Research Article
Modal and Harmonic Analysis of Crankshaft for High-Power Drilling Pump

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Abstract: The crankshaft is very important in high-power drilling pump. The information on the vibration mode and frequency response is critical for the crankshaft to work normally in drilling pump. ANSYS finite element analysis software was used to implement the modal analysis of the crankshaft thus obtaining characteristic frequency and the corresponding vibration mode and then by the harmonic response analysis which was carried out based on the modal analysis, dynamic response of the crankshaft was obtained. The result shows that the working speed of crankshaft can avoid the region of sympathetic vibration effectively, no resonance will happen. The strength and fatigue life of crankshaft are also checked according to the stress results. The analysis also makes a base for the optimized design of dynamic performance of the crankshaft.

Keywords: ANSYS, drilling pump, dynamic response, fatigue life, harmonic response

INTRODUCTION

The high-power drilling pump is made to satisfy the need of the high-pressure, high-duty and superpower drilling pump (Qi *et al*., 1990; Gerzha *et al*., 1991) in the oil field. Special attention should be paid to the design of the crankshaft. The precondition of the design is to make sure it has enough strength, stiffness and stability (Silva, 2003). The damage of the crankshaft is mostly caused by the big vibration stress of the resonance (Su *et al*., 2005). To avoid the structure destroy of the crankshaft by the resonance, dynamics analysis of the crankshaft should be taken. The dynamics analysis mainly consists of two parts. One is the dynamic character analysis of the structure (natural frequency and vibration) (Jonker, 1989) and the other is dynamic response analysis by the harmonic exciting force in certain direction (Wang *et al*., 2010).

The crankshaft is rotating mechanism (Dai *et al*., 2009) and an important component in the drilling pump. It is a spare part which easily ruptures in the accident. The objective of the study is to avoid the destroy by analyzing the relevant property of the crankshaft. To make the crankshaft have enough strength, the strength check should be done by analyzing the dynamic response. The fatigue life should be got in advance to avoid the failure of the structure in the future.

Modal analysis and harmonic response analysis:

The basic theory of modal analysis: The basic vibration unit can be discrete into N degree of freedom linear vibration system with finite mass, damping and elastic element. The relevant dynamics differential equation (Laulagnet and Guyader, 1989) is:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F(t)\} \tag{1}$$

[M] mass matrix; [C] damping matrix; [K] rigidity matrix; F(t) stimulating vector;
$\{\ddot{q}\}, \{\dot{q}\}, \{q\}$ The N dimensional displacement velocity and acceleration vector;

To solve the natural frequency and natural vibration mode, as the damping is relative small and can be omitted. The vibration equation of the systems is:

$$[M]\{\ddot{q}\}+[K]\{q\}=0 \tag{2}$$

It is an equation group with n homogeneous differential equations:

$$\sum_{j=1}^{n}m_{ij}\ddot{q}+\sum_{j=1}^{n}k_{ij}q_j=0 \quad i=1,2,\cdots,n \tag{3}$$

$$q_j(t)=u_jf(t) \quad j=1,2,\cdots,n$$

$$\therefore \ddot{f}(t)\sum_{j=1}^{n}m_{ij}u_j+f(t)\sum_{j=1}^{n}k_{ij}u_j=0 \tag{4}$$

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$$\therefore \frac{\ddot{f}(t)}{f(t)} = \frac{\sum\limits_{j=1}^{n} k_{ij} u_j}{\sum\limits_{j=1}^{n} m_{ij} u_j} \tag{5}$$

The equation shows that the time function can be separated with the space function. The left of the function have no relations with subscript i while the right of the function has no relations with time. Supposing the ratio of two functions is a constant λ, we can have:

$$\ddot{f}(t) + \lambda f(t) = 0 \tag{6}$$

$$\sum_{j=1}^{n}(ki_j - \lambda m_{ij})u_j = 0 \tag{7}$$

The matrix form is:

$$[K]\{u\} - \lambda[M]\{u\} = \{0\} \tag{8}$$

To make the equation has a non-zero solution, the determinant of the coefficient matrix is zero:

$$\|[K] - \lambda[M]\| = 0 \tag{9}$$

The general solution of the function is:

$$\{q(t)\} = \sum_{r=1}^{n}\{q(t)\}_r = \sum_{r=1}^{n} A_r \{u\}_r \sin(\omega_{nr}t + \varphi_r) = [u]\{A\sin(\omega_n t + \varphi)\} \tag{10}$$

The result shows that the natural frequency is decided by the stiffness and the mass of the system. The non-damping vibration with N degree of freedom is a linear combination of n simple harmonic oscillation with natural frequency.

The basic theory of the harmonic response analysis: Any cyclic load will bring the cyclic response (harmonic response) of the structure system. The aim of the analysis is to calculate the response of the structure with different frequency and get the curve of the response value at different frequency. The result can be the bad effect of the forced vibrations. The harmonic response analysis is as follows:

$$[M]\{\ddot{q}\} = -[K]\{q\} + F_0 \cos \omega t \tag{11}$$

$$\therefore \{\ddot{q}\} + [M]^{-1}[K]\{q\} = [M]^{-1} F_0 \cos \omega t \tag{12}$$

Let

$$\omega_0^2 = [M]^{-1}[K] \qquad f_0 = [M]^{-1} F_0$$

$$\{\ddot{q}\} + \omega_0^2 \{q\} = f_0 \cos \omega t \tag{13}$$

We can solve the equation:

$$q(t) = A_0 e^{-\beta t} \cos(\omega t + \delta) + A\cos(\omega_p t + \varphi) \tag{14}$$

The steady state response:

$$\{q(t)\} = A\cos(\omega t + \varphi) \tag{15}$$

The finite element model of the crankshaft: The modal of crankshaft for the finite element analysis is 3NB-2200. According to the real size of the crankshaft for ZY3NB2200 drilling pump, we simplify the solid calculation modal to divide the grid. The finite solid modal and its element modal of the crankshaft are shown in Fig. 1. The grid is divided using SOLID45 unit by the free lattice method (Nath and Ghosh, 1980). The material of the crankshaft is ZG25MnVCu. The material property is shown in Table 1.

The two simple bearings are placed in the shoulder of base bearing. A constraint in the direction of rotation is made in the connecting parts of the gearwheel (Feng *et al.*, 2007).

The modal analysis of the crankshaft: The crankshaft is the high speed rotating member in the drilling pump. We take the modal analysis and solve the natural frequency and the corresponding vibration modal (Kim *et al.*, 2008).

The modal analysis is comprised by the model building, loading, solving and post processing of the extended modal. The Block Lanczo method (Gupta and Lawson, 1988) is adopted as the modal analysis method. The vibration of the structure can be taken as the linear combination of multiple natural mode of vibration. The vibration mode with low order has more dynamic effect than the mode with high order (Khemili and Romdhane, 2008). The dynamic character is depended by the structure of low order. The first eight order natural frequency of the crankshaft is shown in Table 2.

Table 1: Material properties of crank shaft

Elasticity modulus/MPa	Poisson ratio	Density/kg/m³	Yield limit/MPa	Allowable stress/MPa
206000	0.3	7800	550	500

Table 2: The first 8 order natural frequency and period of the crankshaft

Order	1	2	3	4	5	6	7	8
Frequency (Hz)	51.049	243.1100	310.0700	334.8900	358.0700	444.8100	456.3600	493.8300
Period (sec)	0.020	0.0041	0.0032	0.0030	0.0028	0.0023	0.0022	0.0020
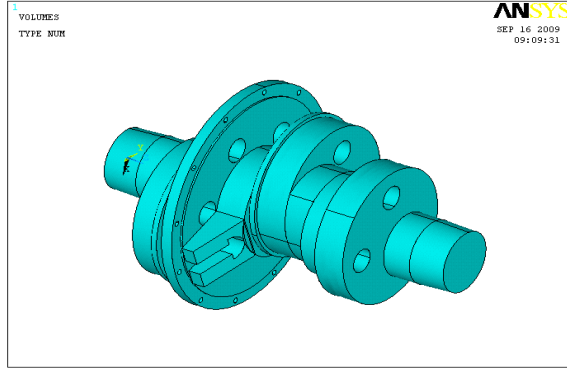
(a) solid modal
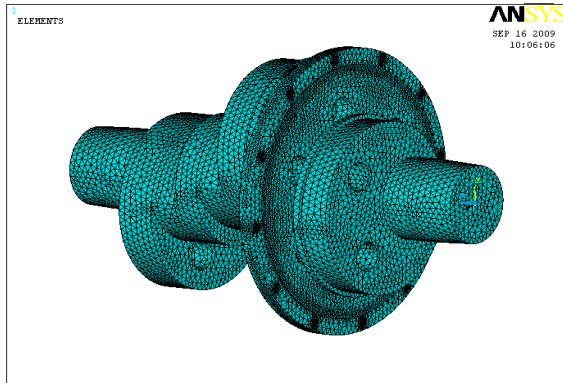
(b) grid modal

Fig. 1: The solid modal and grid modal of the crankshaft
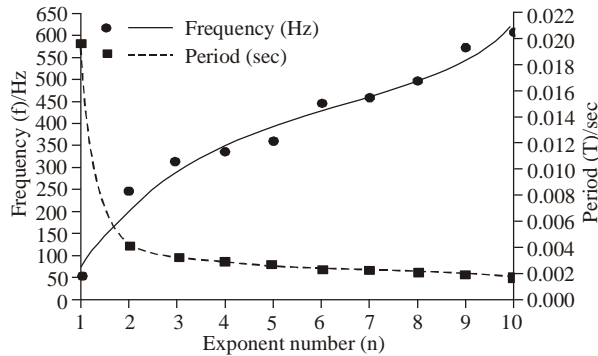

Fig. 2: The relation curve of the order with different frequency period in the free vibration of the crankshaft

By curve fitting of the data in the Table 1, The curves about the order with the frequency and period in the free vibration of connecting rod are shown in Fig. 2.

The order of the vibration is relatively in linear relation with the natural frequency. By curve fitting, we can get:

$$f = -94.35 + 196.9n - 27.02n^2 + 1.45n^3 \tag{16}$$

The order is in exponent relation with the Period, we have:

$$T = 0.002 + 0.005e^{-n/2.16} + 0.43e^{-n/0.29} \tag{17}$$

The correlation coefficient of the Eq. (16) and (17) is R = 0.97 and R = 0.99 relatively. So the two equations can reflect the relation between the order and the frequency. We can anticipate the frequency and period in the free vibration of the crankshaft.

The different orders of the crankshaft vibration mode are shown in Fig. 3

The frequency of stroke for the 3NB-2200 drilling pump is 120 time/min. The excited frequency of the crankshaft is 2 Hz. The first order natural frequency of the crankshaft by modal calculation is 51.049 Hz which is far larger than 2 Hz. So the crankshaft will not have resonance while the drilling pump is working.

Harmonic response analysis: After the modal analysis, we make the harmonic response analysis. The boundary condition of the analysis is the same with the modal analysis (Loannides and Grootenhuis, 1979). The resonance peak on the three-throw crankshaft is 751.4 kN. The place is in the contact point between the crankshaft and the connecting rod. The phase division is 120º. The frequency span is from 0 to 500 Hz. 5 points are taken by the equal interval in the central axis of the crankshaft. The change of the crankshaft displacement can be got from the change of the 5 points. The displacements in the x, y and z direction are calculated apart and the results are shown in Fig. 4.

From the Fig. 4, we can see that when the external load frequency is 50-60 or 230-260 Hz, the crankshaft displacement in x direction is suddenly bigger. It is because that the frequency domain is near the first and second natural frequency. The resonance occurs and the displacement in x direction becomes bigger. When the
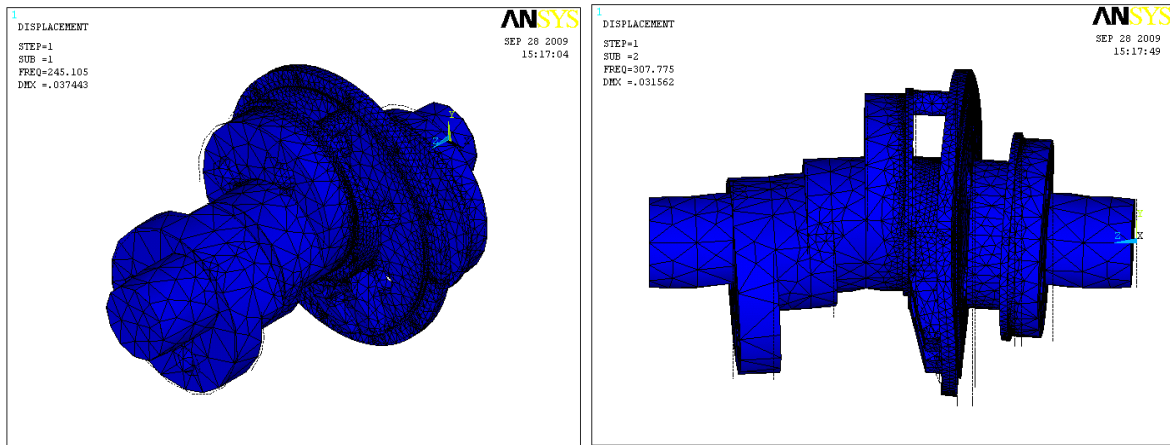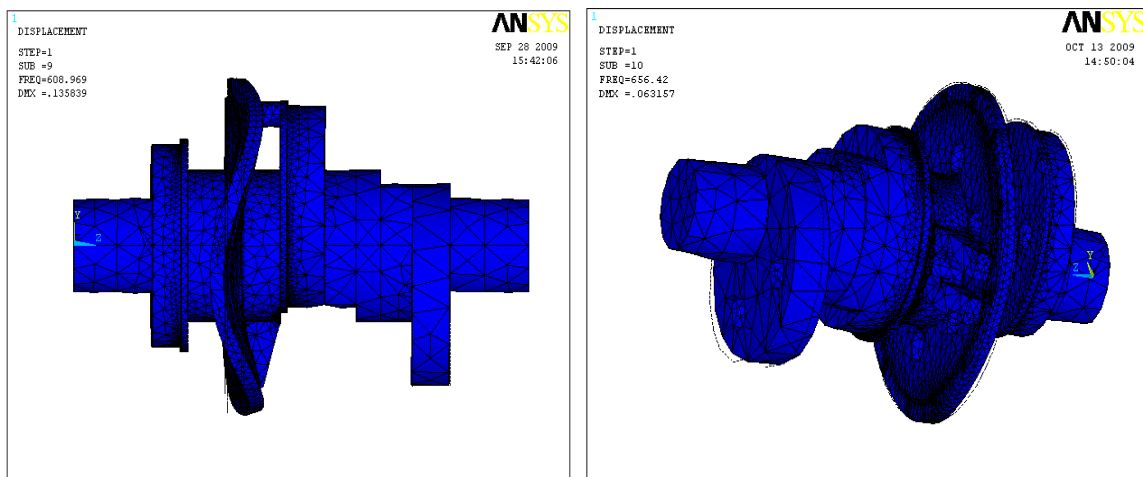
(a) The first order

(b) The second order
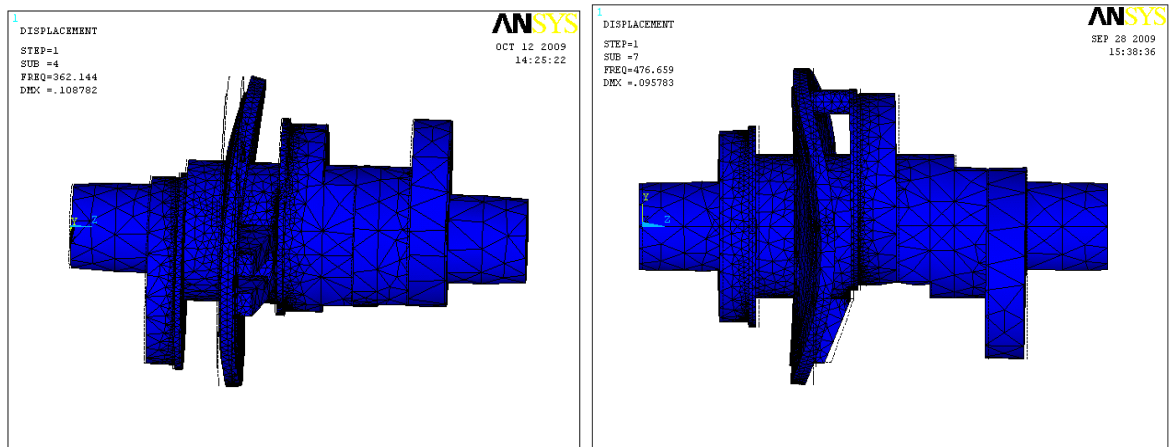
(c) The forth order

(d) The seventh order

(e) The ninth order

(f) The tenth order

Fig. 3: The different order of vibration mode

(a) Displacement in x direction

(b) Displacement in y direction
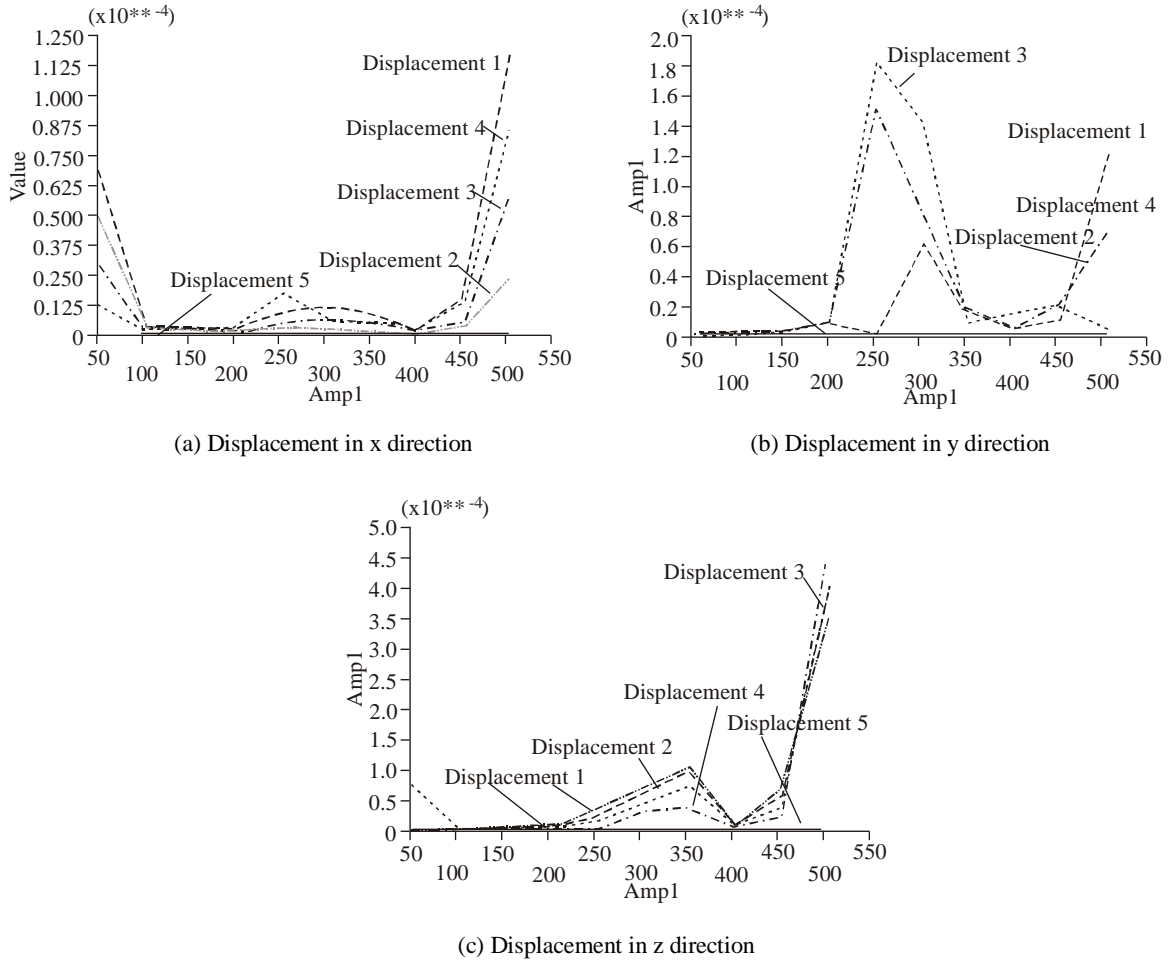

(c) Displacement in z direction

Fig. 4: The curve of crankshaft displacement in different direction as the load frequency changes
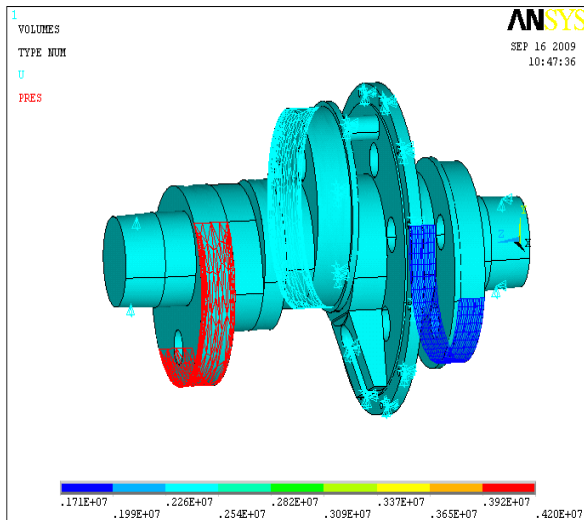

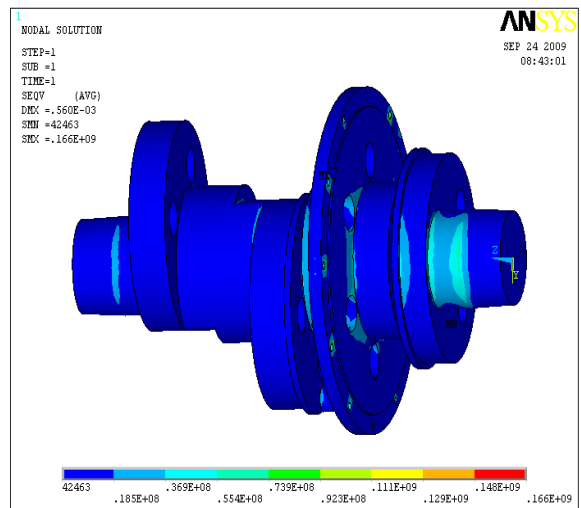Fig. 5: Position of loading and restriction of crank shaft

Fig. 6: Compression stress distribution of crank shaft under work pressure of 60 MPa with rotor angle 90°

external load frequency is 230-260 Hz, the crankshaft displacement in y direction is bigger as the frequency domain is near the first natural frequency and bending resonance occur. Similarly, displacement in z direction is bigger in the frequency domain 340-360 Hz. In reality, these frequency domains should be avoided to prevent the occurrence of resonance (Tsuchiya and Nagashima, 2003).

Strength check and fatigue analysis:
Strength check of the crankshaft: As the structure and stress is relatively complex, the stress of different load case should be checked. The calculation is done every 30° of crank angle. The force during the moving process is exerted in the three crank handle. The details are shown in Fig. 5. When work pressure is 60 MPa and rotor angle 90°, compression stress distribution of crankshaft is shown in Fig. 6

The strength of crankshaft is checked by the fourth strength theory. Two points are picked in each of the three cranks. When the pump pressure is 52 MPa, the

equivalent stress of sampled six points are shown in Table 3.

When work pressure is 52 and 60 MPa, Fig. 7 shows the corresponding maximum equivalent stress of Crank A, B, C and the crankshaft with the crank angle. From Fig. 7, when the work pressure is different, the stress variation tendency is relatively the same. When rotor angle is 180° and 360°, the stress of Crank A has a big leap. This is because the piston changes to discharge process from suction process. The force of the connecting bar acting on the crankshaft becomes bigger. So the maximum stress of Crank A increases suddenly. Similarly, when the piston changes to suction process from discharge process, the maximum stress of Crank A decreases sharply. The stress of Crank C becomes bigger in 60° and smaller in 240°. The reason of change is the same with Crank A.

According to the stress state of different case, the maximum equivalent stress of crankshaft is 166 MPa when work pressure is 60 MPa. The allowable stress of
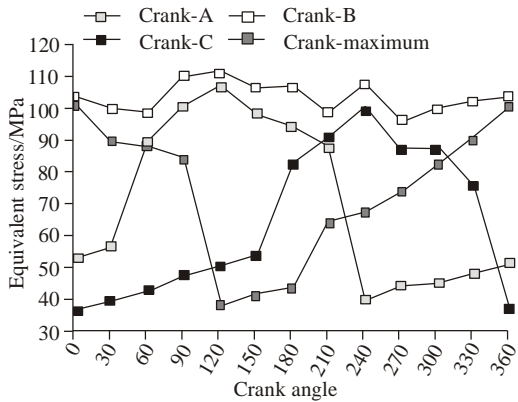
(a) Under pressure 52 MPa
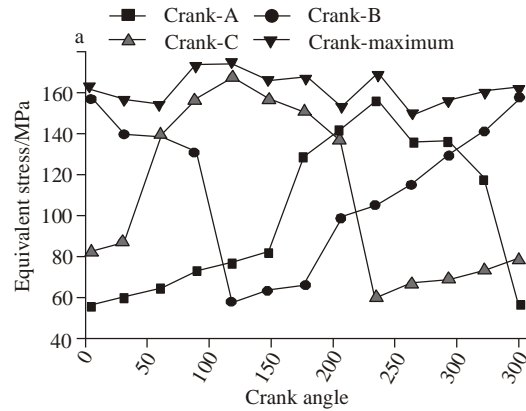
(b) Under pressure 60 MPa

Fig. 7: Variation curve of crankshaft's maximum equivalent stress with the crank angle under different pressure

Table 3: Equivalent stress distribution under work pressure of 52 MPa

Rotor angle	Equivalent stress of A\MPa		Equivalent stress of B\MPa		Equivalent stress of C\MPa		Maximum equivalent stress\MPa
0°	34.5	36.9	97.6	99.800	52.2	52.8	102.5
30°	37.6	39.6	86.5	88.800	55.5	56.1	98.8
60°	42.7	41.3	83.7	87.500	87.0	88.8	97.7
90°	47.4	44.2	81.7	83.300	97.6	99.1	109.1
120°	48.6	50.1	38.6	37.200	105.7	102.9	110.1
150°	51.2	53.1	41.4	40.600	99.1	97.7	105.2
180°	78.2	81.6	43.7	43.300	95.0	93.2	105.6
210°	89.9	45.0	45.9	63.700	87.1	85.3	97.8
240°	98.7	49.6	49.4	67.000	38.4	39.9	106.2
270°	86.2	54.0	53.4	73.610	41.3	43.9	95.1
300°	77.5	86.3	80.9	82.178	43.9	44.8	99.0
330°	70.2	75.3	87.1	89.726	47.9	48.1	101.3
360°	34.5	36.9	97.6	99.892	50.9	51.2	102.5

Table 4: Result of fatigue analysis of crankshaft

Chosen points	Equivalent stress\MPa	Cycle index	Fatigue ratio	Fatigue life
1	166.0	2×10^8	0.06980	2.092×10^9
2	165.1	2×10^8	0.07120	1.839×10^9
3	159.3	2×10^8	0.06030	2.331×10^9

the material is 500 MPa. So the design can satisfy the strength requirement.

Fatigue analysis: According to the results, the fatigue strength of crankshaft need to be checked when the work pressure is 52 MPa. The easily fatigue points in the crankshaft are taken to be analyzed. The results of three points is shown in Table 4.

From the Table 4, the fatigue life which the crankshaft can endure is not more than 1.839×10^9. Supposing the life of the crankshaft is 5 years (Taylor *et al.*, 1997). The working month, working day and working hour are 12 months/year, 20 days/month and 16 h/day, respectively. The theoretic frequency of stroke of the drilling pump is 105 time/min. Then the frequency of load stroke which the crankshaft endures is: $5\times12\times20\times16\times105\times60 = 1.15\times10^8$. So the crankshaft satisfies the need of fatigue life.

CONCLUSION

By the finite element analysis software ANSYS we study the natural vibration character of the crankshaft for the 3NB-2200 drilling pump. We get the first eight order natural frequency of the crankshaft and its vibration mode. The mode calculation truly reflects the natural character of crankshaft and provides the basic theory for improving the crankshaft.

The crankshaft frequency and the periodic data are obtained by the modal analysis and the relationship between the frequency and the order is determined Based on the modal calculation, the response of the crankshaft is analyzed by the harmonic exciting force. The vibration character by the harmonic force is gained

The frequency of stroke for the 3NB-2200 drilling pump is 120 time/min. The excited frequency of the crankshaft is 2 Hz. It is far below the natural frequency and can avoid the resonance frequency domain. So the crankshaft will not have resonance while the pump works normally

According to the fourth strength theory, the maximum equivalent stress of crankshaft is far below the allowable stress. So the design satisfies the strength requirement. The fatigue analysis is done when work pressure is 52 MPa. It is proved that the crankshaft can work normally in 5 years.

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