

Research Article

MIP-Mitigated Sparse Channel Estimation Using Orthogonal Matching Pursuit Algorithm

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Abstract: Wireless communication requires accurate Channel State Information (CSI) for coherent detection. Due to the broadband signal transmission, dominant channel taps are often separated in large delay spread and thus are exhibited highly sparse distribution. Sparse Multi-Path Channel (SMPC) estimation using Orthogonal Matching Pursuit (OMP) algorithm has taken advantage of simplification and fast implementation. However, its estimation performance suffers from large Mutual Incoherent Property (MIP) interference in dominant channel taps identification using Random Training Matrix (RTM), especially in the case of SMPC with a large delay spread or utilizing short training sequence. In this study, we propose a MIP mitigation method to improve sparse channel estimation performance. To improve the estimation performance, we utilize a designed Sensing Training Matrix (STM) to replace with RTM. Numerical experiments illustrate that the improved estimation method outperforms the conventional sparse channel methods which neglected the MIP interference in RTM.

Keywords: Mutual Incoherent Property (MIP), Orthogonal Matching Pursuit (OMP), Sensing Training Matrix (STM), sparse channel estimation

INTRODUCTION

In the last decades, how to overcome the scarcity of spectral resource to meet the ever-growing need for high data rate was a great challenge for communication engineers. One way to achieve a high-rate data is to simply increase the transmission speed. Due to the time delay spread of multi-path channel, the channel impulse response easily spans several hundred symbol intervals. If standard linear channel estimation is used, current training sequence is generally short to provide accurate channel estimation which is shown in Fig. 1. Fortunately, Sparse Multi-Path Channel (SMPC) model as shown in Fig. 2, is frequently encountered in wireless communication applications, such as terrestrial transmission channel of High Definition Television (HDTV) signals (Schreiber, 1995), hilly terrain delay profile of multi-path in the broadband wireless communication (Ghuri and Slock, 2000) and typical underwater acoustic channels (Kocic *et al.*, 1995).

Among the large number of SMPC entries, only a small portion is significantly different from zero. Taking advantage of the sparsity, impulse response of SMPC can be recovered from relatively small number of received data and training data with sparse channel estimation method (Cotter and Rao, 2002) which is shown in Fig. 3. Mathematically, sparsest channel estimation can be obtained by solving the l_0 sparse constraint problem (Donoho, 2006). However, finding the sparsest solution is an NP-Hard combinatorial problem (Donoho, 2006; Candes *et al.*, 2006).

In order to find a suboptimal but sufficient sparse solution, several greedy algorithms (Cotter and Rao, 2002; Carbonelli *et al.*, 2007) and convex relaxation methods (Gui *et al.*, 2008; Bajwa *et al.*, 2008; Taubock and Hlawatsch, 2008) have been proposed. Instead of representing the received signal as accurate as possible by channel impulse response weighted superposition of the transmitted signals, they have available a redundant dictionary and their goal is to obtain not only accurate

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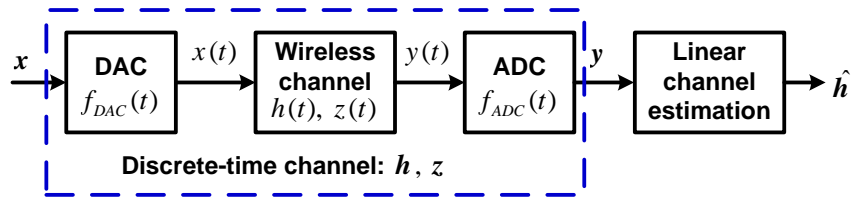


Fig. 1: Framework on traditional linear channel estimation

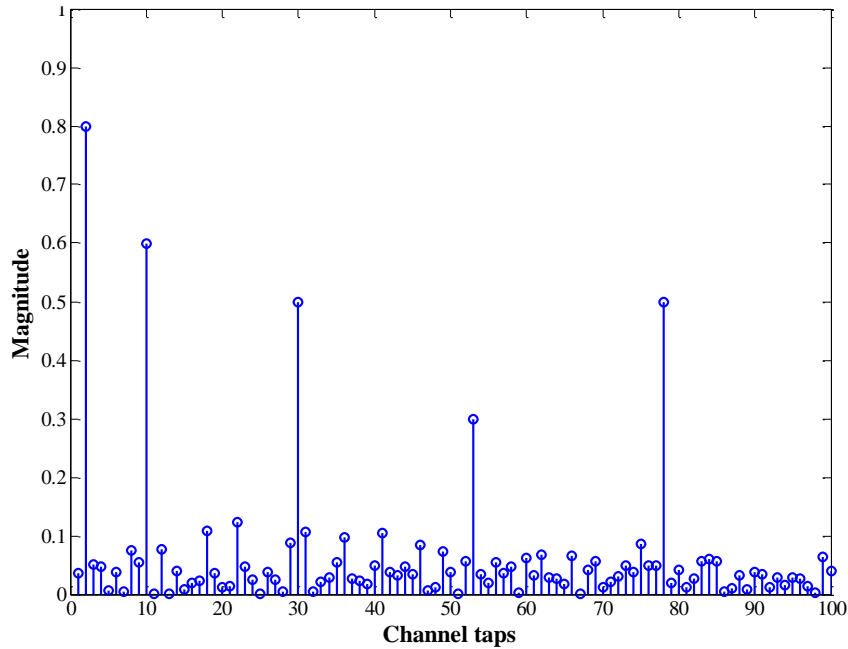


Fig. 2: A typical example of sparse multipath channel

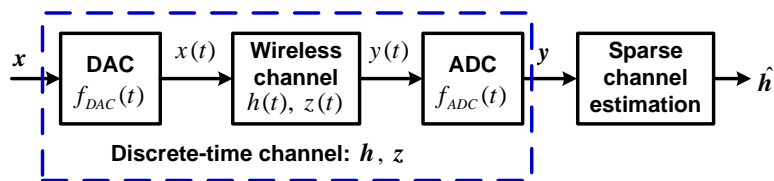


Fig. 3: Framework on sparse channel estimation

but also the sparsest possible representation of the received signal from that over-complete dictionary. Convex relaxation based sparse channel estimation methods are resolved by linear program, their complexity is very high and hard to implement on real time practical wireless communication (Gui *et al.*, 2010). Among these greedy suboptimal methods, Matching Pursuit (MP) algorithm can provide a very fast implementation of sparse approximation (Mallat and Zhang, 1993; Donoho and Huo, 2001). It has been inspiring for many researchers and different variations of this algorithm were proposed. The most famous one

is Orthogonal Matching Pursuit (OMP) algorithm (Tropp and Gilbert, 2007). Using OMP, the convergence problem in MP algorithm based on reselection of the atoms is eliminated. It was also verified that by avoiding the re-selection problem, more accurate channel estimates can be obtained by using the OMP algorithm (Karabulut and Yongacoglu, 2004). However, according to the sufficient condition developed by (Tropp and Gilbert, 2007; Cai and Wang, 2011), both the suboptimal algorithms (i.e., MP and OMP) suffer from Mutual Incoherent Property (MIP) (Donoho and Huo, 2001), interference due to coherency

and redundancy of equivalent training matrix, especially in the case of SMPC with either large time delay spread or relatively small number of training data and received data. As a result of large MIP, we may either choose a false position when the associated entry of SMPC is zero or omit a correct position when the associated entry is nonzero.

Hence, it is necessary to develop a method which can mitigate MIP interference on sparse channel estimation or other sparse signal recovery field. In our previous research (Gui *et al.*, 2011), we have proposed a sensing matrix designing method for sparse signal recovery based on the OMP algorithm in real-valued domain. However, most of the communication systems work in complex-valued domain. Moreover, only the feasibility of our proposed method in Gui *et al.* (2011) was confirmed by computer simulations.

In this study, we propose a MIP-mitigated method to improve the performance of SMPC estimation based on the OMP algorithm. Unlike the traditional OMP based sparse channel estimation, we analysis the sparse channel estimation problem which is interfered by MIP in training matrix. To alleviate the MIP on sparse channel estimation, we design a novel sensing matrix by using convex optimization algorithm. On the taps identification process, the sensing matrix is utilized efficiently to prevent false positions from being selected due to the mitigated MIP. Numerical experiments illustrate that the performance of the proposed method based on MIP mitigation is better than that of the ordinary sparse channel estimation.

System model: Consider a broadband communication system between two single-antenna transceivers over a wireless sparse multipath channel. The input-output system relation is described by:

$$y(t) = \int_0^{\tau_{\max}} h(\tau)x(t - \tau)d\tau + z(t), \quad (1)$$

where, $y(t)$ and $x(t)$ denotes the transmitted and received waveforms, respectively and τ_{\max} is defined as the maximum possible dominant taps delay spread introduced by the channel. And $z(t)$ is a zero-mean complex Additive White Gaussian Noise (AWGN). Commonly, such kinds of communication channels can be characterized as discrete, linear, time-invariant system which is shown in Fig. 1. Hence, the discrete equivalent circulant convolution system model is written as matrix-vector form:

$$\mathbf{y}^{\text{lin}} = \mathbf{x} * \mathbf{h} + \mathbf{z}^{\text{lin}} = \mathbf{X}^{\text{lin}} \mathbf{h} + \mathbf{z}^{\text{lin}}, \quad (1)$$

where, \mathbf{y}^{lin} denotes $(M + N - 1)$ -dimensional received signal vector and \mathbf{z}^{lin} is $(M + N - 1)$ -dimensional Gaussian noise samples with zero mean and variance $\sigma_n^2 \mathbf{I}_{N+M-1}$, \mathbf{h} is N -length sparse channel vector which is supported by K dominant nonzero taps, i.e., $\|\mathbf{h}\|_0 \leq K$, \mathbf{X}^{lin} denotes linear convolution training matrix:

$$\mathbf{X} = \begin{bmatrix} x_1 & 0 & 0 & \cdots & 0 \\ x_2 & x_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ x_M & x_{M-1} & x_{M-2} & \cdots & x_{M-N+1} \\ 0 & x_M & x_{M-1} & \cdots & x_{M-N+2} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x_M & x_{M-1} \\ 0 & 0 & \cdots & 0 & x_M \end{bmatrix} \quad (2)$$

It was worth noting that either the training signal or the impulse response could be rewritten as a linear convolution matrix in the above formulation. Writing it in this way, we can cast the channel estimation problem into the canonical CS framework. The channel estimation problem in this case reduces to reconstructing the unknown impulse response \mathbf{h} from equivalent system model:

$$\mathbf{y} = \mathbf{X} \mathbf{h} + \mathbf{z} \quad (3)$$

where, $\mathbf{y} = [y_1, y_2, \dots, y_M]^T$ is M -length observed signal vector; $\mathbf{z} = [z_1, z_2, \dots, z_M]^T$ denotes an complex additive white Gaussian noise with zero-mean and covariance $\sigma_n^2 \mathbf{I}_N$; $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N]$ is a partial Toeplitz matrix of the form:

$$\mathbf{X} = \begin{bmatrix} x_N & x_{N-1} & \cdots & x_2 & x_1 \\ x_{N+1} & x_N & \cdots & x_3 & x_2 \\ \vdots & \vdots & & \vdots & \vdots \\ x_{N+M-1} & x_{N+M-2} & \cdots & x_{M+1} & x_M \end{bmatrix} \quad (4)$$

Sparse channel estimation using OMP algorithm: In practical environments, most of wireless channels have the inherent sparse structure. By exploiting this sparsity, we can improve the performance or utilize smaller number of training sequence ($M \times N$) to estimate sparse channel, which is corrupted by observation noise. OMP is a canonical greedy algorithm for sparse channel estimation with the over-complete equivalent training matrix given that $M \times N$ (Tropp and Gilbert, 2007). OMP-based sparse channel estimation combines

the simplicity and the fastness. Hence, it is easy to implement in practice. Currently, there exist two kinds of theoretical analysis tools of OMP, namely Mutual Incoherence Property (MIP) (Cai and Wang, 2011) and Restricted Isometry Property (RIP) (Candes, 2008). In this study, we choose the MIP as for analysis tool on OMP-based sparse channel estimation, for the sake of easy be interpreted by numerical simulations. Sparse channel estimation method using OMP algorithm has been proposed in (Karabulut and Yongacoglu, 2004). It selects a column of training matrix which has the most correlation with current residuals at each step. Hence, the chosen tap position is added into the set of selected position. The algorithm updates the residuals by projecting the signal onto the variables which have already been selected and then the algorithm iterates. Commonly, the detail steps of the OMP-based sparse channel estimation are described in Table 1. For the later use, we define all of taps position set as $\Omega = \{1, 2, \dots, N\}$. If the method can identify dominant channel taps subset $K = \text{supp}(\mathbf{h}) = \#\{h_n | > 0, n \in \Omega\}$, then the channel can be easily estimated, for example, by using the Least Squares (LS) to estimate the subset of channel. Hence, accurate channel tap positions are most important on sparse channel estimation using OMP algorithm.

From the Table 1, after k-th iteration, the k-th dominant taps position w_k of the channel $\mathbf{h} = \{h_n, n \in \Omega\}$ can be identified by:

$$\begin{aligned} w_k &= \arg \max_{n \in \Omega} \left| \mathbf{X}^H \mathbf{r}_{k-1} \right| \\ &= \arg \max_{n \in \Omega} \left| \mathbf{X}^H \mathbf{X} \mathbf{h} - \mathbf{X}^H \mathbf{X}_{\Omega_{k-1}} \mathbf{h}_{k-1} + \mathbf{X}^H \mathbf{z} \right| \\ &= \arg \max_{n \in \Omega} \left| \mathbf{X}^H \mathbf{X}_{\Omega \setminus \Omega_{k-1}} \mathbf{h}_{\Omega \setminus \Omega_{k-1}} + \mathbf{X}^H \mathbf{z} \right| \\ &= \arg \max_{n \in \Omega} \left| \mathbf{X}^H \mathbf{x}_{w_k} \mathbf{h}_{w_k} + \mathbf{X}^H \mathbf{X}_{\Omega \setminus \Omega_k} \mathbf{h}_{\Omega \setminus \Omega_k} + \mathbf{X}^H \mathbf{z} \right| \end{aligned} \quad (5)$$

and then update k-th selected subset $\Omega_k = \Omega_{k-1} \cup \{w_k\}$. From above Eq. (6), we can find that the smaller value of $\left| \mathbf{X}^H \mathbf{X}_{\Omega \setminus \Omega_k} \mathbf{h}_{\Omega \setminus \Omega_k} + \mathbf{X}^H \mathbf{z} \right|$, higher accurate probability to identify k-th position w_k , vice versa. The noise interference $\mathbf{X}^H \mathbf{z}$ is decided by the communication environment and transmit power. In this study, we only consider coherence interference on sparse channel estimation under the same noise interference. To evaluate the coherence in X, we introduce a widely used Mutual Incoherence Property (MIP) (Cai and Wang, 2011). For a given $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N]$, its MIP is defined as:

$$\mu(\mathbf{X}) = \max_{n \neq l, n, l \in \Omega} \left| \langle \mathbf{x}_n, \mathbf{x}_l \rangle \right|. \quad (6)$$

The MIP requires the mutual incoherence μ in X to be small. Given a $M \times N$ equivalent training matrix X, where $M = 40$ and $N = 64$, its MIP is depicted in Fig. 4. From the figure, we can find that some MIPs in X is even larger than 0.4 and hence lead to stronger coherent interference to identify dominant channel taps. Herein, in Eq. (6), if $\mathbf{h}_{\Omega \setminus \Omega_k}$ is a zero vector, then MIP interference cannot effect the taps identification and estimation performance. However, if there is a nonzero entry in $\mathbf{h}_{\Omega \setminus \Omega_k}$, the MIP of $\mathbf{X}_{\Omega \setminus \Omega_k}$ will draw the tap position \tilde{w}_k away from its correct position w_k . As a result, we may either select a false tap position if the MIP is large enough. Hence, the improved estimation performance is that how to mitigate the coherent interference of MIP on the performance of OMP algorithm.

MITIGATE MIP TO IMPROVE SPARSE CHANNEL ESTIMATION

In order to identify the correct position in the case of high MIP level, we resort to the OMP based on sensing matrix W and use $w_k = \max_{n \in \Omega} \left| \mathbf{W}^H \mathbf{r}_{k-1} \right|$ rather than $w_k = \max_{n \in \Omega} \left| \mathbf{X}^H \mathbf{r}_{k-1} \right|$ in Eq. (5). Based on the training matrix X, we design a novel sensing matrix W using convex optimization method (Boyd and Vandenberghe, 2004). Consider the $M \times N$ complex training matrix X, where $\mathbf{x}_n, n = 1, 2, \dots, N$ denotes its n-th column vector and assume that each column of X is normalized so that $\|\mathbf{x}_n\|_2 = 1$. We define \mathbf{X}_{Ω_k} as a submatrix of X for any subset \mathbf{X}_{Ω_k} and term \mathbf{x}_k and \mathbf{X}_{Ω_k} as k-th column and selected w_k -th column of X, respectively.

Now the question is how to mitigate this coherent interference in the OMP algorithm. We calculate the coherence between columns in X as shown in Fig. 4. We find that most diagonal coefficients are close to 1 and some of off-diagonal coefficients are larger than 0.4. Therefore, these off-diagonal coefficients easy result in interference while selecting the optimal column in the measurement matrix. Hence, it is necessary to mitigate the coherent interference in taps identification. To mitigate the interference, we design the SMM for OMP in the next section.

Table 1: Sparse channel estimation method using orthogonal matching pursuit algorithm (Karabulut and Yongacoglu, 2004)

Input	An $M \times N$ training matrix X An $M \times 1$ received signal vector y An stopping criterion ξ
Output	An dominant taps position set Ω
Initialization	An $N \times 1$ sparse channel estimator \hat{h} $r_0 = y, h_0 = 0, \Omega^0 = \emptyset, k = 1$
Iteration	Repeat the following steps:
While	$\ r_k\ > \xi, \text{ run } k = k + 1$
Taps identification	$w_k = \arg \max_{n \in \Omega} X_n^H r_{k-1} , \Omega_k = \Omega_{k-1} \cup \{w_k\}$
Residual updating	$h_k = \arg \min \ y - X_{\Omega_k} h\ _2, r_k = y - X_{\Omega_k} h_k, \Omega = \Omega_k$

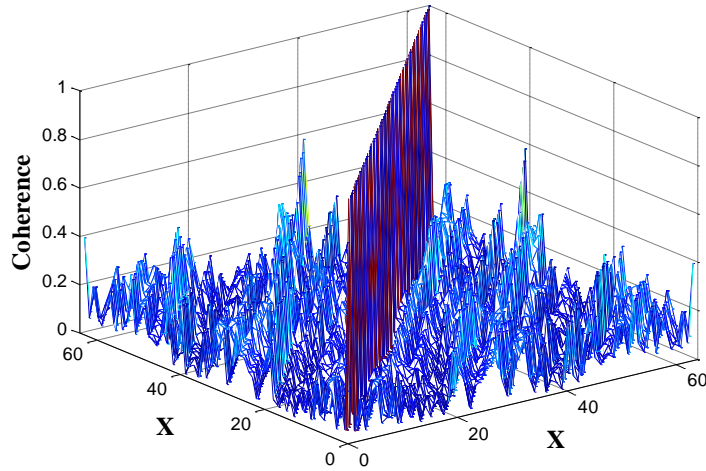


Fig. 4: Coherence in random training matrix X

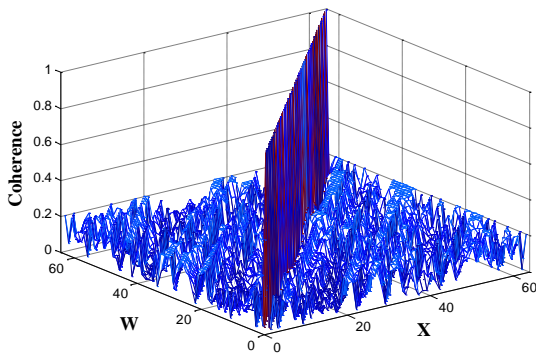


Fig. 5: Coherence in between random training matrix X and sensing matrix W

Table 2: Design a sensing matrix W based on the random training matrix X

Input	An $M \times N$ training matrix X An regularization parameter λ
Initialization	$X = X, Q = W^H X; q = \text{Diag}\{Q\}, G = Q - \text{diag}\{q\}$
Optimization	$\min\{\ G\ _{\infty} + \lambda \ W\ _F\}$ subject to $q = I_{N \times 1}$
Output	An $M \times N$ sensing matrix W .

Table 3: Mitigated MIP sparse channel estimation method using orthogonal matching pursuit algorithm

Input	An $M \times N$ random training matrix X An $M \times 1$ received signal vector y An stopping criterion ξ
Output	An dominant taps position set Ω_k
Initialization	An $N \times 1$ sparse channel estimator \hat{h} $r_0 = y, h_0 = 0, \Omega^0 = \emptyset, k = 1$
Iteration	Repeat the following steps:
While	$\ r_k\ > \xi, \text{ Run } k = k + 1$
Sensing designing	W is given in Table 2.
Taps identification	$w_k = \arg \max_{n \in \Omega} W_n^H r_{k-1} $ $\Omega_k = \Omega_{k-1} \cup \{w_k\}$
Residual updating	$h_k = \arg \min \ y - X_{\Omega_k} h\ _2$ $r_k = y - X_{\Omega_k} h_k, \Omega = \Omega_k$

In order to identify the correct components in a training matrix X , the improved OMP algorithm designs an $M \times N$ complex sensing matrix $W = \{w_n, n = 1, \dots, N\}$ and uses $w_k = \arg \max | \langle W, r_k \rangle |$ rather

than $w_k = \arg \max |\langle X, r_k \rangle|$ in OMP-based sparse channel estimation. Obviously, when $W = X$, the sparse channel estimation using OMP algorithm is a special case of the improved sparse channel estimation.

A good sensing training should have a $\mu(W, X)$ as small as possible. In a straightforward way, we may calculate the correlation between X and W , i.e., the sensing vector $w_n, n \in \Omega$, as the solution to the following convex optimization problem in Table 2. The maximum coherence in off-diagonal between X and W is no more than 0.2 as shown in Fig. 5. Hence, the sensing matrix W can effectively mitigate the coherent interference on sparse channel estimation. The proposed sparse channel estimation using OMP algorithm is summarized as in Table 3.

NUMERICAL SIMULATIONS

To gain some insights into the effect of the proposed improved sparse channel estimation method using OMP algorithm. We evaluate 10000 independent Monte-Carlo trials. The dominant taps of sparse multipath channel h are generated randomly from a complex Gaussian distribution and subject to $\|h\|_2^2 = 1$. The positions of dominant channel taps of h are generated randomly. The channel length is set to $N = 64$ and the length of training sequence x is set to $M = 48$. Hence, the X equivalent to the $M \times N$ partial Toeplitz matrix which is generated by x . Consider the received Signal-to-Noise Ratio (SNR) as:

$$SNR = -10 \log \{P_s / \sigma_n^2\} \tag{8}$$

where, P_s is the average transmitted power. The performance comparison of accurate taps identification

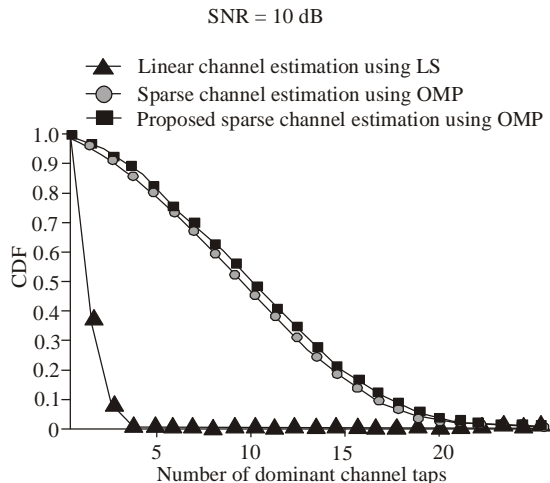


Fig. 6: CDF versus number of dominant channel taps at SNR; 20 dB

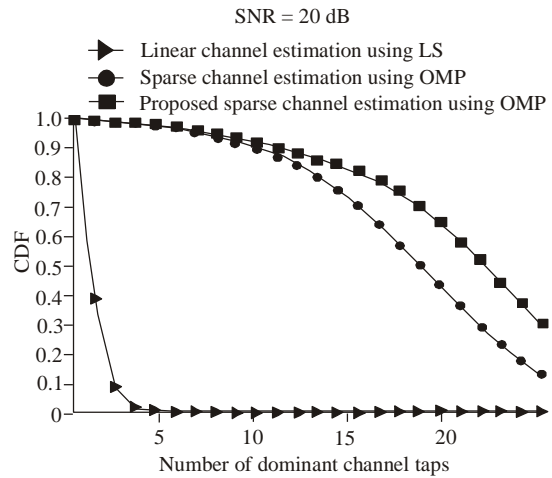


Fig. 7: CDF versus number of dominant channel taps at SNR; 20 dB

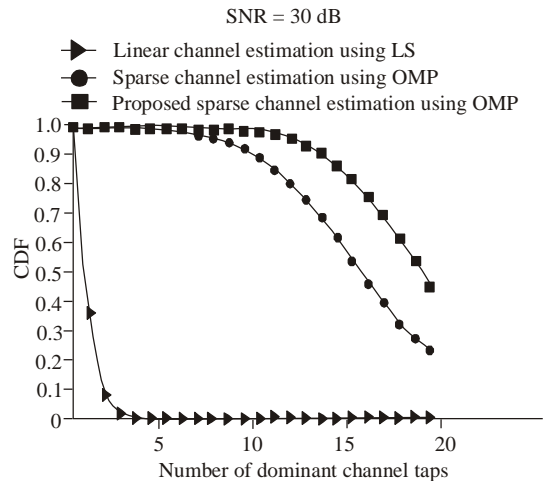


Fig. 8: CDF versus number of dominant channel taps at SNR; 30 dB

via Cumulative Density Function (CDF). From the Fig. 6, 7 and 8, at the different SNR (from 10 to 30 dB), the proposed method has a more accurate probability than ordinary sparse channel estimation method on dominant channel taps. It was worth noting that LS-based linear channel estimation method failed to identify accurate dominant taps under the underdetermined system. Hence, several LS-based taps identifying CDF curves almost overlap in the Fig. 6, 7 and 8.

CONCLUSION

Sparse channel estimation using OMP algorithm is a good candidate for broadband communication systems. However, OMP-based estimation is often degraded due to the large MIP existing in training matrix. In this study,

we proposed a MIP-mitigation method to improve the sparse channel estimation performance especially in the case of sparser and longer SMPC. The numerical experiments indicate that the proposed MIP-mitigation method outperforms the ordinary sparse channel estimation method using OMP algorithm.

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