Research Journal of Applied Sciences, Engineering and Technology 6(24): 4524-4530, 2013
DOI:10.19026/rjaset.6.3461
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: September 10, 2012 Accepted: October 09, 2012 Published: December 25, 2013

Research Article

Intelligent Decentralized Adaptive Controller Design for a Class of Large Scale Nonlinear Non-affine Systems: Nonlinear Observer-based Approach

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Abstract: In this study, an observer based decentralized Fuzzy Adaptive Controller (FAC) is designed for a class of large scale non-affine nonlinear systems with unknown functions of the subsystems and interactions. The proposed controller has the following main characteristics: 1) On-line adaptation of both controller and observer parameters, 2) stabilization of the closed loop system, 3) convergence of both tracking and observer errors to zero, 4) boundedness of all signals involved, 5) being prone to employing experts' knowledge in controller design procedure, 6) chattering avoidance. An illustrative example is given to show the promising performance of the proposed method.

Keywords: Adaptive control, fuzzy system, large scale systems, non-affine nonlinear systems, nonlinear observer, stability

INTRODUCTION

In the control of large-scale systems, one usually faces poor knowledge on the plant parameters and interconnections among subsystems.

As a result of both tunable structure of the FAC and using the experts' knowledge in controller design procedure, Fuzzy Adaptive Controller (FAC) attracted many researchers to develop appropriate controllers for nonlinear systems especially for Large Scale Systems (LSS).

In the recent years, FAC has been fully studied. Initially, the Takagi-Sugeno (TS) fuzzy systems have been used to model nonlinear systems and then TS based controllers have been designed with guaranteed stability (Feng *et al*., 2002). Modeling affine nonlinear systems and designing stable TS fuzzy based controllers have been employed in Hsu *et al*. (2003). Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems were presented in Cheng and Chien (2006). In Park and Park (2004), the non-affine nonlinear function were first approximated by the TS fuzzy systems and then stable TS fuzzy controller and observer have been designed for the obtained model. In these studies, due to the assumption that the systems should be linearizable around some operating points modeling and designing of appropriate controllers could be simply done.

The linguistic fuzzy systems have also been used to design controllers for nonlinear systems.

Jagannathan (1998), Zhang and Bien (2000) and Zhang *et al*. (2002) have considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization and furthermore in Zhang *et al*. (2002) and Zhang and Bien (2000), it has considered that the zero dynamic should be stable. Stable FAC based on sliding mode was designed for affine systems in Labiod *et al*. (2005). Designing FAC for affine chaotic systems were presented in Chen *et al*. (1999). Designing stable FAC and linear observers for class of affine nonlinear systems were discussed in Ho *et al*. (2005), Zhang (2006) and Shaocheng *et al*. (2005). Fuzzy adaptive sliding mode controllers were presented for class of affine nonlinear time delay systems in Yu (2004), Chiang (2005) and Jiang *et al*. (2005). The output feedback FAC for class of affine nonlinear MIMO systems was suggested in Yiqian *et al*. (2004). A robust adaptive fuzzy controller, based on a linear state observer, for a class of affine nonlinear systems has been presented in Hamzaoui *et al*. (2004). In Tong *et al*. (2004), direct and indirect adaptive output-feedback fuzzy decentralized controllers for a class of large-scale affine nonlinear systems have been developed based on linear observer. Vélez-Díaz and Tang (2004) presented fuzzy adaptive controller for a class of affine nonlinear systems. This method guarantied ultimately boundedness of tracking error. A direct adaptive fuzzy controller for a non-minimum phase two-axis inverted pendulum servomechanism has been presented based on real-time stabilization in Wai *et al*. (2008). The main

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drawbacks of these studies are those restricted conditions imposed on the system dynamics. For example, it is assumed the control gain is bounded to some known functions or constant values.

Labiod and Guerra (2007) and Liu and Wang (2007) developed stable FAC for class of non-affine nonlinear systems. The main limitation of these methods is that convergence of tracking errors to zero was not guaranteed. Ghasemi *et al*. (2009) proposed a decentralized fuzzy model reference state tracking controller for a class of canonical nonlinear large scale system. The main limitations of these references are both considering the interaction as a bounded disturbance and availability of all states.

The main contributions of this study are as follows. We propose a new method to design a stable observer based decentralized robust adaptive controller based on fuzzy systems for a class of large scale non-affine nonlinear systems. The controller is robust against uncertainties, external disturbances and approximation errors. Compared with the previous studies which mainly concentrated on observer-based affine SISO systems and observer based affine large scale systems, the proposed method is an observer based non-affine nonlinear large scale systems.

PROBLEM STATEMENT

Consider the following large scale non-affine nonlinear system:

$$\begin{cases} \dot{x}_{i,l} = x_{i,l+1} & l = 1,2,\ldots,n_i - 1 \\ \dot{x}_{i,n_i} = f_i(\underline{x}_i, u_i) + m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) + d_i(t) \\ y_i = C_i^T x_i \end{cases} \quad (1)$$
$$i = 1,2,\ldots,N$$

where,

- $x_{i,l}$: l^{th} state of the i^{th} subsystem
- n_i : The number of states in i^{th} subsystem
- N : The number of subsystems
- $u_i \in R$: The control input
- $y_i \in R$: The system output
- $f_i(\underline{x}_i, u_i)$: An unknown smooth nonlinear function
- $d_i(t)$: A bounded external disturbance
- $m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N)$: An unknown nonlinear interconnection term
- $\underline{x}_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in R^{n_i}$: The state vector of the system which is assumed available for measurement

The control objective is to design an adaptive fuzzy controller for system (1) such that the system output $y_i(t)$ follows a desired trajectory $y_{im}(t)$ while all signals in the closed-loop system remain bounded.

In this study, we will make the following assumptions concerning the system (1) and the desired trajectory $y_{im}(t)$.

Assumption 1: Without loss of generality, it is assumed that the nonzero function $f_u(\underline{x}_i, u_i) = \partial f_i(\underline{x}_i, u_i)/\partial u_i$ satisfies the following condition:

$$f_{iu}(\underline{x}_i, u_i) \geq f_{min} > 0 \quad \forall(\underline{x}_i, u_i) \in R^{n_i} \times R$$
$$\frac{df_{iu}(\underline{x}_i, u_i)}{dt} \geq f_{dm} \quad (2)$$

$f_{min}, f_{dm} \in R$ are known constant parameters and it is defined later. It should be noted that the condition $f_{min} > 0$ does not play a crucial role because as it will be shown later the proposed controller designed, can still work after some modifications if we assume $f_{min} < 0$:

Assumption 2: The desired trajectory $y_{im}(t)$ and its time derivatives $y_{im}^{(j)}(t)$, $j = 1,2,\ldots,n_i$, are all smooth and bounded.

The interactions are considered as external inputs and assumed to be bounded functions of subsystems states. To make it more suitable for the proposed controller derivation, the following assumption is used.

Assumption 3: The interconnection term satisfies the following:

$$\left|m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N)\right| \leq \xi_{i0} + \sum_{j=1}^{N} \xi_{ij}\left(\underline{x}_j, \hat{\underline{x}}_j\right) \quad (3)$$

Each interaction upper bound $\xi_{i0} + \sum_{j=1}^{N} \xi_{ij}\left(\underline{x}_j, \hat{\underline{x}}_j\right)$ is unknown. The scalar terms ξ_{i0} and function $\xi_{ij}\left(\underline{x}_j, \hat{\underline{x}}_j\right)$ are to be estimated properly.

Assumption 4: The external disturbance is bounded as:

$$\|d_i(t)\|_\infty \leq d_{max} \quad (4)$$

Denoting $\hat{\underline{x}}_i(t)$ as an estimation of $\underline{x}_i(t)$, we define the following:

$$\underline{y}_{im} = [y_{im} \ \dot{y}_{im} \ \ldots y_{im}^{(n-1)}]^T$$
$$\underline{e}_i = \underline{y}_{im} - \underline{x}_i = [e_i \ \dot{e}_i \ \ldots e_i^{(n-1)}]^T$$
$$\hat{\underline{e}}_i = \underline{y}_{im} - \hat{\underline{x}}_i = [\hat{e}_i \ \dot{\hat{e}}_i \ \ldots \hat{e}_i^{(n-1)}]^T \quad (5)$$
$$\tilde{\underline{e}}_i = \underline{e}_i - \hat{\underline{e}}_i$$

- $\underline{y}_{im}$ = The reference signal
- $\underline{e}_i$ = The tracking error
- $\hat{\underline{e}}_i$ = The observer error
- $\tilde{\underline{e}}_i$ = The observation error

Consider the following tracking error vector:

$$\underline{e}_i = [e_{i,1}, e_{i,2}, \ldots, e_{i,n_i}]^T \in R^{n_i} \quad (6)$$

where,

$$e_{i,1} = y_{im} - y_i, \quad e_{i,j} = e_{i,1}^{(j)} \tag{7}$$

Taking the n_i^{th} derivative of both sides of the Eq. (7) we have:

$$\begin{aligned} &e_i = y_{im} - x_{i,1} \\ &\dot{e}_i = \dot{y}_{im} - \dot{x}_{i,1} = y_{im} - x_{i,2} \\ &\vdots \\ &e_i^{(n_i)} = y_{im}^{(n_i)} - y_i^{(n_i)} = y_{im}^{(n_i)} - f_i(\underline{x}_i, u_i) \\ &\quad - m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) - d_i(t) \end{aligned} \tag{8}$$

Use Eq. (6) to rewrite the above equation as:

$$\begin{cases} \underline{\dot{e}}_i = A_{i0}\underline{e}_i + b_i\{y_{im}^{(n_i)} - f_i(\underline{x}_i, u_i) \\ \quad - m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) - d_i(t)\} \\ \underline{e}_{iy} = C_i^T \underline{e}_i \end{cases} \tag{9}$$

where, A_{i0} and b_i are defined below:

$$A_{i0} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n_i \times n_i} \tag{10}$$

$$b_i = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T \in R^{n_i}$$

To construct the controller, we define the signal v_i as:

$$v_i = y_{im}^{(n_i)} + k_{ic}^T \underline{\hat{e}}_i + v_i' \tag{11}$$

Consider the vector $k_{ic} = [k_{i,1}, k_{i,2}, \ldots, k_{i,n_i}]^T$ be coefficients of $\psi(s) = s^{n_i} + k_{i,n_i}s^{n_i - 1} + \ldots + k_{i,1}$ and chosen so that the roots of this polynomial are located in the open left-half plane. This makes the matrix $A_{ioc} = A_{i0} - b_i k_{ic}^T$ be Hurwitz.

By adding and subtracting the term $\left(k_{ic}^T \underline{\hat{e}}_i + v_i'\right)$ from the right-hand side of Eq. (9), we obtain:

$$\begin{cases} \underline{\dot{e}}_i = A_{i0}\underline{e}_i - b_i k_{ic}^T \underline{\hat{e}}_i - b_i\{f_i(\underline{x}_i, u_i) - v_i \\ \quad + m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) + d_i(t) + v_i'\} \\ e_{iy} = C_i^T \underline{e}_i \end{cases} \tag{12}$$

Using assumption (1), Eq. (11) and the signal v_i which is not explicitly dependent on the control input u_i, the following inequality holds:

$$\frac{\partial(f_i(\underline{x}_i, u_i) - v_i)}{\partial u_i} = \frac{\partial f_i(\underline{x}_i, u_i)}{\partial u_i} > 0 \tag{13}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_i(\underline{x}_i, u_i) - v_i = 0$ is locally soluble for the input u_i for an arbitrary $(\underline{x}_i, v_i)$. Thus, there exists some ideal controller $u_i^*(\underline{x}_i, v_i)$ satisfying the following equality for a given $(\underline{x}_i, v_i) \in R^{n_i} \times R$:

$$f_i(\underline{x}_i, u_i^*) - v_i = 0 \tag{14}$$

As a result of the mean value theorem, there exists a constant λ in the range of $0 < \lambda < 1$, such that the nonlinear function $f_i(\underline{x}_i, u_i)$ can be expressed around u_i^* as:

$$\begin{aligned} f_i(\underline{x}_i, u_i) &= f_i(\underline{x}_i, u_i^*) + (u_i - u_i^*) f_{iu_\lambda} \\ &= f_i(\underline{x}_i, u_i^*) + e_{u_i} f_{iu_\lambda} \end{aligned} \tag{15}$$

where,

$$f_{iu_\lambda} = \partial f_i(\underline{x}_i, u_i)/\partial u_i|_{u_i = u_{i\lambda}} \text{ and } u_{i\lambda} = \lambda u_i + (1 - \lambda)u_i^*$$

Substituting Eq. (15) into the error Eq. (12) and using (14), we get:

$$\begin{cases} \underline{\dot{e}}_i = A_{i0}\underline{e}_i - b_i k_{ic}^T \underline{\hat{e}}_i - b_i\{e_{iu} f_{iu_\lambda} + v_i' \\ \quad + m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) + d_i(t)\} \\ e_{iy} = C_i^T \underline{e}_i \end{cases} \tag{16}$$

However, the implicit function theory only guarantees the existence of the ideal controller $u_i^*(\underline{x}_i, v_i)$ for system (14) and does not recommend a technique for constructing a solution even if the dynamics of the system are well known. In the following, a fuzzy system and classic controller will be used to obtain the unknown ideal controller.

The output of the system can be rewritten as $y(x) = w(x)^T \theta$ where $\theta = \left[y^1 y^2 \ldots y^M\right]$ is a vector grouping all consequent parameters and $w(x) = [w_1(x) w_2(x) \ldots w_M(x)]^T$ is a set of fuzzy basis functions.

OBSERVER BASED FUZZY ADAPTIVE CONTROLLER DESIGN

In previous Section, it has been shown that there exists an ideal controller for achieving control objectives. Here, we show how to develop a fuzzy system to adaptively approximate this ideal controller. This section deals with an observer based controller designed appropriately for the system given in Eq. (12).

The following nonlinear observer is proposed for the system (12):

$$\begin{cases} \underline{\dot{\hat{e}}}_i = \underbrace{(A_{i0} - b_i k_{ic}^T)}_{A_{ioc}} \underline{\hat{e}}_i + K_{i0} C_i^T \underline{\tilde{e}} \\ \quad + b_i k_{ino}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left|C_i^T \underline{\tilde{e}}_i\right| \\ \hat{e}_{iy} = C_i^T \underline{\hat{e}}_i \end{cases} \tag{17}$$

where, K_{io}, k_{ino} are respectively the linear observer gain and the nonlinear observer gain. K_{io} is selected to make sure that the characteristic polynomial of $(A_{ioo}=A_{io}-K_{i0}C_i^T)$ is Hurwitz. Defining the observation error $\tilde{\underline{e}}_i=\underline{e}_i-\hat{\underline{e}}_i$ and subtracting (16) from (17) yields:

$$\dot{\tilde{\underline{e}}}_i=\underbrace{(A_{i0}-K_{i0}C_i^T)}_{A_{oo}}\tilde{\underline{e}}_i-b_i\{e_{iu}f_{iu_\lambda}+d_i(t)+v_i'+$$
$$+m_i(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)+k_{ino}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)\left|C_i^T\tilde{\underline{e}}_i\right|\} \quad (18)$$
$$\tilde{e}_{iy}=C_i^T\tilde{\underline{e}}_i$$

The output error dynamics of above equation can be written as:

$$\begin{aligned}\tilde{e}_{iy}=H_i(s)\{&e_{iu}f_{iu_\lambda}+m_i(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)\\&+d_i(t)+v_i'+k_{ino}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)\left|C_i^T\tilde{\underline{e}}_i\right|\}\end{aligned} \quad (19)$$

where $H_i(s)$ is defined by:

$$H_i(s)=-C_i^T\left(sI-(A_{i0}-K_{i0}C_i^T)\right)^{-1}b_i \quad (20)$$

$H_i(s)$ is a known stable transfer function. In order to use the SPR-Lyapunov design approach, Eq. (19) is rewritten as:

$$\begin{aligned}\tilde{e}_{iy}=H_i(s)L_i(s)\{&e_{iu}f_{iu_{\lambda f}}+m_{if}(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)\\&+d_{if}(t)+v_{if}'+k_{inof}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)\left|C_i^T\tilde{\underline{e}}_i\right|\}\end{aligned} \quad (21)$$

where, $f_{iu_{\lambda f}}=L_i(s)^{-1}f_{iu_\lambda}$, $k_{inof}=L_i(s)^{-1}k_{ino}$, $v_{if}'=L_i(s)^{-1}v_i'$, $d_{if}(t)=L_i(s)^{-1}d_i(t)$. $L_i(s)$ is chosen so that $L_i(s)^{-1}$ becomes a proper stable transfer function and $H_i(s)L_i(s)$ is a proper Strictly-Positive-Real (SPR) transfer function. We define as $L_i(s)=s^m+b_{i1}s^{m-1}+\cdots+b_{im}$ with $m=n-1$.
The state-space realization of (21) can be written as:

$$\begin{cases}\dot{\tilde{\underline{e}}}_{is}=A_{ioo}\tilde{\underline{e}}_{is}-b_{is}\{e_{iu}f_{iu_{\lambda f}}+d_{if}(t)+v_{if}'\\ \quad+m_{if}(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)+k_{inof}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)\left|C_i^T\tilde{\underline{e}}_{is}\right|\}\\ \tilde{e}_{iy}=C_{is}^T\tilde{e}_{is}\end{cases} \quad (22)$$

where, $b_{is}=[1\ \ b_{i1}\dots\ b_{im}]^T$ and $C_{is}=[1\ 0\dots0]^T$.
The ideal controller can be represented as:

$$u_i^*=f_i(\underline{z}_i) \quad (23)$$

where, $z_i=[x_i,v_i]^T$ and $f_i(\underline{z}_i)=\theta_{i1}^*w_{i1}(\underline{z}_i)+\varepsilon_{iu}$ and θ_{i1}^* and $w_{i1}(\underline{z}_i)$ are consequent parameters and a set of fuzzy basis functions, respectively. ε_{iu} is an approximation error that satisfies $|\varepsilon_{iu}|\le\varepsilon_{\max}$ and $\varepsilon_{\max}>0$. The parameters θ_{i1}^* are determined through the following optimization:

$$\theta_{i1}^*=\arg\min_{\theta_{i1}}\left[\sup\left|\theta_{i1}^Tw_{i1}(\underline{z})-f_i(\underline{z})\right|\right] \quad (24)$$

Denote the estimate of θ_{i1}^* as θ_{i1} and u_{irob} as a robust controller to compensate for approximation error, uncertainties, disturbance and interconnection term. To rewrite the controller given in (23) as:

$$u_i=\theta_{i1}^Tw_{i1}(\underline{z}_i)+u_{irob}+\hat{\underline{e}}_i^TP_{i2}K_{i0} \quad (25)$$

Consider $\xi_{ij}\left(\left|C_{js}^T\tilde{\underline{e}}_{js}\right|\right)=\eta_{ij}\left|C_{sj}^T\tilde{\underline{e}}_{js}\right|\left\|\hat{x}_j\right\|$ and then define u_{irob} by:

$$\begin{aligned}u_{irob}&=sign(C_{is}^T\tilde{\underline{e}}_{is})(\frac{N}{2f_{\min}}\left|C_{is}^T\tilde{\underline{e}}_{is}\right|+\frac{\hat{\xi}_{i0}}{f_{\min}}\\&+\frac{1}{2f_{\min}}\sum_{j=1}^N\hat{\eta}_{ij}\left\|\underline{\hat{x}}_j\right\|^2\left|C_{is}^T\tilde{\underline{e}}_{is}\right|+u_{icom}+\frac{u_{ir}}{f_{\min}}\\&+\frac{\hat{v}_i'}{f_{\min}}+\hat{k}_{inof}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)(\frac{\left|C_{is}^T\tilde{\underline{e}}_{is}\right|}{f_{\min}}+\left|\hat{\underline{e}}_i^TP_{i1}b_i\right|))\end{aligned} \quad (26)$$

In the above, equation $\theta_{i1}^Tw_{i1}(\underline{z})$ approximates the ideal controller, $\hat{\xi}_{i0}+\frac{1}{2}\sum_{j=1}^N\hat{\eta}_{ij}\left\|\underline{\hat{x}}_j\right\|^2\left|C_{is}^T\tilde{\underline{e}}_{is}\right|$ tries to estimate the interconnection term, u_{icom} compensates for approximation errors and uncertainties, u_{ir} is designed to compensate for bounded external disturbances, $\hat{k}_{inof}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)(\left|C_{is}^T\tilde{\underline{e}}_{is}\right|/f_{\min}+\left|\hat{\underline{e}}_i^TP_{i1}b_i\right|)$ tries to estimate the nonlinear gain of the observer and $\hat{v}_i'$ is estimation of v_i'. Define error vector $\tilde{\theta}_{i1}=\theta_{i1}-\theta_{i1}^*$ and use (25) and (26) to rewrite the error Eq. (16) as:

$$\begin{cases}\dot{\underline{e}}_i=A_{i0}\underline{e}_i-b_ik_{ic}^T\hat{\underline{e}}_i-b_i\{(\tilde{\theta}_{i1}^Tw_{i1}(\underline{z}_i)+u_{irob}\\ \quad-\varepsilon_{iu})f_{iu_\lambda}+m_i(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)+d_i(t)+v_i'\}\\ e_{iy}=C_i^T\underline{e}_i\end{cases} \quad (27)$$

Based on Eq. (25) and (26), the state-space realization of Eq. (22) can be written as:

$$\begin{cases}\dot{\tilde{\underline{e}}}_{is}=A_{ioo}\tilde{\underline{e}}_{is}-b_{is}\{(\tilde{\theta}_{i1}^Tw_{i1}(\underline{z}_i)+u_{irob}-\varepsilon_{iu})f_{iu_{\lambda f}}\\ \quad+d_{if}(t)+v_{if}'+k_{inof}(\tilde{\underline{e}}_i,\hat{\underline{e}}_i)\left|C_i^T\tilde{\underline{e}}_i\right|\}\\ \tilde{e}_{iy}=C_{is}^T\tilde{\underline{e}}_{is}\end{cases} \quad (28)$$

Assume that P_{i1} and P_{i2} are respectively positive definite solutions of the following matrix equations:

$$\begin{aligned}&A_{ioo}^TP_{i1}+P_{i1}A_{ioo}=-Q_{i1}\\&A_{ioc}^TP_{i2}+P_{i2}A_{ioc}=-Q_{i2}\\&b_{is}^TP_{i1}=C_{is}\end{aligned} \quad (29)$$

In Eq. (29), Q_{i1} and Q_{i2} are some properly chosen positive definite matrices.
Now, consider the following update laws:

$$\begin{aligned}
\dot{\hat{k}}_{ino} &= \gamma_{iko}\left(\frac{\left|C_{is}^T \underline{\tilde{e}}_{is}\right|^2}{f_{\min}} + \left|C_{is}^T \underline{\tilde{e}}_{is}\right| \left|b_i^T P_{i2} \underline{\hat{e}}_i\right|\right) \\
\dot{\theta}_{i1} &= \Gamma_1 C_{is}^T \underline{\tilde{e}}_{is} w_{i1}(\underline{z}_i) \\
\dot{\hat{\xi}}_{i0} &= \frac{\gamma_{\xi_{i0}}}{f_{\min}} \left|C_{is}^T \underline{\tilde{e}}_{is}\right| \\
\dot{\hat{\eta}}_{ji} &= \frac{\gamma_{\eta_{ji}}}{2 f_{\min}} \left|C_{is}^T \underline{\tilde{e}}_{is}\right| \left\|\underline{\hat{x}}_i\right\|^2 \\
\dot{u}_{ir} &= \gamma_{u_{ir}} \left|C_{is}^T \underline{\tilde{e}}_{is}\right| \\
\dot{u}_{icom} &= \gamma_{u_{icom}} \left|C_{is}^T \underline{\tilde{e}}_{is}\right| \\
\dot{\hat{v}}'_i &= \gamma_{\hat{v}'_i} \left|C_{is}^T \underline{\tilde{e}}_{is}\right|
\end{aligned} \tag{30}$$

where, $\Gamma_1 = \Gamma_1^T > 0, \gamma_{u_{ir}}, \gamma_{\eta_{ji}}, \gamma_{u_{icom}}, \gamma_{\hat{v}'_i}, \gamma_{iko} > 0$ are constant parameters.

To pursue further, the establishment of the following lemma is needed.

Lemma 1: The following inequality holds if:

$$\lambda_{\max}(Q_{i1}) \geq -{f_{dm}}\big/{f_{\min}}\, \lambda_{\max}(P_{i1})$$

$$\frac{1}{f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^T Q_{i1} \underline{\tilde{e}}_{is} + \frac{\dot{f}_{iu_{\lambda f}}}{f_{iu_{\lambda f}}^2} \underline{\tilde{e}}_{is}^T P_{i1} \underline{\tilde{e}}_{is} \geq 0 \tag{31}$$

where, $\lambda_{\max(.)}$ and $\sigma_{\max(.)}$ are maximum eigenvalue and maximum singular value, respectively.

Proof: Refer to Ghasemi *et al*. (2009).

Lemma 2: Based on lemma (1) and Eq. (34), the following inequality holds:

$$\sigma_{\max}(A_{ioo}) \leq -{f_{dm}}\big/{2 f_{\min}} \tag{32}$$

Proof: Refer to Ghasemi *et al*. (2009).

Theorem 2: Consider the error dynamical system given in (17) and (28) for the large scale system (1) satisfying assumption (1), interconnection term satisfying assumption (3), the external disturbances satisfying assumption (4) and a desired trajectory satisfying assumption (2), then the controller structure given in (25), (26) with adaptation laws (30) makes the tracking error and the observer error converge asymptotically to the origin and all signals in the closed loop system bounded.

Proof: Consider the following Lyapunov function:

$$\begin{aligned}
V = \sum_{i=1}^{N} \frac{1}{2}\Big(&\frac{1}{f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^T P_{i1} \underline{\tilde{e}}_{is} + \underline{\hat{e}}_i^T P_{i2} \underline{\hat{e}}_i + \tilde{\theta}_{i1}^T \Gamma_1^{-1} \tilde{\theta}_{i1} \\
&+ \frac{\tilde{\xi}_{i0}^2}{\gamma_{\xi_{i0}}} + \frac{\sum_{j=1}^{N} \tilde{\eta}_{ji}^2}{\gamma_{\eta_{ji}}} + \frac{\tilde{u}_{ir}^2}{\gamma_{u_{ir}}} + \frac{\tilde{u}_{icom}^2}{\gamma_{u_{icom}}} + \frac{\tilde{k}_{ino}^2}{\gamma_{iko}} + \frac{\tilde{v}_i'^2}{\gamma_{\hat{v}'_i}}\Big)
\end{aligned} \tag{33}$$

where, $\tilde{\theta}_{i1} = \theta_{i1} - \theta_{i1}^*$, $\tilde{u}_{ir} = u_{ir} - d_{\max}$, $\tilde{u}_{icom} = u_{icom} - \varepsilon_{\max}$, $\tilde{k}_{ino} = \hat{k}_{ino} - k_{ino}$, $\tilde{\eta}_{ji} = \hat{\eta}_{ji} - \eta_{ji}$, $\tilde{\xi}_{i0} = \hat{\xi}_{i0} - \xi_{i0}$ and $\tilde{v}'_i = \hat{v}'_i - |v'_i|$.
After some manipulation on the time derivative of the Lyapunov function, we have:

$$\begin{aligned}
\dot{V} \leq \sum_{i=1}^{N} \; & -\frac{1}{2 f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^T \left(Q_{i1} + \frac{\dot{f}_{iu}}{f_{iu}} P_{i1}\right) \underline{\tilde{e}}_{is} \\
& -\frac{1}{2} \underline{\hat{e}}_i^T Q_{i2} \underline{\hat{e}}_i
\end{aligned} \tag{34}$$

Use lemma (1) and easily show that $\dot{V} \leq 0$. Using Barbalat's lemma, it is guaranteed the convergence of both the tracking error and the observer error asymptotically to the origin. Furthermore, the boundedness of the coefficient parameters is guaranteed. This completes the proof Q.E.D.

SIMULATION RESULTS

In this section, we apply the proposed observer based decentralized fuzzy model reference adaptive controller to the following large scale system:

$$\begin{aligned}
&\begin{cases}
\dot{x}_{11} = x_{12} \\
\dot{x}_{12} = \sin(x_{11}) + 50\tanh(u_1) + 4\sin(x_{21}) \\
\qquad + d_1(t) \\
y_1 = x_{11}
\end{cases} \\
&\begin{cases}
\dot{x}_{21} = x_{22} \\
\dot{x}_{22} = 40\tanh(u_1) + \sin(x_{11}) + 6\sin(x_{21}) \\
\qquad + d_2(t) \\
y_2 = x_{21}
\end{cases}
\end{aligned} \tag{35}$$

It has been considered that the desired value of the outputs are $y_{1m} = 0.5\sin(2\pi t) + 0.5\sin(4\pi t)$ and $y_{2m} = 0.5\sin(4\pi t) + 0.5\sin(6\pi t)$. Furthermore, it is assumed that $d_1(t) = 0.3\sin(100\pi t)$ and $d_2(t) = 0.5\sin(120\pi t)$.

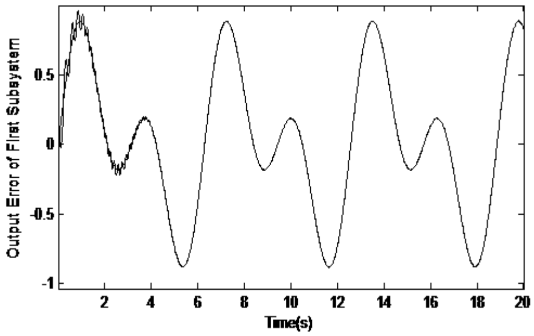

Fig. 1: Performance of the proposed controller in first subsystem

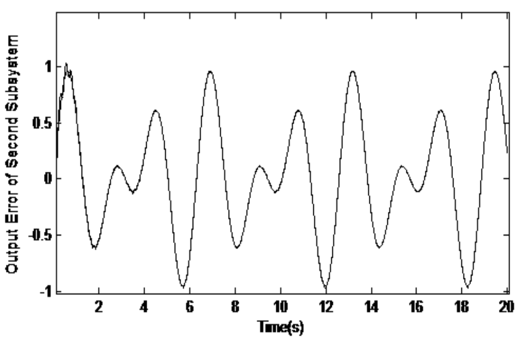

Fig. 2: Performance of the proposed controller in second subsystem

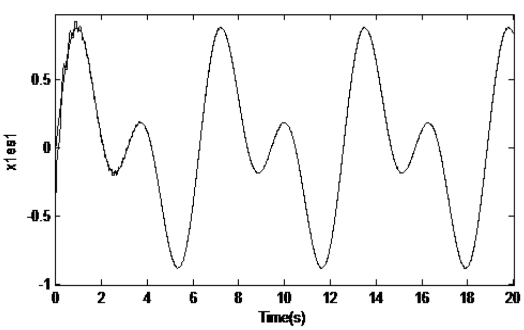

Fig. 3: The estimation of the first state of the first subsystem and the desired value

Now we applied the proposed controller defined in (30), (31) to the system defined in Eq. (35). Based on the experts' knowledge, let to define $x_i = [x_{i1}, x_{i2}]^T$, $z_i = [x_{i1}, x_{i2}, v_i]^T$ and the states of the subsystems are in the range of [5, -5], furthermore v_i are defined over [-45, 45]. For each fuzzy system input, we define 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function:

$$\mu_j(\chi) = \exp\left({(\chi - c)^2}\big/{2\delta^2}\right)$$

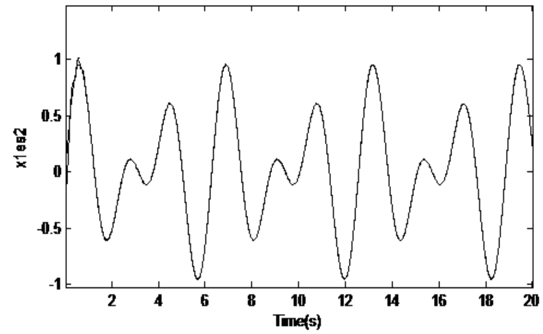

Fig. 4: The estimation of the first state of the second subsystem and the desired value

where c is center of the membership function and δ is its variance. We assume that the initial value of $\theta_{i1}(0)$, $\hat{\xi}_{i0}(0)$, $\hat{\eta}_{ji}(0)$, $u_{ir}(0)$, $u_{icom}(0)$, $\hat{k}_{ino}(0)$ and $\hat{v}_i'(0)$ be zero. Furthermore, it has been assumed that $f_{\min} = 1$, $\Gamma_1 = 10$, $\gamma_{\xi_{i0}} = 2$, $\gamma_{\xi_{ij}} = 2$, $\gamma_{u_{com}} = 5$, $\gamma_{u_r} = 5$, $\gamma_{\hat{v}_i'} = 2$ and $\gamma_{iko} = 10$. In addition, we assume that $\varepsilon = 0.01$. The parameters $f_{dm}, f_{\min}$ and the vector k_{ic} has been chosen so that lemma 2 hold.

In Fig. 1 and 2, it is obvious that the performance of the proposed controller is promising. Furthermore, in the presence of $d_i(t)$ and ε_{iu}, it is evident the controller has still a very promising performance while it is robust against both uncertainties and disturbances. Figure 3 shows the estimation of the first state of the first subsystem with their desired values given in Eq. (17).

The performance of the proposed observer on the second subsystem and their desired trajectories is shown in Fig. 4.

In Fig. 3 and 4, it is obvious that the nonlinear state observer can generate the estimated states and perform exactly. Moreover, it is also clear that the output of the system converge to the desired value. The stability of the closed loop, the convergence of the tracking error and the observer error to zero and robustness against external disturbance and approximation error are the merits of the proposed controller and observer.

CONCLUSION

A new observer based decentralized fuzzy adaptive state tracking controller for a class of large scale non-affine nonlinear systems is presented while the interconnections and the functions of the subsystems are unknown. Both the proposed controller and the designed observer guaranty the stability of the total closed-loop system and asymptotic convergence of the tracking and observer errors to zero. Robustness against external disturbances and approximation errors, relaxing the conditions and using knowledge of experts are the merits of the proposed controller.

ACKNOWLEDGMENT

This study is sponsored by department of electrical engineering, Damavand Branch, Islamic Azad University.

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