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Research Article

Micropolar Fluid Flow with Variable Permeability of the Porous Medium Bounded by A Semi-Infinite Parallel Plate

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Abstract: The aim of the study of the present problem is to study the unsteady flow of an incompressisble micropolar fluid through porous medium bounded by semi infinite plate under the presence of transverse magnetic field. The permeability parameter of the porous medium is taken to be variable. The effect of permeability parameter, magnetic parameter on the flow of the fluid is studied and graphically represented. The expressions for shear stress and temperature gradient are obtained. Some special cases are deduced.

Keywords: Porous medium, rotation vector, variable permeability, AMS classification: 76D, 76S.

INTRODUCTION

Micro polar fluids are fluids with micro structure belonging to a class of fluids with a symmetrically stress Tensor. Physically they represent fluids consisting a randomly oriented particles suspended in a viscous medium (Aero *et al*., 1965; Dep, 1968; Lukaszewicz, 1999). A great number of Darcian flow studies have been performed with out heat transfer in various configurations e.g. in channels and past plates (Takhar and Beg, 1997; Kumari, 1998).

The study of flow through porous medium assumed importance because of the interesting applications in different fields of Science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, irrigation and sanitary engineering and also in the interdisciplinary field such as biomedical Engineering.

The micro polar fluid flow is porous medium has attracted the interest of many researchers in view of its application in a number of engineering problems such as oil exploration chemical catalytic reactors, thermal insulation and geothermal energy extractions.

Soundalgekar (1973) obtained approximate solutions for the two dimensional flow of an incompressible viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate. Raptis (1986) studied the case of time-varying two dimensional natural connective heat transfer of an in compressible electrically conducting viscous fluid through a highly porous medium bounded by an infinite vertical porous plate. Hiremath and Patil (1993) studied the effect of free connection flow on the oscillatory flow of polar fluid in a porous medium which is bounded by vertical plane surface of constant temperature. Kim (2001) studied unsteady convection flow of micro polar fluids past a vertical porous plate embedded in a porous medium. Lminar flow of an incompressible micropolar fluid between two parallel plates with porous liningis studied by Narasimhacharyulu and sunder (2010).

In this study the flow of micro polar fluids to a consist a mean velocity and temperature with a superimposed exponentially small variation with time. The problem under consideration is with out suction or injection. The permeability of the medium is taken to vary from point to point of the porous medium. The plate is impermeable are bounding porous medium.

FORMULATION AND SOLUTION OF THE PROBLEM

We consider the two dimensional unsteady flow of a incompressible micro polar fluid with variable permeability of the porous medium bounded by an semi-infinite parallel plate subjected to the presence of an applied pressure gradient. The plate is having axis as X-axis the physical quantities depend only on Y and time t. and y is perpendicular to the plate. The governing equations in a Cartesian frame of reference is given by:

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$$\frac{\partial v^*}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t} + V^* \frac{\partial u}{\partial y^*} = \frac{-1}{\rho}\frac{\partial p^*}{\partial x^*} + (V + Vr)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T - T_\infty) - v\frac{u^*}{k^*} + 2Vr\frac{\partial w^*}{\partial y^{*2}} \tag{2}$$

$$Pj^*\left(\frac{\partial w^*}{\partial t^*} + v\frac{\partial w^*}{\partial y^*}\right) = r\frac{\partial^2 w^*}{\partial y^{*2}} \tag{3}$$

$$\frac{\partial T}{\partial t^*} + v\frac{\partial T}{\partial y^*} = \alpha\frac{\partial^2 T}{\partial t^{*2}} \tag{4}$$

x^* , $y^*$are dimension less distances u^* ,v^* are dimension less velocities along x^* , y^* directions r is density n is kinematic rotational viscosity, bj is coefficient of volumetric expansion, k* is empirical constant called permeability of porous medium j* micro intertia density, w* is component of angular velocity normal to xy plane g is spin gradient viscosity, T is temperature a is fluid thermal

The non dimensional equations are given by introducing the dimension less variables as follows.

$$u = \frac{u^*}{U_0}, v\frac{v^*}{V_0}, y = \frac{V_0 y^*}{v} \tag{5}$$

$$U_\infty = \frac{u^*_\infty}{U_0}, w = \frac{v}{U_0 V_0} w^*, t\frac{t^* V_0^2}{V}$$

$$\theta = \frac{T - T_\infty}{Tw - T_\infty}, \eta = \frac{n^* v}{V_0^2}, k = \frac{k = V_0^2}{V^2}$$

$$P_r \frac{VpCp}{k} = \frac{V}{\alpha}, G = \frac{VBfg(T_w - T_\infty)}{U_0 V_0^2}, j = \frac{V_0^2}{V^2} j^*$$

$$r = \left(\mu + \frac{A}{2}\right) j^* = \mu j^* \left(1 + \frac{1}{2}\beta\right)$$

$\beta = \dfrac{A}{\mu}$, β is dimensionless viscosity ratio, A is gyro viscosity.

The velocity is taken as:

$$\overline{V} = (u, 0, 0) \tag{6}$$

The appropriate boundary conditions are given by:

$$u^* = 0,\ T = T_w + \in (T_w - T_\infty)e^{n^* t^*},\ w^* = \frac{-1}{2}\frac{\partial U^*}{\partial y^*} at\ \ y = 0$$

$$u^* \to u^*_\infty U_0(1 + \in e^{n^* 1^*}), T \to T_\infty, w^* \to 0\ as\, y^* \to \infty$$

We assume $k = 1 + \varepsilon Ae^{nt}$ where a constant, ÎA is a small quantity. The equation of continuity is satisfied by the choice of the velocity outside the boundary layer equation.

$$-\frac{1}{\rho}\frac{dp}{dx^*} = \frac{dU^*_\infty}{dt^*} + \frac{V}{K^*}U^*_\infty \tag{7}$$

The non dimensional equation of motion are given by :

$$\frac{\partial u}{\partial t} = \frac{dU_\infty}{dt} + (1+\beta)\frac{\partial^2 U}{\partial y^2} + G\theta + \frac{1}{k}(U_\infty - U) + 2\beta\frac{\partial w}{\partial y} \tag{8}$$

Let $k = 1 + \varepsilon Ae^{nt}$ satisfy the above equation we get:

$$\frac{\partial u}{\partial t} = \frac{dU_\infty}{dt} + (1+\beta)\frac{\partial^2 U}{\partial y^2} + G\theta + (1 - \in Ae^{nt})(U_\infty - U) + 2\beta\frac{\partial w}{\partial y} \tag{9}$$

$$\frac{\partial w}{\partial t} = \frac{1}{\eta}\frac{\partial^2 w}{\partial y^2} \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathbf{Pr}}\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

where, $\eta = \dfrac{\mu j^*}{r} = \dfrac{2}{2+\beta}$

The boundary conditions are given by:

$$U^* = 0,\ \theta = 1 + \in e^{nt}, w = \frac{-1}{2}\frac{\partial U^*}{\partial y^*}\ at\, y = 0$$
$$U^* \to U\infty,\ \theta \to 0. w^* \to 0\ as\ y^* \to \infty \tag{12}$$

The solve the above Eq. (9), (10) and (11) we assume:

$$U = U_0(y) + \varepsilon\, e^{nt}\, U_1(y) + o(\varepsilon^2) + ... \tag{13}$$

$$w = w_0(y) + \varepsilon\, e^{nt}\, w_1(y) + o(\varepsilon^2) + ... \tag{14}$$

$$\theta = \theta_0(y) + \varepsilon\, e^{nt}\, \theta_1(y) + o(\varepsilon^2) + ... \tag{15}$$

Substituting (13)-(15) Eq. (9), (10) and (11) and equating the harmonic and non harmonic terms and neglecting the coefficient of 0(Î²) we get pairs of equation for (U_0, w_0 , θ_0) and (U_1, w_1 , θ_1) as:

$$U_0^{\|} - \alpha U_0 = -\alpha(1 + G\theta_0) - 2\alpha\beta w_0^1 \tag{16}$$

$$U_1^{\|} - \alpha_2^2 U_1 = -\alpha_2^2 - \alpha(G\theta_1 + 2\beta w_1^2) \tag{17}$$

$$\frac{d^2w_0}{dy^2} = 0 \qquad (18)$$

$$\theta^{II}_0 = 0 \qquad (19)$$

$$\theta^{II}_0 = \eta \Pr \theta_1 = 0 \qquad (20)$$

The solutions of the above Eq. (16) - (20) subjected to the boundary conditions:

$$U_0 = 0, U_1 = 0, w_0 = \frac{-1}{2}U_0^1, w_1 = \frac{-1}{2}U_1^1, \theta_0 = 1, \theta_1 = 1 \; at \; y = 0 \qquad (21)$$

$$U_0 = 0,\ U_1 = 1,\ w_0 \to 0,\ w_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0 \; as \; y^* \to \infty \qquad (22)$$

are given as under:

$$U_0(y) = -\frac{-(1+G)}{a}\left[1 + e^{-a,y}\right] \qquad (23)$$

$$U_1(y) = -\left[1 + \frac{aa_2}{\eta Pr - a_3^2}\right] e^{-a3y} + \frac{aa_2}{\eta Pr - a_3^2} e^{(-\sqrt{nPr})y} + 1 \qquad (24)$$

$$\alpha_1^2 = \frac{1}{1-\beta}$$

$$\alpha^2 = \frac{1}{1+\beta} \qquad (25)$$

$$\alpha_2^2 = \frac{A+n}{1+\beta}$$

$$\alpha_3^2 = \frac{\alpha_2^2}{1-\alpha\beta}$$

$$\theta_1 = 1 \qquad (26)$$

$$w_0 = \frac{-1}{2}V_0^1 \qquad (27)$$

$$w_1 = \frac{-1}{2}U_1^1 \qquad (28)$$

$$\theta_1 = e(-\sqrt{\eta Pr})y \qquad (29)$$

$$\therefore U(y, t) = U_0(y) + \in e^{nt} U_1(y)$$

$$U(y,t) = \frac{-(1+G)}{a}\left[1 - e^{-a,y}\right] + \in e^{nt} \left[1 + \frac{a.a_2 e^{(-\sqrt{nPr})y}}{nPr - a_3^2}\right] - \left[1 + \frac{a.a_2 e^{-a_3 y}}{nPr - a_3^2}\right] \qquad (30)$$

Shear stress:

$$T_w = \frac{-\alpha_1}{\alpha} + \in e^{nt}$$

$$T_w = \frac{\partial u}{\partial y} / y = 0 \quad \left[\alpha_3 + \frac{\alpha\alpha_2\alpha_3}{np_r - \alpha_3^2} - \frac{\sqrt{np_r}.\alpha\alpha_2}{npr - \alpha_3}\right] \qquad (31)$$

The temperature gradient at y = 0 is given by:

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = - \in e^{nt}.\sqrt{n \Pr} \qquad (32)$$

SPECIAL CASES

Case 1: Newtonian flow through porous medium when there is no rotation of vector normal to xy plane i.e., $\beta = 0$

$$\therefore U_0(y) = \frac{-[1+G]}{a}\left[1 - e^{-a1y}\right] \qquad (33)$$

$$U_1(y) = 1 + \frac{a.a_2}{nPr - a_3^2} e^{-(nPr)y} \left[1 + \frac{a.a_2}{nPr - a_3^2}\right] e^{-a3y} \qquad (34)$$

where,

$$\left.\begin{aligned} \theta_0 &= 1 \\ \theta_1 &= e^{-(\sqrt{nPr})y} \\ w_0 &= \tfrac{-1}{2}U_0^1 \\ w_1 &= \tfrac{-1}{2}U_0^1 \end{aligned}\right\} \qquad (35)$$

and

$$\left.\begin{aligned} a_1^2 &= 1 \\ a^2 &= 1 \\ a_2^2 &= A + n \\ a_3^2 &= a_3^2 \end{aligned}\right\} \qquad (36)$$

Case 2: When $1/k = 1 - \varepsilon\, e^{nt} A$ i.e., $k = \frac{1}{1 - \in e^{nt} A}$ when A is zero the micropolar fluid flow in clear medium is given by

$$U_0(y) = \frac{-[1+G]}{a}\left[1 - e^{-a1y}\right] \qquad (37)$$

$$U_1 = 1 + \frac{a.a_2}{nPr - a_3^2} e^{(-\sqrt{nPr})y} - \left[1 + \frac{a.a_2}{nPr - a_3^2}\right] e^{-a,3y} \qquad (38)$$

where,

$$\left.\begin{aligned} \theta_0 &= 1 \\ \theta_1 &= e^{-(\sqrt{nPr})y} \\ w_0 &= \tfrac{-1}{2}U_0^1 \\ w_1 &= \tfrac{-1}{2}U_1^2 \end{aligned}\right\} \qquad (39)$$

and

$$\left.\begin{aligned} a_1^2 &= \frac{1}{1-\beta} \\ a^2 &= \frac{1}{1+\beta} \\ a_2^2 &= \frac{1}{1+\beta} \\ a_3^2 &= \frac{a_2^2}{1-a\beta} \end{aligned}\right\} \qquad (40)$$

Case 3: Micropolar fluid flow through clear medium with out rotation of molecule of the fluid i.e., A = 0, β = 0.

$$U_0(y) = \frac{-[1+G]}{a}\,[1- e^{-a_1 y}] \qquad (41)$$

$$U_1(y) = 1 + \frac{a.a_2}{nPr - a_3^2}\, e^{(-\sqrt{nPr})y} - \left[1 + \frac{a.a_2}{nPr - a_3^2}\right] e^{-a_3 y} \qquad (42)$$

where,

$$\left.\begin{aligned} \theta_0 &= 1 \\ \theta_1 &= e^{-(\sqrt{\mathrm{nPr}})\mathrm{y}} \\ w_0 &= \frac{-1}{2} U_0^1 \\ w_1 &= \frac{-1}{2} U_0^2 \end{aligned}\right\} \qquad (43)$$

and

$$\left.\begin{aligned} a_1^2 &= 1 \\ a^1 &= 1 \\ a_2^2 &= n \\ a_3^2 = a_3^2 &= n \end{aligned}\right\} \qquad (44)$$

CONCLUSION

The problem of micropolar fluid flow through porous medium bounded by semi infinite plate is investigated under the influence of magnetic field together with heat transfer. The effect of permeability coefficient and magnetic permeability parameter on the flow of the fluid is presented graphically.

The expressions for velocity, rotation vector ω, and temperature and numerically computed and represented graphically. From Fig. 1, the graph shows the rotation vector normal to xy plane the increase in β values shows decrease in profile of the vector.

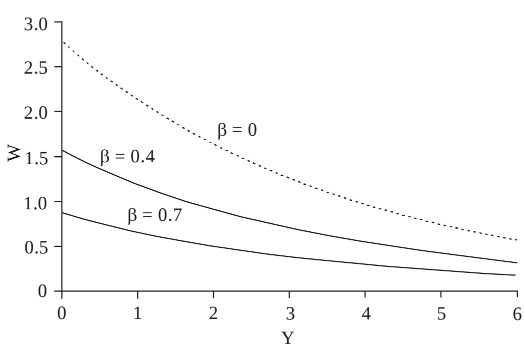

Fig. 1: Variation of rotation vector with b value

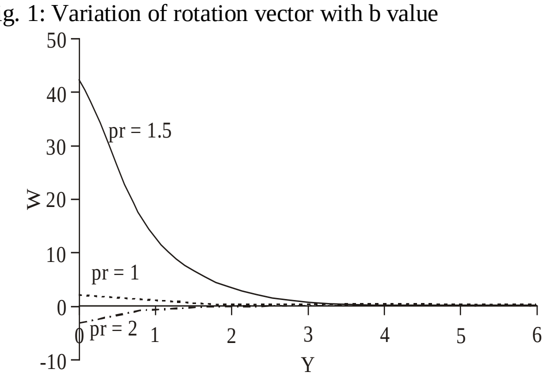

Fig. 2: Variation of rotation vector with prandtl number

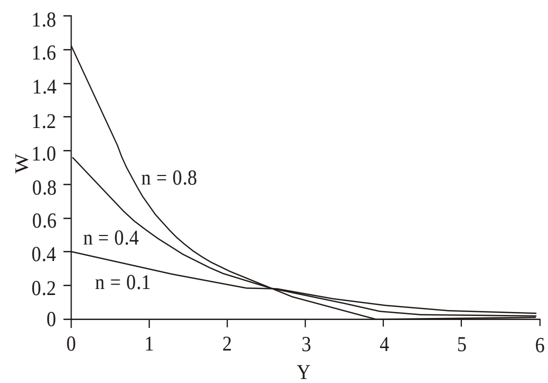

Fig. 3: Variation of rotation vector with a frequency of exponential function

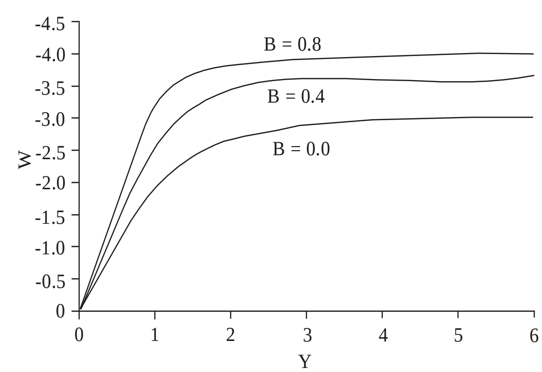

Fig. 4: Variation of velocity with b values

From Fig. 2 Rotation vector against Pr value is plotted we observe that increase in Pr values depicts decrease in profile of the vector.

The frequency of exponential function against the rotation vector shows decrease in the rotation vector profile to the frequency of the velocity (Fig. 3).

The effect of microrotation on velocity shows the decrease is velocity (Fig. 4). Figure 5 shows the velocity profile for Pr values. Effect of increase of

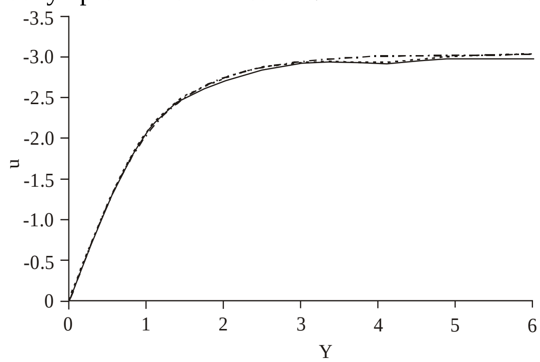

Fig. 5: Variation of velocity for different prandtl numbers

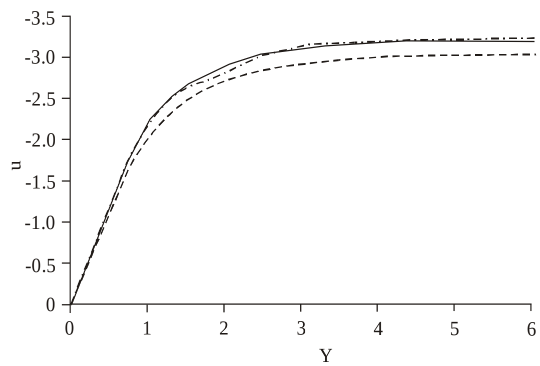

Fig. 6: Variation of velocity for different frequency n values

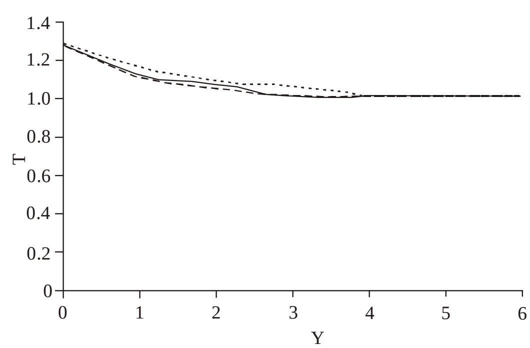

Fig. 7: Variation of temperature for different prandtl numbers

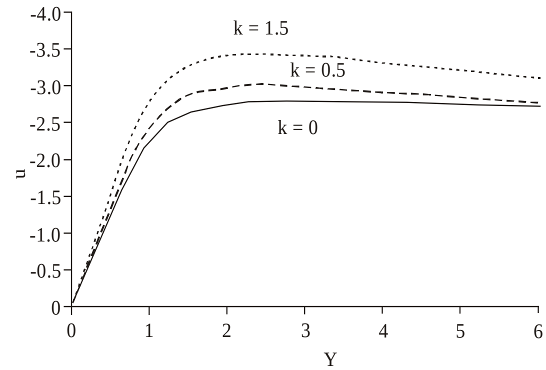

Fig. 8: Variation of velocity for different permeabilities

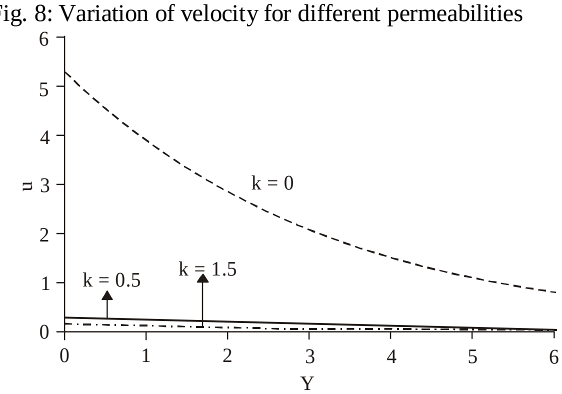

Fig. 9: Variation of rotation vector profile for different permeabilities

frequency of exponential function showing decrease in velocity profile (Fig. 6).

Figure 7 shows frequency of exponential function against temperature. From the Fig. 8 and 9 we can say that as the permeability k is increasing the velocity profile increase and increase in permeability decrease the rotation vector profile.

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