K = The chemical reaction parameter
- b = constant
- Gr = The thermal Grashof number
- Gc = The mass grashof number
- Sc = The Schmidt number
- Pr = The Prandtl number

The imposed boundary conditions are:

$$\left. \begin{aligned} u = 0, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{at } y \rightarrow \infty \end{aligned} \right\} \quad (9)$$

METHOD OF SOLUTION

We assume that ε is small, therefore, we seek the solutions to (10), (11) and (12) having the form:

$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t} \quad (10)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t} \quad (11)$$

$$C(y, t) = C_0(y) + \varepsilon C_1(y) e^{i\omega t} \quad (12)$$

Where, $u_0(y)$, $C_0(y)$, $\theta_0(y)$, $u_1(y)$, $C_1(y)$ and $\theta_1(y)$ are to be determined.

Substituting (10) to (12) into (6) to (8) we obtain the following equations:

$$u_0'' + su_0' = -Gr\theta_0 - GcC_0 \quad (13)$$

$$\theta_0'' + Pr bs\theta_0' - Pr F\theta_0 = 0 \quad (14)$$

$$C_0'' + sScC_0' - KScC_0 = 0 \quad (15)$$

$$u_1'' + Nu_1' = -Gr\theta_1 - GcC_1 - su_1' \quad (16)$$

$$\theta_1'' + Pr bs\theta_1' - Pr(F + i\omega)\theta_1 = -Pr bs\theta_0' \quad (17)$$

$$C_1'' + sScC_1' - (K + i\omega)ScC_1 = -sScC_0' \quad (18)$$

All primes denote differentiation with respect to y .

The boundary conditions are:

$$\left. \begin{aligned} u_0 = 0, \theta_0 = 1, C_0 = 1 & \quad \text{on } y = 0 \\ u_1 = 0, \theta_1 = 1, C_1 = 1 & \quad \text{on } y = 0 \\ u_0 \rightarrow 0, \theta_0 \rightarrow 0, C_0 \rightarrow 0 & \quad \text{as } y \rightarrow \infty \\ u_1 \rightarrow 0, \theta_1 \rightarrow 0, C_1 \rightarrow 0 & \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (19)$$

Solving Eq. (13) to (15) subject to the boundary conditions (20) we obtain:

$$\begin{aligned} u_0(y) = & \left[Gr \left(\frac{1}{H_1} + \frac{H_2^2}{H_1^2 - H_2^2 H_1} \right) + Gc \left(\frac{1}{s_1} + \frac{s_2^2}{s_1^3 - s_2^2 s_1} \right) \right] e^{-\sigma y} \\ & - Gr \left(\frac{1}{H_1} + \frac{H_2^2 Gr}{H_1^2 - H_2^2 H_1} \right) \cosh(\lambda y) e^{-\beta y} + \frac{H_2 Gc}{H_1^2 - H_2^2} \sinh(\lambda y) e^{-\sigma y} \\ & - Gc \left(\frac{1}{s_1} + \frac{s_2^2 Gc}{s_1^3 - s_2^2 s_1} \right) \cosh(\eta y) e^{-\beta y} + \frac{s_2 Gc}{s_1^2 - s_2^2} \sinh(\eta y) e^{-\sigma y} \end{aligned} \quad (20)$$

$$\theta_0(y) = \cosh(\lambda y) e^{-\sigma y} \quad (21)$$

$$C_0(y) = \cosh(\eta y) e^{-\beta y} \quad (22)$$

Solving Eq. (16) to (18) subject to the boundary conditions (20), we obtain:

$$\begin{aligned} u_1(y, t) = & M^* e^{-\sigma y} \\ & + \left(\frac{-Gr\Omega_1}{P_1} - \frac{(GrP_1P_2\Omega_2 + GrP_2^2\Omega_1)}{P_1^3 - P_2^2P_1} \right) \cosh(\lambda y) e^{-\sigma y} \\ & + \left(\frac{GrP_1\Omega_2 + GrP_2\Omega_1}{P_1^2 - P_2^2} \right) \sinh(\lambda y) e^{-\sigma y} \\ & + \left(\frac{-Gr\Omega_3}{F_1} - \frac{F_2^2 Gr\Omega_3}{F_1^3 - F_2^2 F_1} \right) \cosh(\sigma y) e^{-\sigma y} \\ & + \left(\frac{F_2 Gr\Omega_3}{F_1^2 - F_2^2} \right) \sinh(\sigma y) e^{-\sigma y} \\ & + \left(\frac{-Gr\Omega_3}{F_1} - \frac{F_2^2 Gr\Omega_3}{F_1^3 - F_2^2 F_1} \right) \cosh(\sigma y) e^{-\sigma y} \\ & + \left(\frac{F_2 Gr\Omega_3}{F_1^2 - F_2^2} \right) \sinh(\sigma y) e^{-\sigma y} \\ & + \left(\frac{-Gc\alpha_1}{R_1} - \frac{(GcR_1R_2\alpha_2 + GcR_2^2\alpha_1)}{R_1^3 - R_2^2R_1} \right) \cosh(\eta y) e^{-\beta y} \\ & + \left(\frac{GcR_1\alpha_2 + GcR_2\alpha_1}{R_1^2 - R_2^2} \right) \sinh(\eta y) e^{-\beta y} \\ & + \left(\frac{-Gc\alpha_3}{L_1} - \frac{L_2^2 Gc\alpha_3}{L_1^3 - L_2^2 L_1} \right) \cosh(\rho y) e^{-\beta y} \\ & + \left(\frac{L_2 Gc\alpha_3}{L_1^2 - L_2^2} \right) \sinh(\rho y) e^{-\beta y} + \frac{sW_1}{s - N} e^{-\sigma y} \\ & + \left(\frac{-s(fW_2 + \lambda W_3)}{G_1} - \frac{G_1 G_2 s(\lambda W_2 + fW_3) + G_2^2 s(fW_2 + \lambda W_3)}{G_1^3 - G_2^2 G_1} \right) \cosh(\lambda y) e^{-\beta y} \\ & + \left(\frac{G_1 s(\lambda W_2 + fW_3) + G_2 s(fW_2 + \lambda W_3)}{G_1^2 - G_2^2} \right) \sinh(\lambda y) e^{-\beta y} \\ & + \left(\frac{-s(fW_4 + \eta W_5)}{I_1} - \frac{I_1 I_2 s(\eta W_4 + fW_5) - I_2^2 s(fW_4 + \eta W_5)}{I_1^3 - I_2^2 I_1} \right) \cosh(\eta y) e^{-\beta y} \\ & + \left(\frac{sI_1(\eta W_4 + fW_5) + sI_2(fW_4 + \eta W_5)}{I_1^2 - I_2^2} \right) \sinh(\eta y) e^{-\beta y} \end{aligned} \quad (23)$$

$$\begin{aligned} \theta_1(y, t) = & \left(1 - \frac{X_1}{z_1} - \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1} \right) \cosh(\sigma y) e^{-\sigma y} \\ & + \left(\frac{X_1}{z_1} + \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1} \right) \cosh(\lambda y) e^{-\sigma y} \\ & - \frac{(z_1 X_2 + z_2 X_1)}{z_1^2 - z_2^2} \sinh(\lambda y) e^{-\sigma y} \end{aligned} \quad (24)$$

$$C_1(y, t) = \left(1 - \frac{Y_1}{T_1} - \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1}\right) \cosh(\rho y) e^{-f y} + \left(\frac{Y_1}{T_1} + \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1}\right) \cosh(\eta y) e^{-f y} - \left(\frac{T_1 Y_2 + T_2 Y_1}{T_1^2 - T_2^2}\right) \sinh(\eta y) e^{-f y} \quad (25)$$

where,

$$a = \frac{\text{Pr } bs}{2}, \lambda = \sqrt{\frac{(\text{Pr } bs)^2 + 4 \text{Pr } F}{4}},$$

$$\sigma = \sqrt{\frac{(\text{Pr } bs)^2 + 4 \text{Pr } (F + i\omega)}{4}},$$

$$X_1 = \text{Pr } bsa, X_2 = \text{Pr } bs\lambda,$$

$$\mu = \text{Pr } bs, m_1 = \text{Pr } (F + i\omega),$$

$$z_1 = \lambda^2 - a\mu + a^2 - m_1,$$

$$z_2 = -2a\lambda + \mu\lambda, \eta = \sqrt{\frac{(sSc)^2 + 4KSc}{4}}$$

$$f = \frac{sSc}{2}, \rho = \sqrt{\frac{(sSc)^2 + 4Sc(K + i\omega)}{4}},$$

$$Y_1 = sScf, Y_2 = sSc\eta,$$

$$\mu_2 = sSc, m_2 = Sc(K + i\omega)$$

$$T_1 = \eta^2 - f\mu_2 + f^2 - m_2,$$

$$T_2 = -2f\eta + \mu_2\eta,$$

$$H_1 = \lambda^2 - as + a^2,$$

$$H_2 = -2a\lambda + \lambda s$$

$$s_1 = \eta^2 - fs + f^2, s_2 = -2f\eta + s\eta,$$

$$N = s - i\omega, \Omega_1 = \frac{X_1}{z_1} + \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1}$$

$$\Omega_2 = \frac{z_1 X_2 + z_2 X_1}{z_1^2 - z_2^2}$$

$$P_1 = \lambda^2 + a^2 + aN,$$

$$P_2 = -2a\lambda - \lambda N,$$

$$\Omega_3 = \left(1 - \frac{X_1}{z_1} - \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1}\right),$$

$$F_1 = \sigma^2 + a^2 + aN$$

$$F_2 = -2a\sigma - \sigma N,$$

$$\alpha_1 = \frac{Y_1}{T_1} + \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1},$$

$$\alpha_2 = \frac{T_1 Y_2 + T_2 Y_1}{T_1^2 - T_2^2},$$

$$R_1 = \eta^2 + f^2 + fN,$$

$$R_2 = -2f\eta - \eta N$$

$$\alpha_3 = \left(1 - \frac{Y_1}{T_1} - \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1}\right),$$

$$L_1 = \rho^2 + f^2 + fN,$$

$$L_2 = -2f\rho - \rho N$$

$$W_1 = \left(\frac{Gr}{H_1} + \frac{GrH_2^2}{H_1^3 - H_2^2 H_1} + \frac{Gc}{s_1} + \frac{Gcs_2^2}{s_1^3 - s_2^2 s}\right),$$

$$W_2 = \left(\frac{Gr}{H_1} + \frac{GrH_2^2}{H_1^3 - H_2^2 H_1}\right),$$

$$W_3 = \left(\frac{H_2 Gr}{H_1^2 - H_2^2}\right)$$

$$W_4 = \left(\frac{Gc}{s_1} + \frac{Gcs_2^2}{s_1^3 - s_2^2 s}\right),$$

$$W_5 = \left(\frac{s_2 Gc}{s_1^2 - s_2^2}\right),$$

$$G_1 = \lambda^2 + f^2 - fN,$$

$$G_2 = -2f\lambda + \lambda N,$$

$$I_1 = \eta^2 + f^2 - fN$$

$$I_2 = -2f\eta + \eta N,$$

$$M^* = \frac{Gr\Omega_1}{P_1} + \frac{GrP_1 P_2 \Omega_2 + GrP_2^2 \Omega_1}{P_1^2 - P_2^2 P_1}$$

$$+ \frac{Gr\Omega_3}{F_1} + \frac{F_2^2 Gr\Omega_3}{F_1^3 - F_2^2 F_1}$$

$$+ \frac{Gc\alpha_1}{R_1} + \frac{GcR_1 R_2 \alpha_2 + GcR_2^2 \alpha_1}{R_1^3 - R_2^2 R_1}$$

$$+ \frac{Gc\alpha_3}{L_1} + \frac{L_2^2 Gc\alpha_3}{L_1^3 - L_2^2 L_1} - \frac{sW_1}{s - N}$$

$$+ \frac{s(fW_2 + \lambda W_3)}{G_1} +$$

$$\frac{G_1 G_2 s(\lambda W_2 + fW_3) + G_2^2 s(fW_2 + \lambda W_3)}{G_1^3 - G_2^2 G_1}$$

$$+ \frac{s(fW_4 + \eta W_5)}{I_1} +$$

$$\frac{I_1 I_2 s(\eta W_4 + fW_5) - I_2^2 s(fW_4 + \eta W_5)}{I_1^3 - I_2^2 I_1}$$

Substituting (20) to (25) into (10) to (12) we have the solution for velocity, temperature and concentration as follows:

$$u(y, t) = \left(\frac{Gr}{H_1} + \frac{GrH_2^2}{H_1^3 - H_2^2 H_1} + \frac{Gc}{s_1} + \frac{Gcs_2^2}{s_1^3 - s_2^2 s}\right) e^{-sy} - \left(\frac{Gr}{H_1} + \frac{GrH_2^2}{H_1^3 - H_2^2 H_1}\right) \cosh(\lambda y) e^{-f y}$$

$$\begin{aligned}
 & + \left(\frac{H_2 Gr}{H_1^2 - H_2^2} \right) \sinh(\lambda y) e^{-fy} - \left(\frac{Gc}{s_1} + \frac{Gc s_2^2}{s_1^3 - s_2^2 s} \right) \cosh(\eta y) e^{-fy} \\
 & + \left(\frac{s_2 Gc}{s_1^2 - s_2^2} \right) \sinh(\eta y) e^{-fy} + \varepsilon \left[M^* e^{-sy} \right. \\
 & + \left. \left(\frac{-G\Omega}{P_1} - \frac{(GrP_1\Omega_2 + GrP_2\Omega_1)}{P_1^3 - P_2^2 P_1} \right) \cosh(\lambda y) e^{-\sigma y} + \left(\frac{GrP_1\Omega_2 + GrP_2\Omega_1}{P_1^3 - P_2^2 P_1} \right) \sinh(\lambda y) e^{-\sigma y} \right. \\
 & + \left. \left(\frac{-Gr\Omega_3}{F_1} - \frac{F_2^2 Gr\Omega_3}{F_1^3 - F_2^2 F_1} \right) \cosh(\sigma y) e^{-\sigma y} + \left(\frac{F_2 Gr\Omega_3}{F_1^3 - F_2^2 F_1} \right) \sinh(\sigma y) e^{-\sigma y} \right. \\
 & + \left. \left(\frac{-Gc\alpha_1}{R_1} - \frac{(GcR_1R_2\alpha_2 + GcR_2^2\alpha_1)}{R_1^3 - R_2^2 R_1} \right) \cosh(\eta y) e^{-fy} \right. \\
 & + \left. \left(\frac{GcR_1\alpha_2 + GcR_2\alpha_1}{R_1^3 - R_2^2 R_1} \right) \sinh(\eta y) e^{-fy} \right. \\
 & + \left. \left(-\frac{Gc\alpha_3}{L_1} - \frac{L_2^2 Gc\alpha_3}{L_1^3 - L_2^2 L_1} \right) \cosh(\rho y) e^{-fy} \right. \\
 & + \left. \left(\frac{L_2 Gc\alpha_3}{L_1^2 - L_2^2} \right) \sinh(\rho y) e^{-fy} + \frac{sW_1}{s - N} e^{-sy} \right. \\
 & + \left. \left(\frac{-s(fW_2 + \lambda W_3)}{G_1} - \frac{G_1 G_2 s(\lambda W_2 + fW_3) + G_2^2 s(fW_2 + \lambda W_3)}{G_1^3 - G_2^2 G_1} \right) \cosh(\lambda y) e^{-fy} \right. \\
 & + \left. \left(\frac{G_1 s(\lambda W_2 + fW_3) + G_2 s(fW_2 + \lambda W_3)}{G_1^3 - G_2^2 G_1} \right) \sinh(\lambda y) e^{-fy} \right. \\
 & + \left. \left(\frac{-s(fW_4 + \eta W_5)}{I_1} - \frac{I_1 I_2 s(\eta W_4 + fW_5) - I_2^2 s(fW_4 + \eta W_5)}{I_1^3 - I_2^2 I_1} \right) \cosh(\eta y) e^{-fy} \right. \\
 & + \left. \left(\frac{sI_1(\eta W_4 + fW_5) + sI_2(fW_4 + \eta W_5)}{I_1^3 - I_2^2 I_1} \right) \sinh(\eta y) e^{-fy} \right] e^{i\omega t} \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 \theta(y, t) &= \cosh(\lambda y) e^{-ay} \\
 & + \varepsilon \left(\left(1 - \frac{X_1}{z_1} - \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1} \right) \cosh(\sigma y) e^{-ay} \right. \\
 & + \left(\frac{X_1}{z_1} + \frac{z_1 z_2 X_2 + z_2^2 X_1}{z_1^3 - z_2^2 z_1} \right) \cosh(\lambda y) e^{-ay} \\
 & - \left. \frac{(z_1 X_2 + z_2 X_1)}{z_1^2 - z_2^2} \sinh(\lambda y) e^{-ay} \right) e^{i\omega t} \quad (27)
 \end{aligned}$$

$$\begin{aligned}
 C(y, t) &= \cosh(\eta y) e^{-fy} \\
 & + \varepsilon \left(\left(1 - \frac{Y_1}{T_1} - \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1} \right) \cosh(\rho y) e^{-fy} \right. \\
 & + \left(\frac{Y_1}{T_1} + \frac{T_1 T_2 Y_2 + T_2^2 Y_1}{T_1^3 - T_2^2 T_1} \right) \cosh(\eta y) e^{-fy} \\
 & - \left. \left(\frac{T_1 Y_2 + T_2 Y_1}{T_1^2 - T_2^2} \right) \sinh(\eta y) e^{-fy} \right) e^{i\omega t} \quad (28)
 \end{aligned}$$

RESULTS AND DISCUSSION

Heat and mass transfer over a vertical plate with periodic suction and heat sink has been formulated, analysed and solved analytically. In order to point out the effects of physical parameters namely; suction parameter s , Heat sink F , thermal Grashof number Gr , mass Grashof number Gc , Prandtl number Pr , Schmidt number Sc , time t and chemical reaction parameter K . on the flow patterns, the computation of the flow fields are carried out. The value of the Prandtl number Pr is chosen to represent air ($Pr = 0.71$). The value of Schmidt number is chosen to represent water vapour ($Sc = 0.6$). The values of velocity, temperature and concentration are obtained for the physical parameters as presented below.

The velocity profiles has been studied and presented in Fig. 1 to 7. The velocity profiles is studied for different values of Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$) and is presented in Fig. 1. It is observed that velocity increases with decreasing Schmidt number. The velocity profiles is studied for various values of Prandtl number ($Pr = 0.71, 0.85, 1$) and is presented in Fig. 2. It is observed that Pr velocity increases with decreasing Prandtl number.

The velocity profiles is studied for different values of suction parameter ($s = 0.1, 0.2, 0.3, 0.4$) and is presented in Fig. 3. It is observed that velocity increases with increasing suction parameter. The velocity profiles

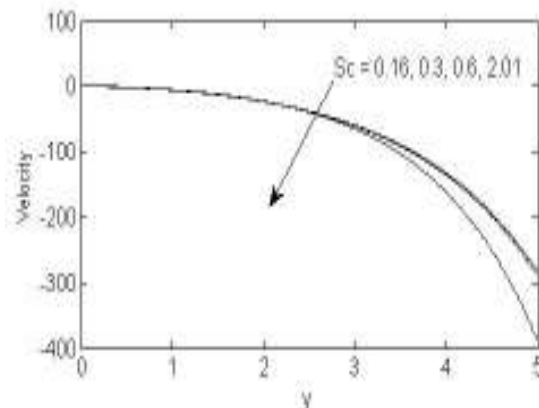


Fig. 1: Velocity profiles for different values of Sc

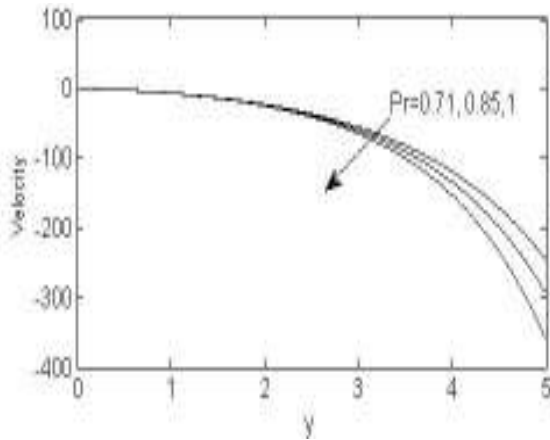


Fig. 2: Velocity profiles for different values of Pr

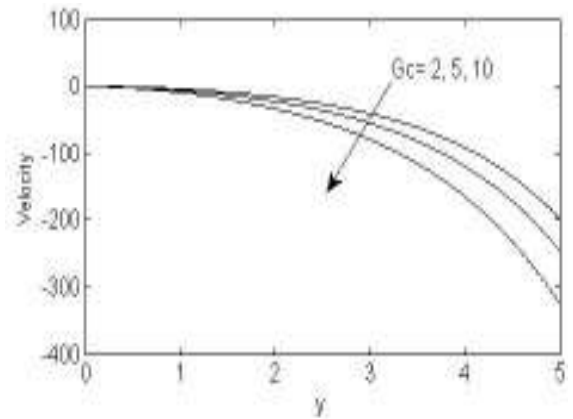


Fig. 5: Velocity profiles for different values of Gc

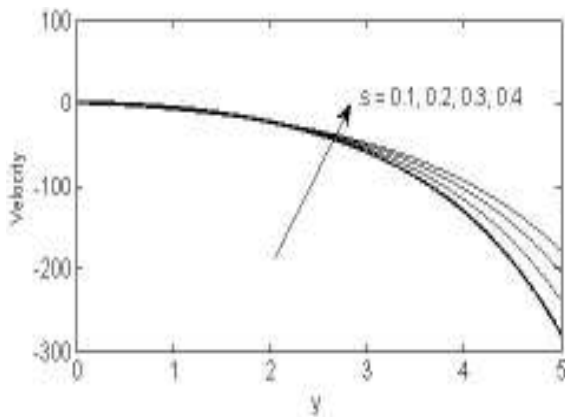


Fig. 3: Velocity profiles for different values of s

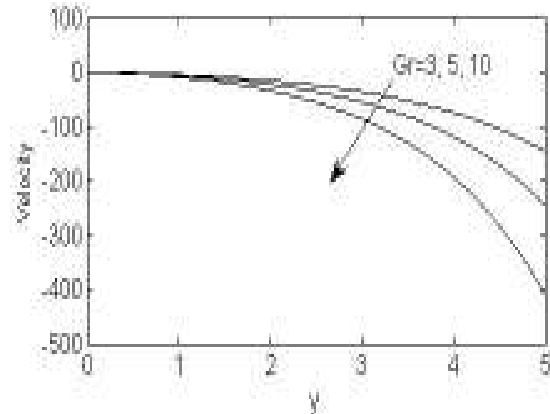


Fig. 6: Velocity profiles for different values of Gr

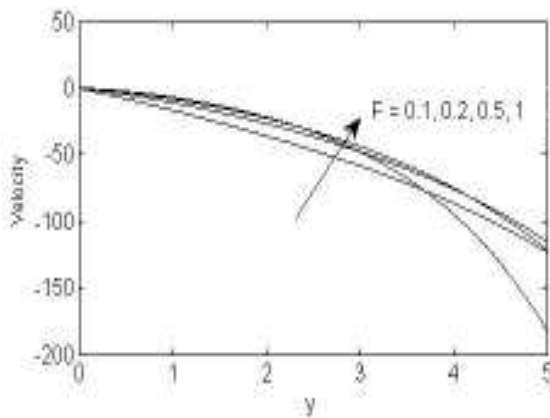


Fig. 4: Velocity profiles for different values of F

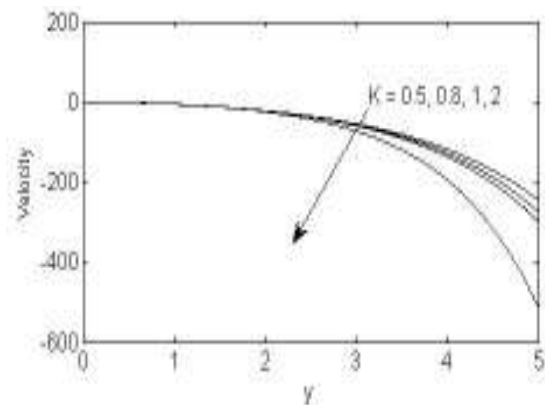


Fig. 7: Velocity profiles for different values of K

is studied for various values of heat sink ($F = 0.1, 0.2, 0.5, 1.0$) and is presented in Fig. 4. It is observed that velocity increases with increasing heat sink. The velocity profiles is studied for different values of mass and thermal Grashof number ($Gc = 2, 5, 10$) and ($Gr = 3, 5, 10$) are presented in Fig. 5 and 6, respectively. It is observed that velocity increases with decreasing Gc and Gr , respectively. The velocity profiles is studied for

various values of chemical reaction parameter ($K = 0.5, 0.8, 1, 2$) and is presented in Fig. 7. It is observed that velocity increases with decreasing values of K .

The temperature profiles has been studied and presented in Fig. 8 to 10. The effects of temperature for different values of heat sink ($F = 0.1, 0.2, 0.5, 1.0$) is presented in Fig. 8 from the graph, it shows that temperature increases with increasing F . The effects of temperature for different values of Prandtl number (Pr

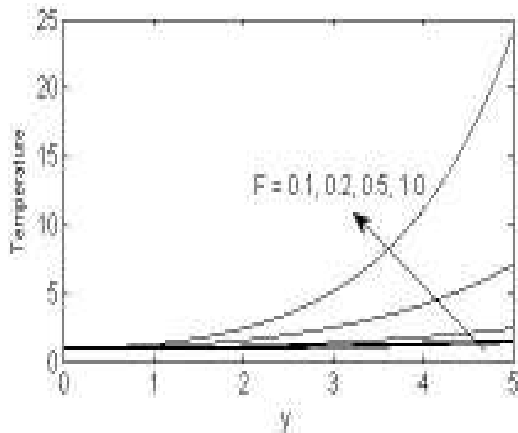


Fig. 8: Temperature profiles for different values of F

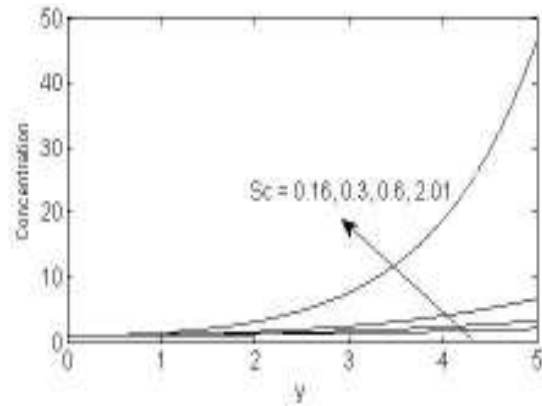


Fig. 11: Concentration profiles for different values of Sc

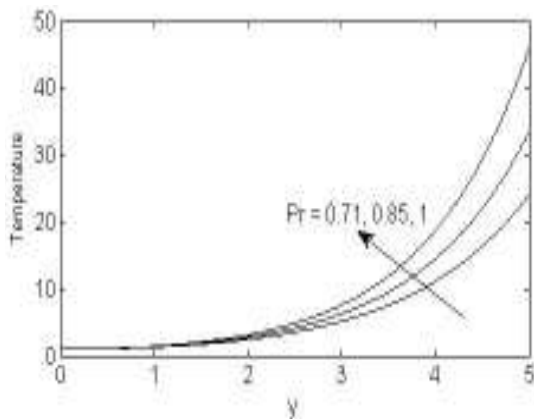


Fig. 9: Temperature profiles for different values of Pr

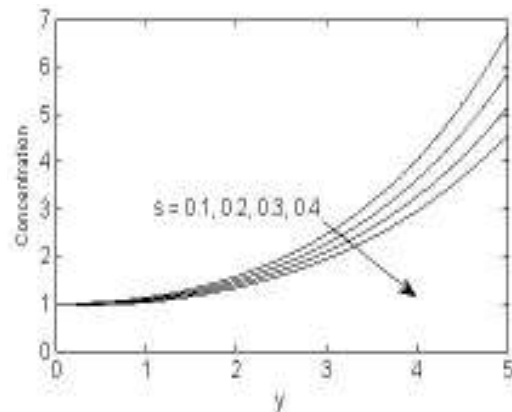


Fig. 12: Concentration profiles for different values of s

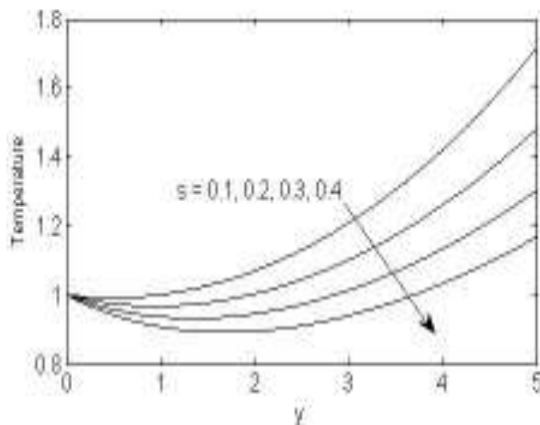


Fig. 10: Temperature profiles for different values of s

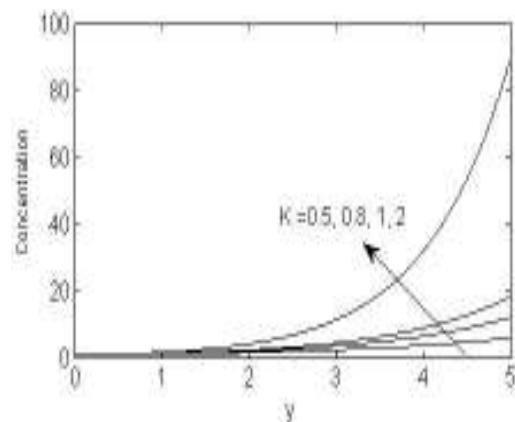


Fig. 13: Concentration profiles for different values of K

$= 0.71, 0.85, 1$) are computed and presented in Fig. 9. It is observed that the temperature of the flow increases with increasing values Pr. The effects of temperature for different values of suction parameter ($s = 0.1, 0.2, 0.3, 0.4$) are computed and presented in Fig. 10. It is observed that the temperature of the flow increases with decreasing values of s.

The concentration profiles has been studied and presented in Fig. 11 to 13. The concentration profiles for different values of Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$) is presented in Fig. 11. It is observed that the concentration increases with increasing Schmidt number. The concentration profiles for different values of suction parameter ($s = 0.1, 0.2, 0.3, 0.4$) is presented in Fig. 12. It is observed that the concentration

Table 1: Skinfriction τ

Gr	Gc	F	S	ϵ	ω	Pr	K	Sc	τ
2	3	0.1	0.2	0.001	0.1	0.71	0.5	0.6	-6.7498
2	5	0.1	0.2	0.001	0.1	0.71	0.5	0.6	-7.1490
3	5	0.1	0.2	0.001	0.1	0.71	0.5	0.6	-10.2248
5	5	0.2	0.4	0.002	0.2	0.85	0.5	0.6	-16.2202
5	5	0.2	0.4	0.002	0.2	0.85	1	0.3	-12.3868
5	10	0.2	0.4	0.002	0.2	0.85	1	0.3	-12.3896

Table 2: Nusselt number Nu

F	s	ϵ	ω	Pr	Nu
0.1	0.2	0.001	0.1	0.71	-0.0711
0.1	0.3	0.001	0.2	0.71	-0.1067
0.2	0.4	0.001	0.1	0.71	-0.1421
0.2	0.2	0.001	0.1	0.85	-0.0851
0.2	0.4	0.002	0.2	0.85	-0.1701
0.2	0.4	0.002	0.2	1	-0.2002

Table 3: Sherwood number Sh

K	s	ϵ	ω	Sc	Sh
0.5	0.2	0.001	0.1	0.6	-0.0601
0.5	0.2	0.001	0.2	0.16	-0.0160
0.5	0.4	0.001	0.1	0.6	-0.1201
1	0.2	0.001	0.1	0.22	-0.0220
1	0.2	0.002	0.2	0.3	-0.0301
1	0.4	0.002	0.2	0.3	-0.0601

increases with decreasing suction parameter. The concentration profiles for different values of chemical reaction parameter ($K = 0.5, 0.8, 1, 2$) is presented in Fig. 13. It is observed that the concentration increases with increasing K.

Table 1 to 3 gives the Skin friction, Nusselt number and Sherwood number, respectively.

Table 1 shows the effect of physical parameters Gr, Gc, F, s, Sc, K, ϵ , ω and Pr on the skin friction. Its observed that shear stress increases when Gr, Gc, F, s, Sc, K, ϵ , ω and Pr increases.

Table 2 shows the variation of physical parameters F, s, Pr, ϵ and ω on the nusselt number which determines the rate of heat transfer. It's observed that rate of heat transfer increases when F, s, Pr, ϵ and ω increases.

Table 3 shows the effect of physical parameters K, s, Sc, ϵ and ω the Sherwood number. It's observed that mass transfer increases when K, s, Sc, ϵ and ω increases.

CONCLUSION

Heat and mass transfer over a vertical plate with periodic suction and heat sink has been studied. The dimensional governing equations are solved by perturbation technique. In order to point out the effects of physical parameters namely; suction parameter, Heat sink, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and chemical reaction parameter K are presented graphically and tabulated on the flow patterns. It is observed that velocity profile increases with increasing s and F.

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