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Research Article

Research on Consistency Test in Group Decision Making based on Different Preference Information

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Abstract: The aim of study is to improve the consistency of fuzzy complementary judgment matrices. Group decision making problems with different forms of p reference information are discussed. Firstly, four forms of p reference information (i.e., preference ordering, utility value, AHP judgment matrix and fuzzy judgment matrix) are introduced and the computing formulas are given to uniform different forms of preference information into the form of fuzzy judgment matrix. Then, the paper discusses the method of testing the complete consistency of the fuzzy complementary judgment matrices. On the basis of some theories, a new algorithm is proposed to improve the fuzzy complementary judgment matrices consistency. And then the paper analyzes the feasibility of the algorithm theoretically. The algorithm is not only simple and effective, but also can provides a new idea for the research on attributes sorting or some references for expert s to modify the original judgment matrix.

Keywords: Consistency, fuzzy complementary judgment matrices, group decision making, information synthesis

INTRODUCTION

In group decision making, the different preference information structures of the comparison matrix, utility value, and preference order are probably adopted owing to the decision-making difference on knowledge structures, individual preference and judgment level of the decision maker. In addition, the fuzzy judgment matrix that decision-makers give has incomplete consistency (Switalski, 2001), so the fuzzy judgment matrix must be adjusted. In this case, the aggregation approach of how to aggregate different structure preference from single decision maker into group preference should be studied (Kacprzyk *et al*., 1992; Romero, 2001). The literature (Switalski, 2001) summarized the progress in field of uncertainty decision making, which only limited to single preference information. If the fuzzy judgment matrix given by decision-makers has incomplete consistency, then it needs to be revised. But the revision will change the original judgment matrix to a certain extent, which will result the inconsistency of the final results with the opinion of actual experts. In order to guarantee the credibility and accuracy of the final result, the degree of consistency of judgment matrix must meet certain requirements and the judgment matrix that does not satisfy with the consistency requitement must be adjusted.

In this paper, Group decision making problems with different forms of preference information are studied. The computing formulas are given to uniform different forms of preference information into the form of fuzzy judgment matrix (Finan and Hurley, 1997; Xu and Wei, 1999) and then how to test consistency of the fuzzy judgment matrix is discussed, On the basis of some theories, a new algorithm is proposed to improve the fuzzy complementary judgment matrices consistency. So, the aim of study is to improve the consistency of fuzzy complementary judgment matrices.

LITERATURE REVIEW

In group decision-making, there are a finite set of decision-making scheme $X = \{x_1, x_2,\ldots, x_n\}$, $n\geq 2$. And there are many decision makers $D = \{d_1, d_2, \ldots, d_m\}$, $m\geq 2$. The decision-makers may provide four kinds of preference information: utility value, the value of order relations, AHP and Fuzzy judgment matrix. The four different forms of preference information are transformed into fuzzy comparison matrix. Specific conversion method can be described as follows:

- For the utility vector $u^k = (u^k{}_1, u^k{}_2,\ldots, u^k{}_n)^T$

$$p_{ij}^k = \zeta^1(u_i^k, u_j^k) = \begin{cases} \dfrac{u_i^k}{u_i^k + u_j^k}, (u_i^k, u_j^k) \neq (0,0) \\ 0.5, (u_i^k, u_j^k) = (0,0) \end{cases}, i \neq j \qquad (1)$$

- For the order relation vector:

$$o^k = (o_1^k, o_2^k, \cdots, o_n^k)^T$$

$$p_{ij}^k = \zeta^2(o_i^k, o_j^k) = 0.5(1 + \frac{o_j^k - o_i^k}{n-1}), i \neq j \qquad (2)$$

- For the AHP Judgment matrix:

$$p_{ij}^k = \zeta^3(a_{ij}^k) = 0.5 + \log_c a_{ij}^k \quad (3)$$
$$c = 81$$

From the above analysis, we can see that the different forms of preference information can be transformed into each other. Therefore, the following discussion is based on the reciprocal form of the comparison matrix.

Complementary judgment matrix converted from the order relation and the utility value is not required for consistency analysis. Complementary judgment matrix itself and converted from the reciprocal judgment matrix is needed to identify the consistency.

Definition 1: If fuzzy judgment matrix $P = (p_{ij})_{n\times n}$ meets:

$$(p_{ik} - \frac{1}{2}) + (p_{kj} - \frac{1}{2}) = p_{ij} - \frac{1}{2} \quad i, j, k = 1, 2, \cdots, n$$

then it is consistent. The test method of consistency of fuzzy judgment matrix is simple, matrix, as follows**:**

Theorem 1: If the difference of corresponding elements in two rows (columns) is a constant, then the fuzzy judgment matrix is a consistency matrix

If P is not the consistent fuzzy judgment matrix and how to measure the degree of its consistency?

Suppose $P = (p_{ij})_{n\times n}$ is consistent fuzzy judgment matrix, $\forall i, j, k \in N$, $p_{ij} = p_{ik} - p_{jk} + 1/2$, that is:

$$p_{ij} = p_{ik} + p_{kj} - \frac{1}{2}$$

So

$$\sum_{k=1}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right| = 0$$

that is

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right| = 0$$

In fact, because of the complexity of problem and the inconsistency of the decision makers thinking and other reasons, the fuzzy judgment matrix given by the decision makers is often not consistent. So, for a given $p_{ij}(i \neq j)$, $\sum_{k=1}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right|$, is not equal to zero. When $k = i$ or $k = j$, $[p_{ij} - (p_{ii}+p_{ij}-1/2] = [p_{ij}-(p_{ij}+p_{jj}-1/2] = 0$, Therefore, $N(p_{ij}) = 1/n\text{-}2 \sum_{k=1, k\neq i,,j}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right|$ can express the inconsistent degree of fuzzy judgment matrix. Because $p_{ij} = 1 - p_{ij}$ So,

$$\rho = \frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{\substack{k=1\\k\neq i,j}}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right|$$

Theorem 2: The necessary and sufficient condition that fuzzy judgment matrix is consistent is the consistency index $\rho = 0$.

As for the fuzzy judgment matrix that is inconsistent, its consistency index $\rho>0$ and the greater ρ, the worse consistency degree. In practical application, decision maker gives a threshold $\varepsilon>0$ (for example, $\varepsilon = 0.2$), when $\rho<\varepsilon$, then t the fuzzy judgment matrix has a satisfactory agreement.

THE METHOD OF IMPROVING THE CONSISTENCY OF FUZZY JUDGMENT MATRIX

The key of correcting inconsistent fuzzy complementary judgment matrix is to amend certain elements by using the information provided by the judgment matrix:

$\forall\ i, j, k \in N$ let $\delta_{ijk} = |p_{ij}\text{-}(p_{ik}+p_{kj}\text{-}1/2)|$

Theorem 3: As for the fuzzy judgment matrix, deviation δ_{ijk}, δ_{jik}, δ_{ikj}, δ_{kij}, δ_{kji} is equal to each other.

Theorem 4: If only a pair of elements p_{ij}, p_{ji} in the matrix is amended, then the deviations that subscript contains i, j may change, the remaining deviations do not change.

For any non-zero deviation, the direct method decreasing its value is to amend $p_{ij}(p_{ji})$ or p_{ik} (p_{ki}) or p_{jk} (p_{kj}). To preserve the preference information of experts, a basic principle of correction is to adjust a pair of elements and the changing deviation shall meet the following condition:

$$\left(p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2})\right) \cdot \left(p_{ij}' - (p_{ik}' + p_{kj}' - \frac{1}{2})\right) \geq 0 (k \in N, k \neq i, j)$$

Therefore, if $(p_{ij}\text{-}(p_{it}+p_{tj}\text{-}1/2))$. $(p_{ij}\text{-}(p_{ik}+p_{kj}\text{-}1/2) > 0$, then deviation δ_{ijk} $(k \in N,\ k\neq i, j)$ will decrease; If $(p_{ij}\text{-}(p_{it}+p_{tj}\text{-}1/2))$. $(p_{ij}\text{-}(p_{ik}+p_{kj}\text{-}1/2) \leq 0$, then deviation $\delta_{ijk}(k \in N, k\neq i, j)$ will increase.

Obviously, if the revised deviation is smaller than the original deviation, then a single amendment is effective:

$$\sum_{1\leq i<j<k\leq n} \delta_{ijk}' - \sum_{1\leq i<j<k\leq n} \delta_{ijk} = \sum_{\substack{k\neq i,j\\k\in N}} \left| p_{ij}' - (p_{ik} + p_{kj} - 0.5) \right| - \sum_{\substack{k\neq i,j\\k\in N}} \left| p_{ij} - (p_{ik} + p_{kj} - 0.5) \right|$$

$$= \sum_{k \in \Delta_{ij}^{(1)}} \left(p_{ij}' - p_{ij} \right) + \sum_{k \in \Delta_{ij}^{(2)}} \left(p_{ij} - p_{ij}' \right)$$
$$= n_{ij}^{(1)} \left(p_{ij}' - p_{ij} \right) + n_{ij}^{(2)} \left(p_{ij} - p_{ij}' \right)$$
$$= \left(n_{ij}^{(1)} - n_{ij}^{(2)} \right) \left(p_{ij}' - p_{ij} \right)$$

Among:

$$\Delta_{ij}^{(1)} = \left\{ k \left| p_{ij} - (p_{ik} + p_{kj} - 0.5) > 0, k \neq i, j \right. \right\}$$
$$\Delta_{ij}^{(2)} = \left\{ k \left| p_{ij} - (p_{ik} + p_{kj} - 0.5) \leq 0, k \neq i, j \right. \right\}$$

$n_{ij}^{(1)}$ Represents the number of deviations that the deviation value in the matrix reduces;
$n_{ij}^{(2)}$ Represents the number of deviations that the deviation value in the matrix increases;

Obviously, if $\left(n_{ij}^{(1)} - n_{ij}^{(2)} \right) \left(p_{ij}' - p_{ij} \right) < 0$, then this revision is effective.

Therefore, the steps of improving consistency of fuzzy judgment matrix are as follows:

Step 1: As for the initial fuzzy judgment matrix, given the critical value of consistency ε (usually ε = 0.2).
Step 2: Calculating the consistency index ρ of fuzzy judgment matrix, if $\rho<\varepsilon$, then turn to step 7, otherwise, turn to step 3.
Step 3: Finding out the elements from the current matrix that satisfy of the following conditions:

(1) $S^+(p_{ij})$ or $S^-(p_{ij})$ is greater than zero.
Among, $S^+(p_{ij}) = n^{(1)}_{ij} - n^{(2)}_{ij}$ $S^-(p_{ij}) = n^{(2)}_{ij}$-$n^{(1)}_{ij}$
(2) $S^+(p_{ij})>0$ and $p_{ij} \neq 0.1$; Or S-(p_{ij})>0, $p_{ij} \neq 0.9$

Step 4: Identifying the elements to be amended and the principle is that the correction element can minish the consistency index as large as possible. pi^*j*= $\max\{S(p_{ij})|p_{ij}`-p_{ij}|\}$,if $p^{i^*}j^*$ is single, then let $r = i^*$, $s = j^*$.

Otherwise, in order to minish the deviation as much as possible, selecting the greatest one of $S(p_{i^*j^*})$. Among:

$$S(p_{ij}) = \begin{cases} S^+(p_{ij}), S^+(p_{ij}) > 0 \\ S^-(p_{ij}), S^-(p_{ij}) > 0 \end{cases}$$

$$p_{ij}' = \begin{cases} \max\{0.1, \max\{p_{ik} + p_{kj} - 0.5 \left| p_{ij} - (p_{ik} + p_{kj} - 0.5) > 0, k \in N, k \neq i, j \right.\}\}, S^+(p_{ij}) > 0 \\ \min\{0.9, \min\{p_{ik} + p_{kj} - 0.5 \left| p_{ij} - (p_{ik} + p_{kj} - 0.5) > 0, k \in N, k \neq i, j \right.\}\}, S^-(p_{ij}) > 0 \end{cases}$$

Step 5: Amendment (p_{rs}, p_{sr}) to (p_{rs}\`, $p_{sr}$\`), among, P_{sr}\` = 1-$p_{rs}$\`
Step 6: Repeating step 2 to step 5.
Step 7: The end of adjustment and the current judgment matrix has satisfied consistency.

CALCULATION

Suppose the fuzzy judgment matrix:

$$P_0 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{pmatrix} \quad \varepsilon = 0.2$$

Step 1: Calculating the consistency index ρ:

$$\rho = \frac{1}{n(n-1)(n-2)} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{\substack{k=1 \\ k \neq i,j}}^{n} \left| p_{ij} - (p_{ik} + p_{kj} - \frac{1}{2}) \right| = 0.5$$

So, the fuzzy judgment matrix has no satisfied consistency. The specific revising process is as follows:

$$P_1 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.9 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.1 & 0.1 & 0.5 \end{pmatrix} \quad \rho = 0.4 > \varepsilon$$

$$P_2 = \begin{pmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.9 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.1 & 0.4 & 0.5 \end{pmatrix} \quad \rho = 0.15 < \varepsilon$$

CONCLUSION

In this paper, Group decision making problems with different forms of preference information are studied. Firstly, different forms of preference information are conformed to the information expressed by fuzzy judgment matrix, then the test method of the consistency of fuzzy judgment matrix is introduced. Finally, the method of improving consistency of fuzzy judgment matrix is given. This research study enriches and develops the theory and methods of fuzzy judgment matrix and numerical examples illustrate the feasibility and effectiveness of the Act. It can be seen, this method has strong maneuverability and practicability. And then the paper analyzes the feasibility of the algorithm theoretically. The algorithm is not only simple and effective, but also can provides a new idea for the research on attributes sorting or some references for expert s to modify the original judgment matrix.

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