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Research Article

Research on the Robustness of an Adaptive PID Control of a Kind of Supersonic Missile

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Abstract: In this study, the dynamic characteristic of missile system is viewed as a two-loop system, such as inner loop and outer loop and we design an adaptive PID control strategy for the pitch channel linear model of supersonic missile. The robustness of a double PID controller is analyzed by changing the aerodynamic coefficients. The control law is testified to be stable even the aerodynamic coefficients are changed between 0.7 and 1.7 times of its standard value and the control effect is compared with the sliding mode control strategy. Also the advantage and defect of both control strategy are summarized at the end of this study.

Keywords: Adaptive, missile, robustness, sliding mode, supersonic

INTRODUCTION

Robustness is one of most important characteristics of a controller designed for high speed air-vehicles. It not only demands a controller to be stable in the situation of standard aerodynamic coefficients, but also the controller should have good performance if the real aerodynamic coefficients are largely different from the standard value (Gao and James, 1993; Polycarpou and Ioannou, 1996; Seung-Hwan *et al*., 2004; Tang and Ling, 2005; Elabbasy *et al*., 2006; Qian and Yanan, 2000; Tian-Bo, 2010). It really happens when an air-vehicle is flying in bad air conditions or the weather is totally different from the designers assumed.

It is not easy to analyze the robustness of a real controller applied in aircrafts since there is not enough theory available to analyze even the stability of a complex nonlinear system (Johansson and Wanhammar, 1999; Patil *et al*., 2008; Fujita *et al*., 1998). As the development of computer technology, it becomes easier to apply numerical simulations to testify the robustness of a control system. Simulation is widely used in engineering and it is still very important to choose a proper method to perform the tasks.

In this study, based on the multi-loop design method, the dynamic characteristic of supersonic missile system is viewed as a two loops system such as inner loop and outer loop. A double PID control strategy is designed for the pitch channel linear model of missile. The robustness of a double PID controller is analyzed by changing the aerodynamic coefficients.

PROBLEM DESCRIPTION

Consider the pitch channel model of anti-ship missile (Qian and Yanan, 2000):

$$\begin{cases} \dot{\alpha} = \omega_z - \dfrac{1}{mV}(P\sin\alpha + Y - mg\cos\theta) \\ \dot{\omega}_z = \dfrac{M_z}{J_z} \\ n_y = \dfrac{P\sin\alpha + Y}{mg} \end{cases} \tag{1}$$

where, α, θ, V are attack angle, trajectory inclination angle and velocity of the missile, respectively. For definition of other symbols, refer to Qian and Yanan (2000).

A theorem in Qian and Yanan (2000) shows that any linear feedback control law that can stabilize the linear approximate system can also stabilize the original nonlinear system only if the linear approximate system of the nonlinear system is gradual stable. Thus we can study the linear approximate system and then find a control law which can stabilize the original nonlinear system.

The linear approximate system can be described as follows:

$$\begin{cases} \dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \\ \dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z \\ n_y = \dfrac{V}{g}a_{34}\alpha + \dfrac{V}{g}a_{35}\delta_z \end{cases} \tag{2}$$

where, δ_z is the output angle of the elevating rudder and a_{34}, a_{35}, a_{22}, a_{24}, a_{25} are dynamic coefficients defined as follows:

$$a_{34} = \frac{g}{V}\cdot\frac{\partial n_y}{\alpha},\ a_{35} = \frac{\partial Y}{\delta_z}$$

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$$a_{22} = \frac{M_z^{\omega_z}}{J_z},\ a_{24} = \frac{M_z^{\alpha}}{J_z},\ a_{25} = \frac{M_z^{\delta_z}}{J_z}$$

The control objective is to design the input of elevating rudder, u, such that the overload of the system, n_y, can track the desired value n^d_y.

ADAPTIVE PID CONTROL STRATEGY

Through some transformations, the above model can be written as:

$$\begin{aligned} \dot{n}_y &= \frac{V}{g} a_{34}\omega_z - a_{34}\frac{V}{g} a_{34} f(n_y, \delta_z) \\ &\quad - a_{35}\frac{V}{g} a_{34}\delta_z + \frac{V}{g} a_{35}\dot{\delta}_z \\ \dot{\omega}_z &= a_{22}\omega_z + a_{24} f(n_y, \delta_z) + a_{25}\delta_z \end{aligned} \tag{3}$$

where, $f(n_y, \delta_z)$ can be solved by the above output equation.
For the above subsystem, define a new variable as:

$$e_1 = n_y - n_y^d$$

The error can be written as:

$$\dot{e}_1 = \frac{V}{g} a_{34}\omega_z - a_{34}\frac{V}{g} a_{34} f(n_y, \delta_z) - a_{35}\frac{V}{g} a_{34}\delta_z + \frac{V}{g} a_{35}\dot{\delta}_z \tag{4}$$

Define a new variable as:

$$f_1 = -a_{34}\frac{V}{g} a_{34} f(n_y, \delta_z) - a_{35}\frac{V}{g} a_{34}\delta_z + \frac{V}{g} a_{35}\dot{\delta}_z \tag{5}$$

Assume there exist two parameters d_{11} and d_{10}, such that:

$$|f_1| \le d_{11}|e_1| + d_{10} \tag{6}$$

Then design the virtual control as ω_z^d :

$$\omega_z^d = -k_{11}\operatorname{sgn}(a_{34})e_1 - k_{12}\operatorname{sgn}(a_{34})\int e_1 dt \tag{7}$$

Then it holds:

$$e_1\dot{e}_1 = -\frac{V}{g}|a_{34}|k_{11}e_1^2 - \frac{V}{g}|a_{34}|k_{11}e_1\int e_1 dt + f_1 e_1 + \frac{V}{g} a_{34} e_2 \tag{8}$$

Define an error variable as:

$$e_2 = \omega_z - \omega_z^d$$

the second subsystem can be written as:

$$\dot{e}_2 = a_{22}\omega_z + a_{24} f(n_y, \delta_z) + a_{25}\delta_z - \dot{\omega}_z^d \tag{9}$$

Similarly, a new variable can be defined as:

$$f_2 = a_{22}\omega_z + a_{24} f(n_y, \delta_z) + a_{25}\delta_z - \dot{\omega}_z^d \tag{10}$$

Assume there exists two positive constants d_{21} and d_{30}, such that:

$$|f_2| \le d_{21}|e_2| + d_{20} \tag{11}$$

With the same way, the virtual control is defined as:

$$\delta_z^d = -k_{21}\operatorname{sgn}(a_{22})e_2 - k_{22}\operatorname{sgn}(a_{22})\int e_2 dt \tag{12}$$

Then it holds:

$$e_2\dot{e}_2 = f_2 e_2 - k_{21}|a_{22}|e_2^2 - k_{22}|a_{22}|e_2\int e_2 dt + a_{22}e_3 \tag{13}$$

Then define e_3 as:

$$e_3 = \delta_z - \delta_z^d$$

For the actuator subsystem, it holds:

$$\dot{e}_3 = au - a\delta_z - \dot{\delta}_z \tag{14}$$

Since it is a certain system, it is easy to design the PID control law as:

$$u = -\frac{1}{a}(a\delta_z - k_{31}e_3 - k_{32}\int e_3 dt) \tag{15}$$

The control law proposed above is somewhat complex. Since the response of rudder is more quick than the other two variables, the controller for the above linear model can be simplified to a double PID control strategy as follows:

$$e_1 = n_y - n_y^d \tag{16}$$

$$\omega_z^d = -k_{11}\operatorname{sgn}(a_{34})e - k_{12}\operatorname{sgn}(a_{34})\int e dt \tag{17}$$

$$e_2 = \omega_z - \omega_z^d \tag{18}$$

$$\delta_z^d = -k_{21}\operatorname{sgn}(a_{22})e_2 - k_{22}\operatorname{sgn}(a_{22})\int e_2 dt \tag{19}$$

ROBUSTNESS OF CONTROLLER

In order to testify the robustness of the double PID control strategy, two kinds of experiments are performed as follows:

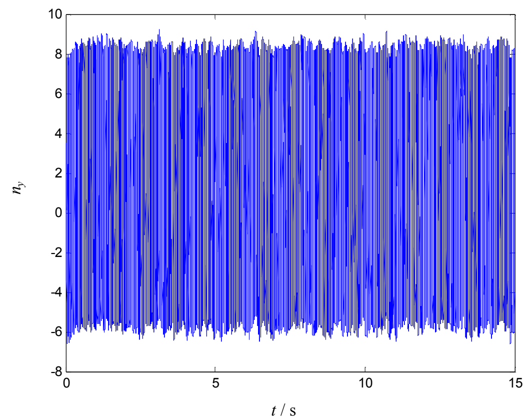

Fig. 1: Overload response for k = 1.8

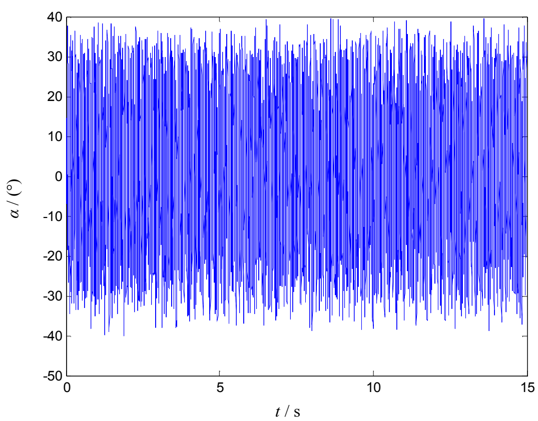

Fig. 2: Attack angle response for k = 1.8

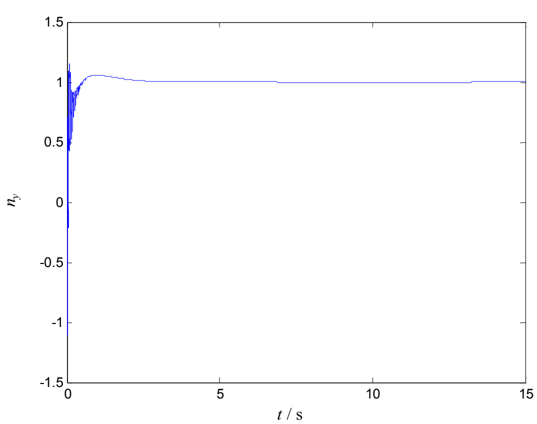

Fig. 3: Overload response for k =1.7

Consider the situation of change of aerodynamic parameters and use the same double PID control parameters and do the numerical simulation to check the performance of the controller.

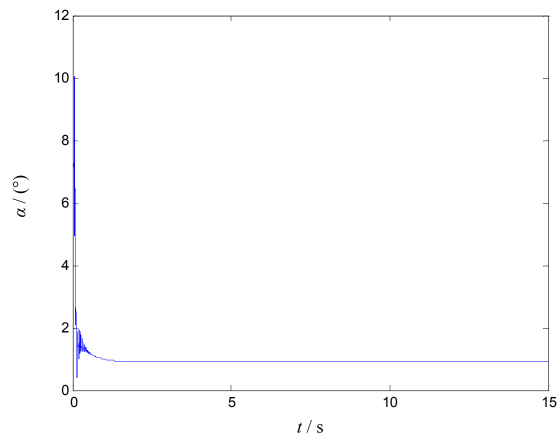

Fig. 4: Attack angle response for k = 1.7

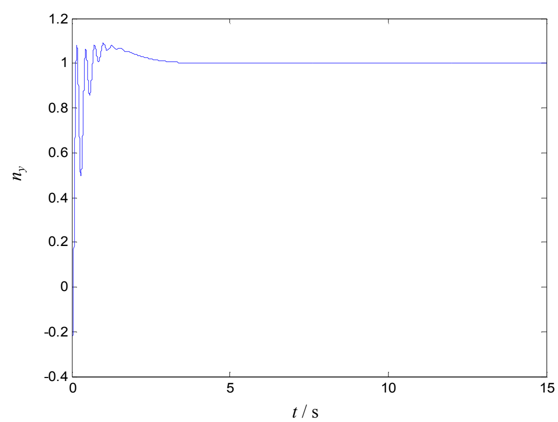

Fig. 5: Overload response for k = 1

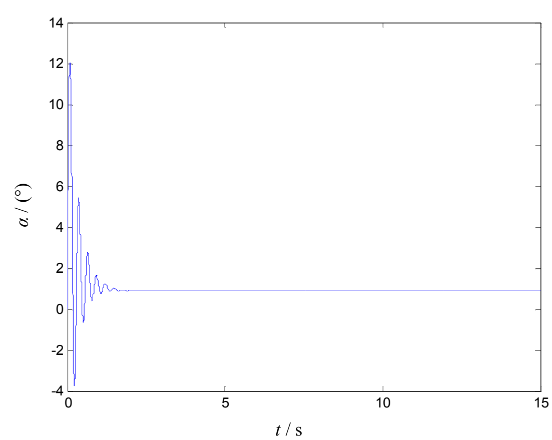

Fig. 6: Attack angle response for k = 1

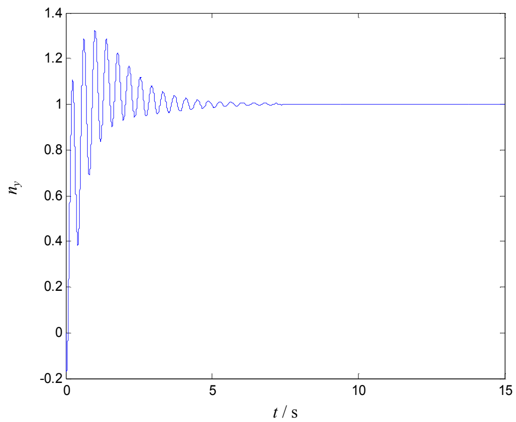

Fig. 7: Overload response for k = 0.7

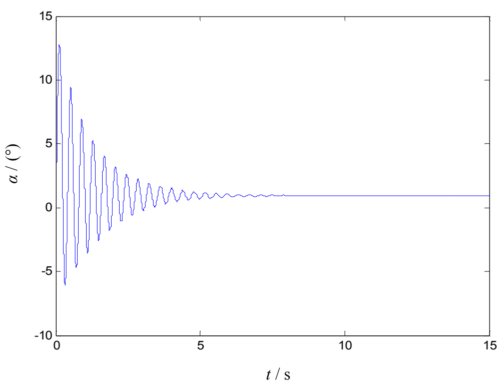

Fig. 8: Attack angle response for k = 0.7

The dynamic parameters of supersonic missile are chosen as follows: V = 749.65, g = 9.8, a = 50, a_{25} = 155k, a_{35} = 0.18k, a_{24} = 0.170k, a_{34} = 0.9k, a_{22} = −2.1k and the PID control is constructed with parameters as c_{d1} = 1.5, c_{i1} = 40, c_{p1} = 30, c_{k1} = −0.0008, c_{d2} = 0, c_{i2} = −6, c_{p2}, = −1, c_{k2}, = −5, choose the k as follows: k = 1.8, k = 1.7, k = 1 k = 0.7.

The simulation result of k = 1.8 is shown in Fig. 1 and 2. The simulation result of k = 1.7 is shown in Fig. 3 and 4. The simulation result of k = 1 is shown in Fig. 5 and 6. The simulation result of k = 0.7 is shown in Fig. 7 and 8.

Simulation results lead to the following conclusion: if the aerodynamic coefficients are increased as 1.8 times as its standard value, the control strategy will be unstable. And the control law is stable if aerodynamic coefficients are changed between 0.7 to 1.7.

COMPARISON WITH SLIDING MODE METHOD

Define a sliding mode surface as:

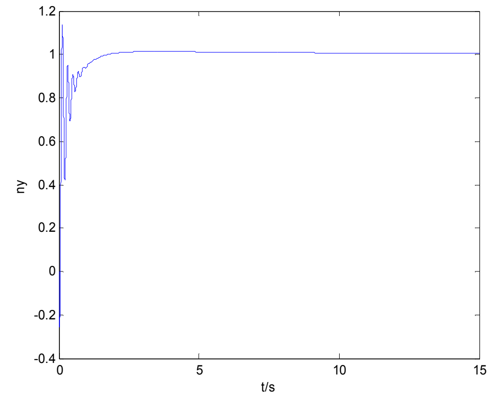

Fig. 9: Overload response for sliding mode control

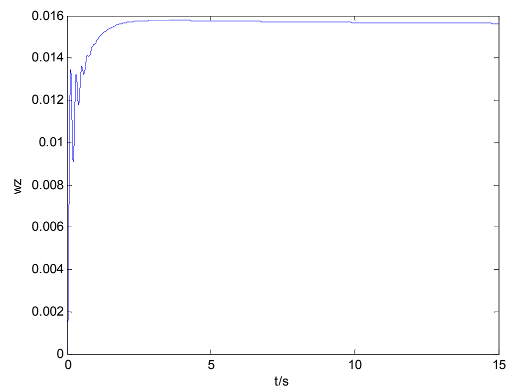

Fig. 10: Angle rate response for sliding mode control

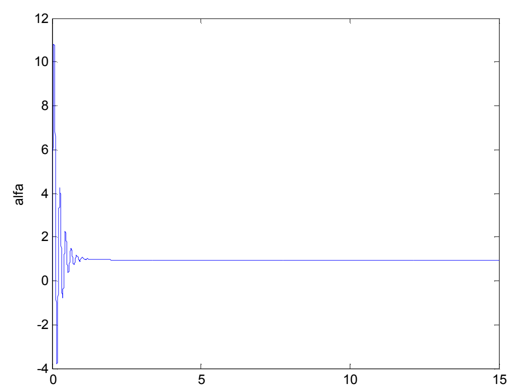

Fig. 11: Attack angle response for sliding mode control

$$s = c_1 e_1 + c_2 \int e_1 dt + c_3 \omega_z \tag{20}$$

In order to reduce the steady-state error, an integral Item is introduced into the control law, then a PISS control law is formed, where ‘P’ means proportional control, ‘I’ means integral control, the first ‘S’ means

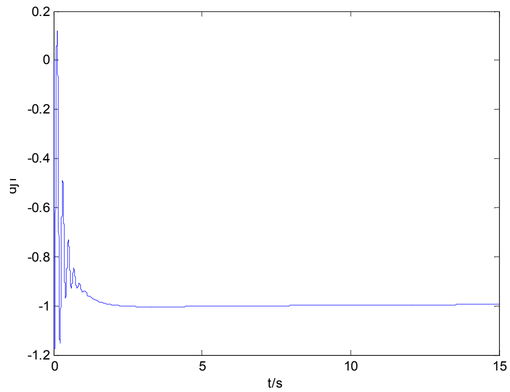

Fig. 12: Rudder response for sliding mode control

the sign function and the second 'S' means soft function. The hybrid control law can be written as:

$$\delta_z = -\operatorname{sgn}(c_3 a_{25}) k_1 s - \operatorname{sgn}(c_3 a_{25}) - k_3 \operatorname{sgn}(s) - \operatorname{sgn}(c_3 a_{25}) k_2 \frac{s}{|s| + \varepsilon} - \operatorname{sgn}(c_3 a_{25}) k_5 \int s dt \tag{21}$$

Remember that:

$$s \int s dt = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \left[\left(\int s dt \right)^2 \right] \tag{22}$$

Then choose the Lyapunov function as:

$$V = \frac{1}{2} \Upsilon_1 s^2 + \frac{1}{2} \Upsilon_2 \left[\left(\int s dt \right)^2 \right] \tag{23}$$

It is easy to prove that:

$$\mathrm{V} \leq 0 \tag{24}$$

So the stability of the control system is proved. Figure 9 to 12 show the control effect of sliding mode control law.

In comparison to the simulation results of sliding mode control, a conclusion can be drawn that the adaptive PID control method has a strong robustness, while the sliding mode control method has a quick response speed, but both its input and output response are unsmooth and chattering phenomenon is serious.

CONCLUSION

An adaptive PID control strategy is proposed to control a pitch channel model of a kind a supersonic missiles. The control strategy is simple but the simulation result shows that it has a strong robustness. Also a kind of sliding mode design method is given as an example and numerical simulation shows the control effect of sliding mode method. The comparison between the two methods is done and also a conclusion is made as follows: the adaptive PID control has a strong robustness but the sliding mode control has a quick response speed. Also the sliding mode control has a defect that the output and control response are not smooth and serious chattering problem is caused.

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