Research Journal of Applied Sciences, Engineering and Technology 5(1): 42-48, 2013
DOI:10.19026/rjaset.5.5082
ISSN: 2040-7459; E-ISSN: 2040-7467

Submitted: January, 14, 2012 Accepted: February 17, 2012 Published: January 01, 2013

Research Article

Nonlinear Phenomena in Buck-Boost Power Factor Correction Converter

[1]Mehrnoosh Vatani, [2]Soodabeh Soleymani and [3]Mehrdad Ahmadi Kamarposhti
[1]Department of Electrical Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran
[2]Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[3]Department of Electrical Engineering, Jouybar Branch, Islamic Azad University, Jouybar, Iran

Abstract: Buck-Boost power-factor-correction (PFC) converter with average-current-model (ACM) control is a nonlinear circuit because of the multiplier using and large change in the duty cycle, so its stability analysis must be studied by nonlinear model. In this paper double averaging method is used for describing the model of this converter. By this model we would be able to explain the low frequency dynamics of the system and identify stability boundaries according to circuit parameters and also nonlinear phenomena of this converter are detected.

Keywords: Averaged model, bifurcation, buck-boost converter, double averaging, power factor correction

INTRODUCTION

In traditional switching power supplies the input ac voltage is rectified by the usual capacitor filter following the input bridge rectifier, caused severely distorted input line current waveform. These lines current consists of very narrow spikes with fast rise and fall time. These kind of pulses cause so many problems such as radio frequency interface (RFI) problems and higher temperature rise and decreased reliability in filter capacitor (Pressman Abraham, 1998). Therefore the aim of power-factor-correction (PFC) is to maximize or correct the power factor by forcing the input line current to be sinusoidal, and in phase with the input line voltage, and as free from line harmonics as possible. Many researchers considered boost converter to achieve unity power factor because of its simplicity (Orabi and Ninomiya, 2003a, b). Apart from boost converter in many applications other dc-dc converter may be used for PFC circuits (Huliehel *et al*., 1992), for instance buck-boost converter. Buck-boost converter can be used for PFC application, because it has simple circuit and also its output voltage can be either higher (like a boost converter) or lower (like a buck converter) than the input voltage. In this paper we study buck-boost converter under average current mode (ACM) control. Lack of the consideration on the nonlinear model of buck-boost PFC converter made us to develop nonlinear model in order to identify the low frequency dynamics. This analytical nonlinear model is based on double averaging method, which first, applies the standard averaging over the switching period and then applies generalized averaging over the mains period (Chu *et al*., 2007). After applying averaging twice, over the switching period and the mains period, double averaged model will be obtained, therefore we are capable to identify stability boundaries according to circuit parameters.

PFC buck-boost converter under ACM control: The system under study is a buck-boost PFC converter under ACM control which consists of inductor L, diode D, switch Q and capacitor C connected in parallel to load R. The switch Q and the diode D are always in complementary operating states during the continuous-conduction-mode (CCM) operation. It means that the switch is in ON state until the diode is in OFF state and vice versa.

The control loop circuit is constructed of feedback and feed forward loop. We select ACM control because it is less sensitive to commutation noises, due to current filtering (Rossetto *et al*., 1994). Figure 1 shows the schematic of the buck-boost PFC converter under average current control (Yun *et al*., 2004).

By using ACM control we achieve near unity power factor, so the input current follow the input voltage and the “ideally shaped” input current waveform can be expressed as:

$$i_L(t) = \frac{p(t).\sqrt{2}}{v_{in,rms}} |\sin \omega_m t| \qquad (1)$$

where w_m is the mains angular frequency and p(t) is the feedback variable which is derived from the output voltage.

Corresponding Author: Mehrnoosh Vatani, Department of Electrical Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran

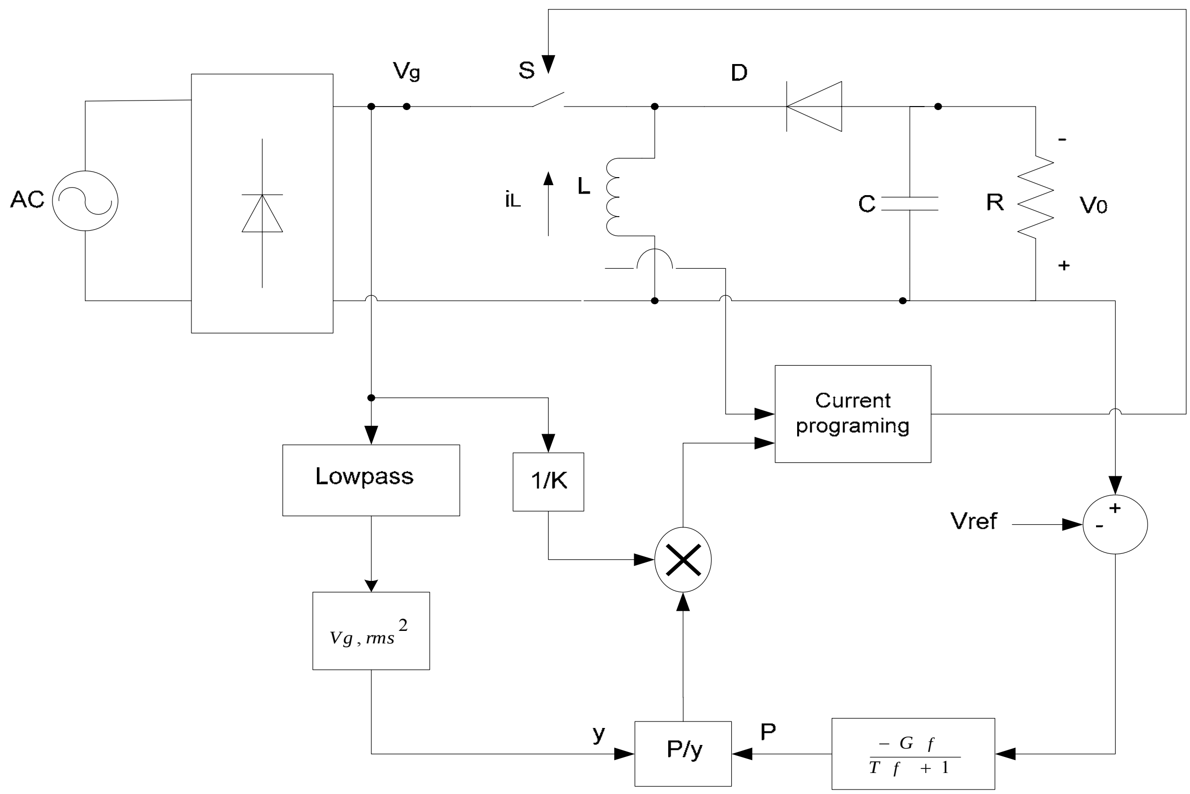


Fig. 1: Buck-boost pfc converter under acm control

$$\nu_{in}(t) = \sqrt{2}V_{in}|\sin\omega_m t| \tag{2}$$

$$i_L(t) = \frac{p(t)}{V_{in}^2}\nu_{in}(t) \tag{3}$$

The feedback loop model in frequency domain is:

$$\frac{P(s)}{V_o(s)} = \frac{-G_f}{\tau_f s + 1} \tag{4}$$

where Gf and τf is the voltage error amplifier dc gain and the cut-off frequency. In the time domain, "4" can be written as:

$$\tau_f \frac{dp(t)}{dt} + p(t) = -G_f(\nu_o - V_{ref}) \tag{5}$$

where Vref is the reference output voltage.

Nonlinear analytical model: In this section we find nonlinear analytical model of buck-boost PFC converter.

Standard averaging: Since the power semiconductors will either be conducting or blocking, the time-dependent switching function d(t) can be used to describe the allowed switch states of each structure, (e.g., d (t) = 1 for the on state circuit and d(t) = 0 for the off state circuit). By assuming the duty cycle d as the average value of d(t), and operating in continuous conduction mode, and supposing the power semiconductors as controlled ideal switches, the state space averaged model as a function of d can be written as (Silva, 2001):

$$\begin{bmatrix} \dot{i}_L \\ \dot{\nu}_o \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-d}{L} \\ \frac{1-d}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ \nu_o \end{bmatrix} + \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} [\nu_{in}] \tag{6}$$

The state-space averaged model "6" is also the state-space model of the circuit represented in Fig. 2. Hence, this circuit is designated as the standard averaged equivalent circuit of the converter and allows the determination, under small ripple and slow variations of the average equivalent circuit of the converter switching cell (power transistor plus diode).

We write down Kirchhoff's law equations over this model:

$$L\frac{di_L}{dt} + \nu_o = d(\nu_o + \nu_{in}(t)) \tag{7}$$

$$-C\frac{d\nu_o}{dt} - \frac{\nu_o}{R} + i_L = di_L \tag{8}$$

After combining "7" and "8" and eliminating duty cycle we get:

$$\frac{C}{2}\frac{d\nu_o^2}{dt} + C\nu_{in}\frac{d\nu_o}{dt} = -\frac{\nu_o^2}{R} - \frac{\nu_{in}\nu_o}{R} + \nu_{in}i_L \tag{9}$$

The power flowing through inductor L is normally much smaller than that in capacitor C operating near the line frequency. So by ignoring the dynamics of the inductor and replacing "1" in "9" we have:

$$\frac{C}{2}\frac{d\nu_o^2}{dt} + C\nu_{in}\frac{d\nu_o}{dt} + \frac{\nu_o^2}{R} + \frac{\nu_{in}\nu_o}{R} = p(t)(1-\cos 2\omega_m t) \tag{10}$$

Generalized averaging: In this section, we take generalized averaging over the mains period (Sanders *et al*., 1991), In generalized averaging method, we take Fourier series expansion of every state variable with the fundamental frequency being the line frequency, w_m. For any variable x(t), we have the following form:

$$x(t) = a_0 + a_1 e^{j\omega_m t} + \left[a_1 e^{j\omega_m t}\right]^* + a_2 e^{j2\omega_m t} + \left[a_2 e^{j2\omega_m t}\right]^* \tag{11}$$

$$a_n = \frac{\omega_m}{2\pi}\int_{t-\frac{2\pi}{\omega_m}}^{\tau} x(\tau).e^{-jn\omega_m \tau} d\tau \tag{12}$$

where n = 0, 1, 2 and superscript * denotes complex conjugation, and a_0 is the dc component, a_1 is the fundamental frequency component at w_m, and a_2 is the second harmonic component at $2w_m$.

The time derivative of the nth coefficient is computed to be:

$$\left[\frac{da(t)}{dt}\right]_k = \frac{\omega_m}{2\pi}\int_{t-\frac{2\pi}{\omega_m}}^{t} \frac{da(\tau)}{d\tau} e^{-jk\omega_m \tau} d\tau = \frac{da_k(t)}{dt} + jk\omega_m a_k(t) \tag{13}$$

Besides the relations in (Wong *et al*., 2006), we obtained the following relations:

$$\left[a(t).\left|\sin\omega_m t\right|\right]_k = \frac{j}{2}(a_{k-1} - a_{k+1}) \tag{14}$$

$$\left[\frac{da(t)}{dt}\left|\sin\omega_m t\right|\right]_k = \frac{j}{2}\frac{da_{k+1}}{dt} - \frac{(k+1)}{2}\omega_m a_{k+1} - \frac{j}{2}\frac{da_{k-1}}{dt} + \frac{(k-1)\omega_m a_{k-1}}{2} \tag{15}$$

Double averaged model: After applying averaging twice, over the switching period and the mains period, the nonlinear model based on double averaging will be obtained. To avoid confusion due to mix-up of subscripting indices, we define x as vo and y as p (t).

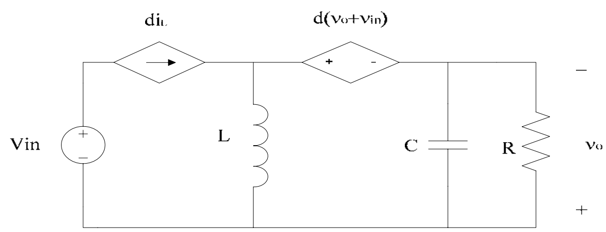


Fig. 2: Standard averaged model, where d is defined by the acm control

By applying foregoing averaging to "5" and "10" we get three equations from each of them. Now by applying generalized averaging to "10" we have "16":

$$\frac{C}{2}\frac{d}{dt}(x_0^2 + 2x_{1r}^2 + 2x_{1i}^2 + 2x_{2r}^2 + 2x_{2i}^2) + \sqrt{2}.C.V_{in}.(\frac{j}{2}.\frac{dx_1}{dt} - \frac{\omega_m}{2}x_1 - \frac{j}{2}.\frac{dx_1^*}{dt} - \frac{\omega_m}{2}x_1^*) + \frac{x_0^2 + 2x_{1r}^2 + 2x_{1i}^2}{R} + \frac{2x_{2r}^2 + 2x_{2i}^2}{R} + \frac{\sqrt{2}}{2}.j.\frac{V_{in}}{R}(x_1 - x_1^*) = y_0 - y_{2r} \tag{16}$$

$$\frac{C}{2}\frac{d}{dt}(2x_0x_1 + 2(x_{1r}x_{2r} + x_{1i}x_{2i} + j(x_{1r}x_{2i} - x_{1i}x_{2r}))) + (\frac{j\omega_m C}{2} + \frac{1}{R}).(2x_0x_1 + 2(x_{1i}x_{2i} + x_{1r}x_{2r} + j(x_{1r}x_{2i} - x_{1i}x_{2r}))) + \sqrt{2}.C.V_{in}.(\frac{j}{2}.\frac{dx_2}{dt} - \omega_m x_2) + \frac{\sqrt{2}}{2}.\frac{jv_{in}}{R}(x_2 - x_0) = y_1 - \frac{y_1^*}{2} \tag{17}$$

$$\frac{C}{2}\frac{d}{dt}((x_1^*)^2 + 2x_0x_2) + \sqrt{2}.C.v_{in}(-\frac{j}{2}\frac{dx_1}{dt} + \frac{1}{2}\omega_m x_1) + (j\omega_m C + \frac{1}{R})((x_1^*)^2 + 2x_0x_2) - \sqrt{2}\frac{jV_{in}}{2}x_1 = y_2 - \frac{y_0}{2} \tag{18}$$

By applying this averaging to "5" this equation is written as:

$$\tau_f\frac{dy_0}{dt} + y_0 = -G_f(x_0 - V_{ref}) \tag{19}$$

$$\tau_f \frac{dy_1}{dt} + (j\omega_m \tau_f + 1) y_1 = -G_f x_1 \tag{20}$$

$$\tau_f \frac{dy_2}{dt} + (j2\omega_m \tau_f + 1) y_2 = -G_f x_2 \tag{21}$$

Thus “16-21” are nonlinear analytical model based on double averaging method for PFC buck-boost converter under ACM control.

Steady state analysis: In steady state analysis we put all time derivative to zero and with the assumption of xo $=V_{ref}$ and also we put all the second harmonic components to zero so we have:

$$\begin{bmatrix} x_{1r} \\ x_{1i} \end{bmatrix} = \frac{R}{x_0(4+\omega_m{}^2 C^2 R^2)} \begin{bmatrix} 1 & \frac{3}{2}\omega_m RC \\ -\frac{1}{2}\omega_m RC & 3 \end{bmatrix} \cdot \begin{bmatrix} y_{1r} \\ y_{1i} \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} y_{1r} \\ y_{1i} \end{bmatrix} = \frac{-1}{1+\tau_f{}^2 \omega_m} \begin{bmatrix} G_f & G_f \tau_f \omega_m \\ -G_f \tau_f \omega_m & G_f \end{bmatrix} \cdot \begin{bmatrix} x_{1r} \\ x_{1i} \end{bmatrix} \tag{23}$$

From “22” and “23” the signal transfer function is “24”. The magnitude of the total loop gain $|Tx|$ is obtained from the eigen value of the matrix M. “24” and “25” are shown below:

$$M = \frac{-1}{1+\tau_f{}^2 \omega_m} \begin{bmatrix} G_f & G_f \tau_f \omega_m \\ -G_f \tau_f \omega_m & G_f \end{bmatrix} \frac{R}{x_0(4+\omega_m{}^2 C^2 R^2)} \begin{bmatrix} 1 & \frac{3}{2}\omega_m RC \\ -\frac{1}{2}\omega_m RC & 3 \end{bmatrix} \tag{24}$$

$$T_{x_1} = \frac{-\frac{3}{8} G_f R}{x_0 \left(1 - \frac{1}{2}\omega_m{}^2 CR\tau_f + \frac{1}{2}\sqrt{1 - 4\omega_m{}^2 CR\tau_f - 3\omega_m{}^2 \tau_f{}^2 + \frac{1}{4}\omega_m{}^2 C^2 R^2 (\omega_m{}^2 \tau_f{}^2 - 3)}\right)} \tag{25}$$

Stability boundaries curves: Our purpose in this section is to identify stability boundaries according to circuit parameters. The input voltage is a rectified sine wave repeating at $2w_m$. So we expect that all variables in the system repeat at this frequency (Orabi *et al*., 2002c). If the variables change its operation and repeat at half of the expected frequency, i.e., w_m the operation would be undesirable and device stresses would violently changed. In fact “period-doubling” bifurcation is occurred (Banerjee and Verghese, 2001).
In order to have stable operation $|Tx|<1$, so the condition Tx<1 is equivalent to “26”:

$$x_0 > \frac{G_f R \left(\omega_m{}^2 CR\tau_f - 2 + \sqrt{1 - 4\omega_m{}^2 CR\tau_f - 3\omega_m{}^2 \tau_f{}^2 + \frac{1}{4}\omega_m{}^2 C^2 R^2 (\omega_m{}^2 \tau_f{}^2 - 3)}\right)}{(4+\omega_m{}^2 C^2 R^2)(1+\omega_m{}^2 \tau_f{}^2)} \tag{26}$$

Although, double averaged model for buck-boost PFC converter is different from the boost PFC converter, the stability criterion for buck-boost PFC converter is exactly the same equation which is derived in (Wong *et al*., 2006) for the PFC boost converter. To indicate the above results we plot boundary curves from “25”. Figure 3 shows stability boundaries according to circuit parameters:

- The y-axis is vo and x-axis is load resistance. We observe the following results from Fig. 3.
- The lower limit vo increases as Gf increases.
- The lower limit of vo increases as τf decreases.
- The lower limit of vo increases as the output capacitance decreases.

It shows clearly that, below the lower limits of vo, the system will become unstable and fail to operate at the frequency $2w_m$. Thus the period-doubled operation will begin (Orabi *et al*., 2002a).

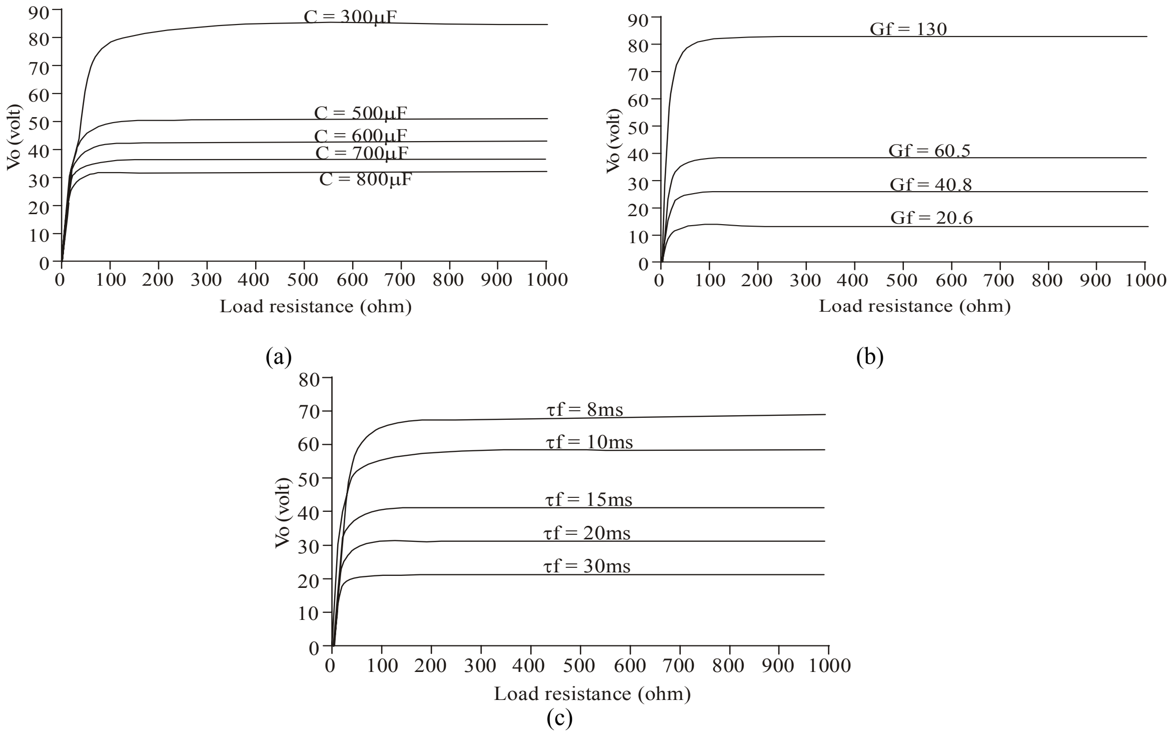


Fig. 3: Stability boundaries of buck-boost PFC converter with: (a) Gf = 50A and τf = 20 ms, $w_m = 2\pi$ (60) rad/s, (b) C = 800 mF and τf = 20 ms, $w_m = 2\pi$ (60), (c) Gf = 50A and C = 800 mF, $w_m = 2\pi$ (60) rad/s

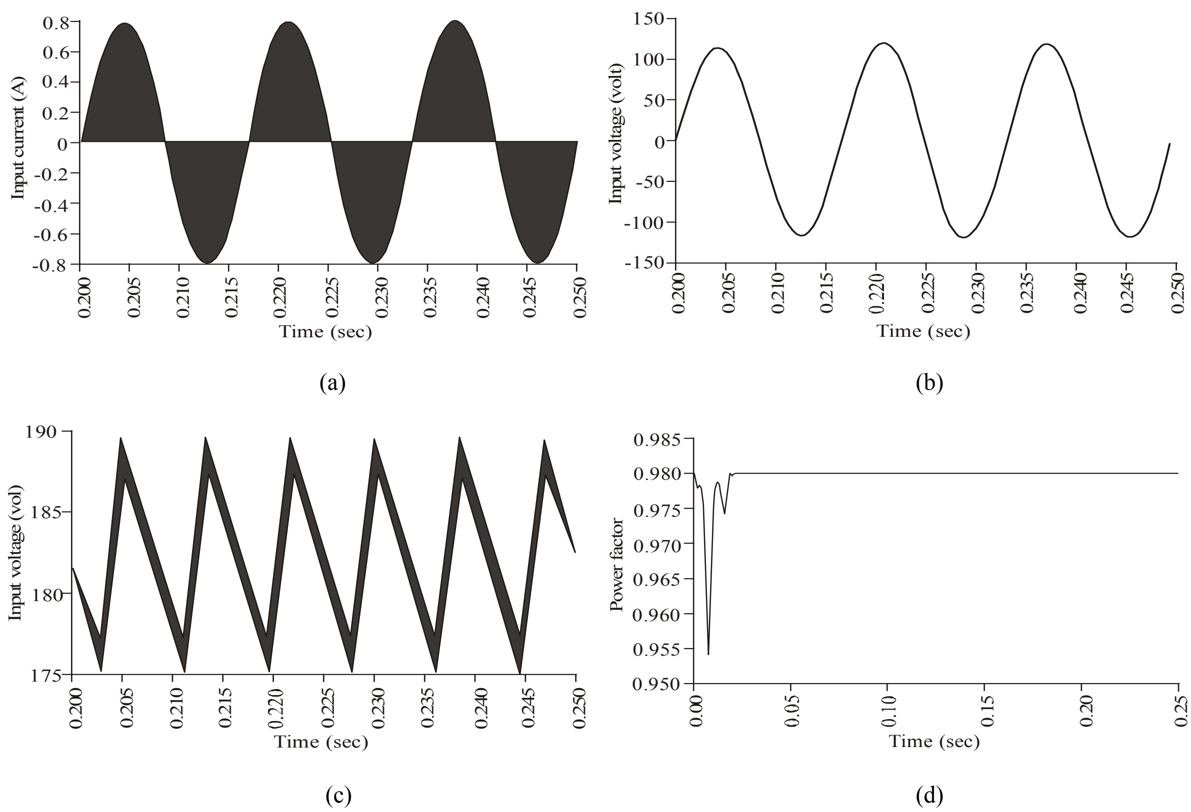
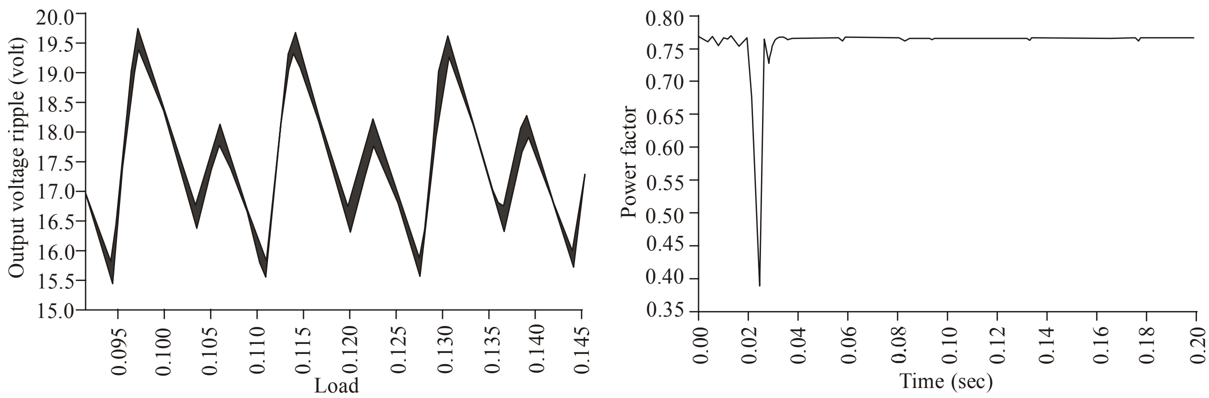


Fig. 4: (a) The input current, (b) The input voltage, (c) The output voltage ripple, (d) The power factor



(a) (b)

Fig 6: (a) The output voltage ripple in period doubling bifurcation case (simulation result), (b) The power factor

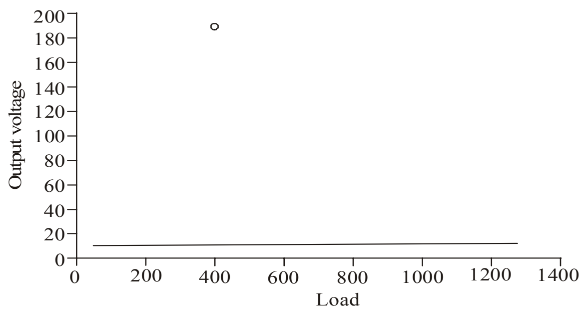


Fig. 5: Simulation data point is shown as circle and solid curve is from analytical expressions

Table 1: Out put voltage loop in different loads (the system operate in unstable condition)

Resistance load (ohm)	Output voltage (volt)
325	19.5
425	20
525	20.4
625	20.8
725	21.3
825	22
950	22.2

Table 2: Out put voltage loop in different loads (the system operate in unstable condition)

Resistance load (ohm)	Output voltage ripple
300	55
400	56
500	57
600	58
700	58.8
800	59.6
900	60.2

SIMULATION RESULTS

A PFC buck-boost converter under ACM control has been simulated in MATLAB SIMULINK for verification results. The circuit schematic is shown in Fig. 1. We test the simulation for 3 different conditions such as:

$G_f = 0.31$, $\tau_f = 8ms$, $C = 100\mu F$

First of all the operation is examined over the above conditions which is shown in Fig. 4, the input current is periodic at the line voltage frequency and the output voltage ripple is periodic at the double line frequency. So the system is operating in the stable condition. The power factor is very high (0.98).

This simulation has been performed to verify the stability area predicted by double averaged model. As shown in Fig. 5. At the top of boundary curve system is stable.

- $G_f = 5.8$, $\tau_f = 8ms$, $C = 300\mu F$

In the second condition system moves to be unstable as shown in Fig. 5, the output ripple voltage became periodic with the double period in stable case. This is period doubling bifurcation instability. The important point that is highlight for industrial view is the low result power factor (lessens to 0.77) that means that the PFC converter is lost its operation. Figure 6 shows the output voltage ripple at 324 resistance load.

We test the system at this condition ($G_f = 0.31$, $\tau_f = 8ms$, $C = 100\ \mu F$) in different resistance load. The following result in Table 1 is obtained.

- $G_f = 16.3$, $\tau_f = 8ms$, $C = 300\mu F$

This condition is the same as previous condition and the system moves to be unstable as shown in Fig. 7.

Also the system is simulated in different load condition so the output voltage values are in Table 2.

In Fig. 8 the stability boundaries that derived from analytical model are verified by simulation results. The data which is obtained from the simulation match well with the analytical results.

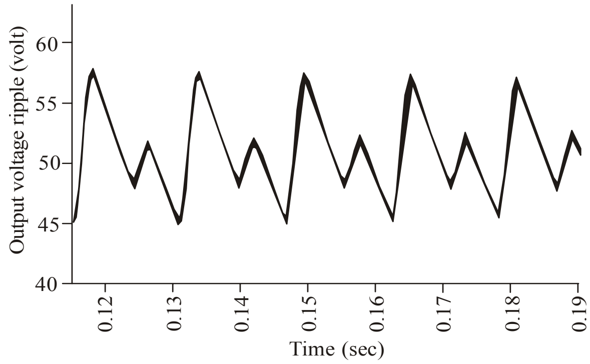


Fig. 7: The output voltage ripple in bifurcation case

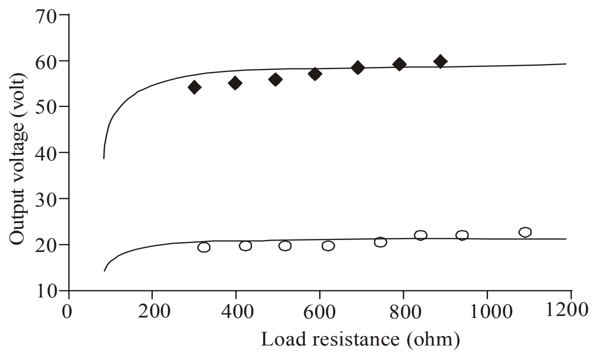


Fig. 8: Comparison between simulations results and analytical model, simulation data points are plotted as circles for Gf = 5.8, τf = 8ms, C = 300μF and plotted as xs for Gf = 16.3. τf = 8ms, C = 300μF, Solid curves are from analytical expressions

CONCLUSION

PFC converter treats as nonlinear circuit system, and so the stability analysis must be studied from the nonlinearity view point (Orabi *et al*., 2002b). Since there weren't enough stability information about buck-boost PFC converter, in this paper we applied double averaging method to produce nonlinear analytical model for buck-boost PFC converter. We derived closed-form stability conditions from this model. Also we identified the stability boundaries. Finally, the simulation results proved the analytical results with a good matching.

REFERENCES

Banerjee, S. and G. Verghese, 2001. Nonlinear Phenomena in Power Electronic: Attractors, Bifurcations, Chaos and Nonlinear Control. IEEE Press, New York.

Chu, G., K.T. Chi and S.C. Wong, 2007. A model for stability study of pfc power supplies. IEEE Power Electronics Specialists Conference, (PESC), 17-21 June, Hong Kong, pp: 1298-1303.

Huliehel, F.A., F.C. Lee and B.H. Cho, 1992. Small-signal modeling of the single-phase boost high power factor converter with constant frequency control. Virginia Power Electronics Center seminar VPEC, pp: 93-100.

Orabi, M., T. Ninomiya and J. Chunfeng, 2002a. A novel modeling of Instability phenomena in PFC converter. 24th Annual International Telecommunications Energy Conference (INTELEC), Fukuoka, Japan, pp: 566-573.

Orabi, M., T. Ninomiya and C. Jin, 2002b. Nonlinear dynamics and stability analyzed of boost power-factor-correction circuit. Proceeding of International Conference on Power Syst. Tech., pp: 600-6005.

Orabi, M., T. Ninomiya and C. Jin, 2002c. New formulation for stability analysis of power factor correction converters. Proceeding of CIEP, Oct., pp: 33-38.

Orabi, M. and T. Ninomiya, 2003a. Nonlinear dynamics of power-factor-correction converter. IEEE Trans. Ind. Electron., 50(6): 1116-1125.

Orabi, M. and T. Ninomiya, 2003b. Numerical and experimental study of instability phenomena of a boos PFC cnverter. Power Electronic Conference, Japan, pp: 854-859.

Pressman Abraham, I., 1998. Switching Power Supply Design. 2nd Edn., McGraw-Hill, New York.

Rossetto, L., G. Spiazzi and P. Tenti, 1994. Control Techniques for power factor correction converters. Proceeding of PEMC'94, Sept., pp: 1310-1318.

Sanders, S.R., J.M. Noworolski, X.Z. Liu and G. Verghes, 1991. Generalized averaging method for power conversion circuits. IEEE Trans. Power Electron., 6(2): 251-259.

Silva, J.F., 2001. Control Methods for Power Converters. In: Rashid, M.H. (Ed.), Power Electronics Handbook. Academic, New York.

Wong, S.C., C.K. Tse, M. Orabi and T. Ninomiya, 2006. The method of double averaging an approach for modeling power-factor-correction switching converters. IEEE Trans. Circuit Syst. I, 53(2): 454-462.

Yun, W.L., F. Guang and L. Yan-Fei, 2004. A large signal dynamic model for dc-to-dc converters with average current control. IEEE Power Electronics Conference, pp: 797-803.