

Research Article

Optimized Reconfigurable Control Design for Aircraft using Genetic Algorithm

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Abstract: In this study, we propose a Genetic Algorithm (GA) based modular reconfigurable control scheme for an over-actuated non-linear aircraft model. The reconfiguration of the flight controller is achieved for the case of control surface faults/failures using a separate control distribution algorithm without modifying the base-line control law. The baseline Multi-Input Multi-Output (MIMO) Linear Quadratic Regulator (LQR) is optimized using GA to produce desired moment commands. Then, a GA based weighted pseudo-inverse method is used for effective distribution of commands between redundant control surfaces. Control surface effectiveness levels are used to redistribute the control commands to healthy actuators when a fault or failure occurs. Simulation results using ADMIRE aircraft model show the satisfactory performance in accommodating different faults, which confirm the efficiency of optimized reconfigurable design strategy.

Keywords: Control allocation, genetic algorithm, linear quadratic regulator, non-linear aircraft model, pseudo-inverse method

INTRODUCTION

In advanced safety critical systems, e.g., civil and modern combat aircraft, physical redundancy is ensured in the design by integrating multiple redundant control surfaces with Fly-By-Wire (FBW) technology (Brière and Traverse, 1993; Forssell and Nilson, 2005). In essence, these aircrafts have more actuating surfaces to control the same three rotational degrees of freedom (pitch, roll and yaw). Thus, the advance fight control schemes are utilizing available actuator redundancy for aircraft safety, survivability and maneuverability which on the other hand, increases cost, weight and design complexity. However, mixed control strategy allows the usage of already available control effectors to generate moments along other axes e.g., differential movement of elevator can be used to produce a rolling moment. Among the recent popular approaches to manage the actuator redundancy is the modular approach where a separate control allocator is introduced with base-line control strategy for handling actuator faults or failures (Harkegard, 2003; Shertzer *et al.*, 2002).

There is extensive literature available on control allocation strategies without consideration of an optimized baseline controller (Bordigon, 1996; Enns, 1998). These optimization based control allocation methods have varying performance characteristics with respect to computational requirement, constraint handling, allocation efficiency and ease of implementation in open and closed-loop (Bodson, 2001; Page and Steinberg, 2000). In recent years,

researchers have used control allocation for fault tolerant and reconfigurable control in modular design (Joosten *et al.*, 2007; Raza and Silverthorn, 1985; Ducard, 2009). However, none of these studies have discussed a powerful and fast natural evolution based optimization technique for reconfigurable modular flight control design.

In this research study, base-line controller is optimized using GA with control allocation strategy for reconfigurable control in compensating actuator faults (Fig. 1). An optimal allocation strategy whose volume of the attainable subset (V_{ψ}) is maximized through GA and compared with geometric constrained CA known as direct control allocation method proposed by Durham (1993, 1994) is used. The control surface deflection to angular acceleration relationship is treated as linear and presented in a control effectiveness matrix estimated at trim conditions. This complex problem of a desired moment commands distribution between multiple coupled control surfaces requires a fast, efficient and optimal solution. Improved performance of the modular design approach because of a global optimization algorithm which is highly adaptable for parallel processing is demonstrated in normal and fault conditions.

The objective of this study is to optimally design and implement a control law and a CA strategy using evolutionary algorithm for nonlinear model of a generic aircraft (ADMIRE). In view of simplicity, robustness and reasonable performance, we have used LQR as a base-line controller with control allocation approach

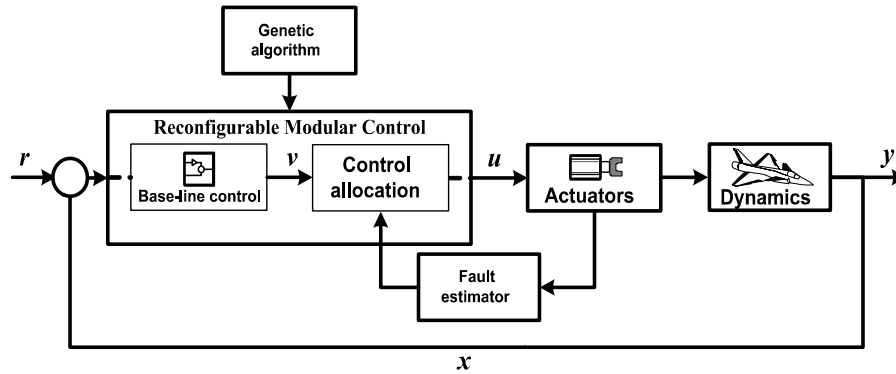


Fig. 1: Structure of GA optimized modular flight control

based on pseudo-inverse method (Bodson, 2001). The optimized modular control strategy uses the identification of effectiveness level of the actuators to redistribute the control to the remaining actuators when faults occur. The novelty of this study as compared to previous study (Shi *et al.*, 2010; Ahmad *et al.*, 2009) lies in consideration of flight dynamics and actuator faults using optimal controller and control allocation approach tuned with an intelligent optimization algorithm for reconfigurable control.

PROBLEM STATEMENT AND MODULAR CONTROL SCHEME

Consider the linearized aircraft dynamics at a trim condition in state-space form as:

$$\dot{x}(t) = Ax(t) + B_u u(t), \quad y(t) = Cx(t) \quad (1)$$

where,

- A = The $n \times n$ state matrix
- B_u = The $n \times m$ input control matrix
- C = The $p \times n$ output matrix
- $x \in \mathfrak{R}^n$ = The system state vector
- $u \in \mathfrak{R}^m$ = The control input vector
- $y \in \mathfrak{R}^p$ = The system output vector to be controlled without excessive expense of control "effort"

Here, we assume that all states are measurable and the system is full-state feedback system.

Now we consider the actuator faults and failures in (1) and rewrite the state-space equation as:

$$\dot{x}(t) = Ax(t) + B_u K u(t) \quad (2)$$

where, $K = \text{diag}(k_1, \dots, k_m)$ is the actuator effectiveness gain matrix or actuator fault/failure identification matrix and k_i are scalars having values from 0 to 1. If k_i

= 1 the i^{th} actuator is functioning perfectly whereas if $0 < k_i < 1$, a fault is present in the actuator and if $k_i = 0$ the actuator has failed completely. In this study, information about K will be used in control allocation algorithm for reconfigurable control design.

To implement a control allocation strategy in modular approach we use virtual command concept (Harkegard, 2003) for redundant actuators. Control allocation is usually used for over-actuated systems, where the control devices are greater than the variables to be controlled) $\text{len}(v) < \text{len}(u)$. Let assume that $\text{rank}(B_u) = l < m$ in (1). So, B_u can be factorized as:

$$B_u = B_v B_e \quad (3)$$

where $B_u \in \mathfrak{R}^{n \times m}$, $B_v \in \mathfrak{R}^{n \times l}$ and $B_e \in \mathfrak{R}^{l \times m}$ are the control, virtual control and control effectiveness matrices respectively.

The alternate state equation form of (1) can be given as:

$$\dot{x}(t) = Ax(t) + B_v v(t) \quad (4)$$

$$v(t) = B_e u(t) \quad (5)$$

where, $v \in \mathfrak{R}^l$ is the total control effort produced by the actuators and commanded by the base-line controller. Here, we consider that the number of virtual control (v) equals the number of outputs to be controlled (y), ($l = p$). Normally, the actuator dynamics are much faster than the aircraft dynamics. So, the control allocation process has a linear relationship between constrained control command (u) and virtual command (v) in (5). The control $u(t)$ is limited by:

$$\underline{u} \leq u \leq \bar{u} \quad (6)$$

where, \underline{u} and \bar{u} are the lower and upper physical position deflection limits of actuators. The modular

structure of suggested scheme with control allocator and GA optimization is shown in Fig. 1. Here, the virtual command vector consists of, roll (C_l), pitch (C_m) and yaw (C_n) moments and u represents the commanded actuator positions:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} \quad (7)$$

Base-Line Control law (LQR): Considering optimal control theory, we determine the virtual command by minimizing the following objective function:

$$J = \int_0^{\infty} ((x - x^*)^T Q (x - x^*) + (v - v^*)^T R (v - v^*)) dt \quad (8)$$

where, Q (positive semi-definite) and R (positive definite) are weighting matrices and x^* , v^* solve:

$$Ax + B_v v = 0 \quad (9)$$

$$Cx = r \quad (10)$$

where, $r \in \mathfrak{R}^l$ is the reference input and the optimized virtual command v is given by:

$$v = L_r r - Lx \quad (11)$$

$$L_r = G_0^{-1} \quad (12)$$

$$L = R^{-1} B_v^T S \quad (13)$$

where,

$$G_0 = C(B_v F - A)^{-1} B_v \quad (14)$$

Here L , L_r and S are the feedback gain matrix, feed-forward gain matrix and the unique positive semi-definite and symmetric matrix solution to:

$$A^T S + SA - SB_v R^{-1} B_v^T S + Q = 0 \quad (15)$$

The above expression is the famous Algebraic Riccatic Equation (ARE). Now, the optimized closed-loop poles are the eigenvalues of $A - B_v L$.

The best LQR controller performance can be achieved by proper selection of Q and R weighting matrices. There are several methods available for determining weighting matrices, with closed loop poles

placement in complex left half plane. The new poles placement improves the stability index and minimizes the control effort. The selection of Q and R weighting matrices was generally done intuitively. Whereas, different poles locations, because of weighting matrices and gains correspond to varying system performance. Thus employing intelligent optimization techniques for searching Q and R is more impressive. The weighting matrices $Q = \text{diag} (18.46 \ 0.72 \ 0.74 \ 6.78 \ 1.48)$ and $R = \text{diag} (0.15 \ 20 \ 3.5)$ are searched using genetic algorithm. The feedback gain matrix L derived using (13) is given as:

$$L = \begin{bmatrix} 0.1344 & -2.6365 & 1.1041 & 0.0661 & -0.4991 \\ 4.8530 & 0.0123 & 0.0005 & 2.5069 & -0.0042 \\ -0.0487 & -0.9156 & -0.0220 & -0.0240 & 1.2092 \end{bmatrix}$$

Cascaded Generalized Inverse (CGI) method:

Generalized inverse (GI) based solutions are very popular in control allocation. Where a control mixing matrix is used to satisfy $B_e P = I_l$, where I_l is $l \times l$ identity matrix and $P \in \mathfrak{R}^{m \times l}$. The cascaded generalized inverse is an improved version of redistributed pseudo-inverse (Bordigon, 1996; Virning and Bodden, 1994) is an optimization base linear control allocation method which exploits the use of generalized inverse, where the cost function is:

$$u = \arg \min_{u_{\min} \leq u \leq u_{\max}} \|W_u (u - u_d)\|_2 \quad (16)$$

Subject to $B_e u = v$.

and the solution of (16) is given as:

$$u = P v, \quad P = W_u^{-1} (B_e W_u^{-1})^\dagger \quad (17)$$

where ‘ \dagger ’ is the left pseudo-inverse operator. The CGI allocator efficiency depends on the optimized Weighted Pseudo-Inverse (WPI) matrix P , where $W_u \in \mathfrak{R}^{m \times m}$ is the diagonal weighting matrix which allows the designer to penalize the control commands.

The procedure of the CGI is described as follows. Initially, a pseudo inverse is computed which distribute the controls given in response of desired moments from base-line controller. If the control inputs exceed the respective position limits the pseudo-inverse solution is limited to its respective maximum or minimum value and removed from the optimization and its effect is subtracted from the desired virtual command. Then, again the pseudo-inverse based control allocation is performed for remaining unsaturated controls to achieve the desired moments. Keep, this process continues until the desired response is achieved. The

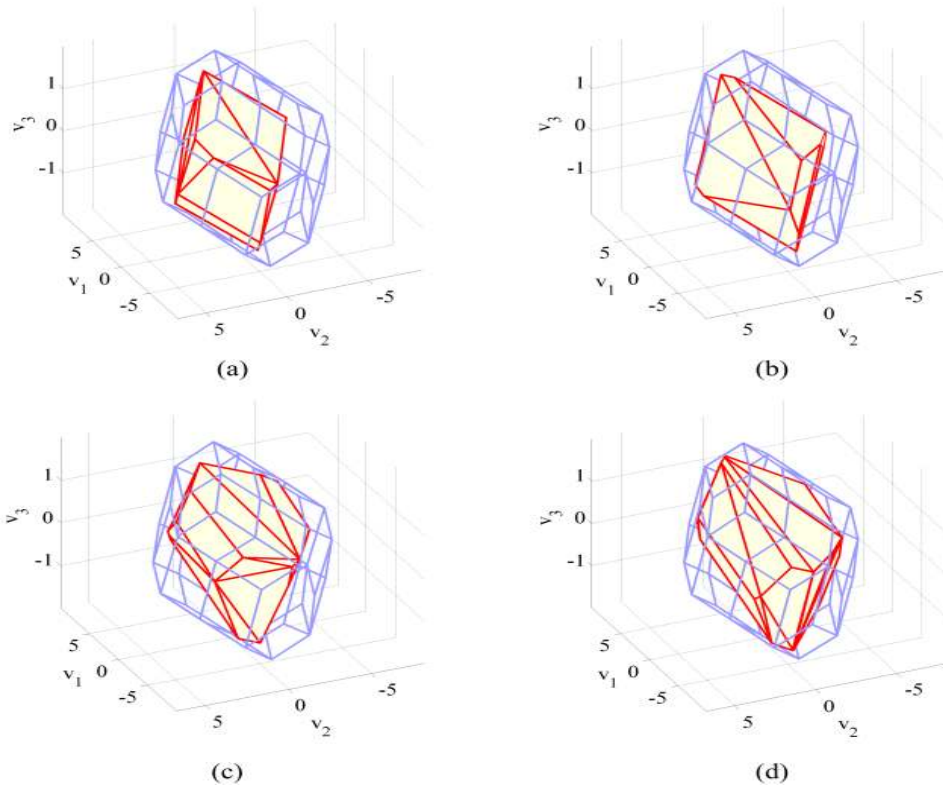


Fig. 2: Increase in volume of subset of moments attainable (a) 30% (b) 34% (c) 40% (d) 46%

cascaded generalized inverse method is simple and computationally efficient. The GA optimized pseudo-inverse solution efficiency in our study is approximately 46%. The subset of attainable moments of WPI is shown in Fig. 2. Where, the allocation efficiency of pseudo-inverse solution is improved from 30% to 46%. The optimized WPI matrix calculated using (17) is:

$$P = \begin{bmatrix} 0.0164 & 0.2209 & -0.2232 \\ -0.0154 & 0.2126 & 0.2430 \\ -0.0539 & -0.0574 & -0.0807 \\ -0.0489 & -0.0780 & -0.0940 \\ 0.0473 & -0.0817 & 0.1129 \\ 0.0578 & -0.0542 & 0.0740 \\ 0.0312 & -0.0026 & -0.4277 \end{bmatrix}$$

OPTIMIZATION USING GA

Genetic algorithms are widely used in optimization of engineering systems e.g., (Mohammadi and Akbar Nasab, 2011; Farhani *et al.*, 2011). For an appropriate design of closed loop flight control system, where we want to achieve the desired response output vector $Y_d = [Y_{1d} \dots Y_{id}]^T$, whereas the actual response of the system, is $Y = [y_1 \dots y_i]^T$. So, it is necessary to use some intelligent optimization technique for closed loop

system performance improvement by varying output Y as close to desired output Y_d , through closed loop gain tuning and poles placement (Khan *et al.*, 2011, 2012).

The optimization problem is to minimize the difference between desired output vector Y_d and real output Y . Considering the 2-norm of the difference vector, which sometimes referred to as minimum norm solution, we can relate the optimal index with the desired vector (Y_d) as described:

$$\min \|Y_d - Y\|_2 \quad (18)$$

In modular flight control, one of the methods for improving control allocation efficiency for over-actuated systems is to maximize the volume of attainable moment subset of allocation scheme (II) as compare to the volume of Attainable Moment Subset (AMS) Φ through optimization. Here, by searching the best possible generalized inverse solution improve the efficiency of weighted pseudo-inverse based CGI allocator. Optimization objective for control allocator also involves finding a vector of control variable in the AMS that produce moment of same magnitude and direction as that of desired moment.

Genetic algorithm is one of the most widely employed stochastic search and optimization technique

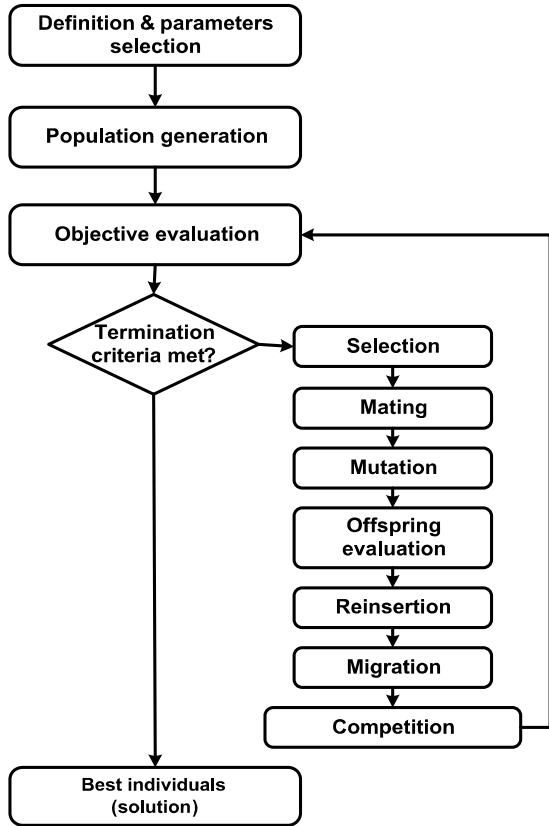


Fig. 3: Flow chart of multi-population GA (modified from (Haupt and Haupt, 2004))

for evolutionary computation which is a developing area of artificial intelligence. The GA starts with randomly initialized population of chromosomes and evolves towards the objective by utilizing evolution operators occurring in nature. The GA has been used in search, machine learning and optimal control (Goldberg, 1989). Several advantages of GA includes simultaneous searching, large number of variables handling and providing multiple optimum solutions where traditional optimization techniques fail. In this study we utilize the GA to find best weighting matrices for closed loop poles placement.

The GA was introduced and developed by John Holland and proved by one of his students, David Goldberg, as a strong method for optimizing large

complex control systems. GA is a searching technique based on the process of natural genetics, selection, recombination and mutation. GA operates on the population of chromosomes based on the principle of fitness to produce best possible solution and selects chromosomes for crossover and mutation. After definition of fitness function and selection of GA parameters, the algorithm proceeds as shown in Fig. 3.

1. Select an initial, random population of chromosomes (elements of diagonal weighting matrices) of specified size.
2. Evaluate these chromosomes for Continuous Algebraic Riccati using (15) for feedback gain L .
3. Simulation is performed with fittest individual set of the population. If the termination criterion is met, then go to 9.
4. Reproduce next generation using probabilistic method.
5. Implement crossover operation on reproduced chromosomes.
6. Perform mutation operation with offspring evaluation.
7. Execute reinsertion and migration.
8. Repeat step 2 until best weighting matrices and control gain L matrix is achieved.
9. End.

This intelligent and systematic approach of determining optimized Q and R matrices (Fig. 4) has great precision and advantage over the time-consuming trial-error approach. Through this method we can achieve best possible closed loop response by placing poles of the system. The new gain matrix L increases the system stability to perturbations. Also, by employing the CGI allocator to efficiently distribute the input command vector among the control surfaces even in the case of partial or total failure of some actuating device improve overall system performance to faults/failures and make the whole system active fault tolerant. In this study, we have used the Control System and GA toolboxes with modifications.

The optimization objective function for control allocation efficiency improvement is given as:

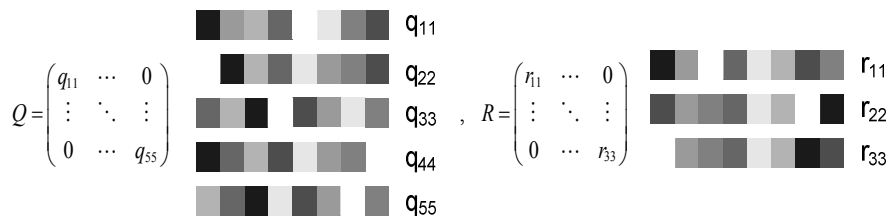


Fig. 4: Chromosomal representation of Q and R matrices

$$\eta = \frac{V_{\psi}}{V_{\Phi}} \times 100 \% \quad (19)$$

where,

- V_{ψ} = The volume of the subset of $\partial(\Omega)$ which maps to $\partial(\Phi)$
- V_{Φ} = The volume of the subset of attainable moments
- η = The ratio between the two volumes which will be maximized through GA.

The details of finding maximum volume Attainable Moment Subset (AMS) can be found in our related previous publications (Shi *et al.*, 2010; Khan *et al.*, 2011, 2012).

ADMIRE FLIGHT CONTROL

For evaluation of our purposed strategy we used generic aircraft model (ADMIRE) (Forssell and Nilson, 2005), developed by the Swedish Defense Research Agency (FOI). It is a nonlinear, 6-DOF simulation model of a delta-canard configured, small single seated and single engine fighter aircraft with twelve control actuators. ADMIRE is freely available environment for flight control research, developed from Generic Aero-data Model (GAM). ADMIRE flight envelop is valid up to Mach 1.2 and altitudes below 6 km with additional varying constraints on angle-of-attack, angle of sideslip and control surface due to aero-data.

In Fig. 5, ADMIRE configuration with control effectors are: left and right canards (δ_{lc} and δ_{rc}), left inner and outer elevons (δ_{lie} and δ_{loe}), right inner and outer elevons (δ_{rie} and δ_{roe}), leading edge flaps (δ_{le}) and rudder (δ_r), as well as leading gear (δ_{ldg}), horizontal and vertical thrust vectoring (δ_{th} and δ_{tv}) and air brake (δ_{ab}). Both canards and elevons are used for pitch control, rudder is used for yaw control and rudder with elevons is used for roll control. There is a

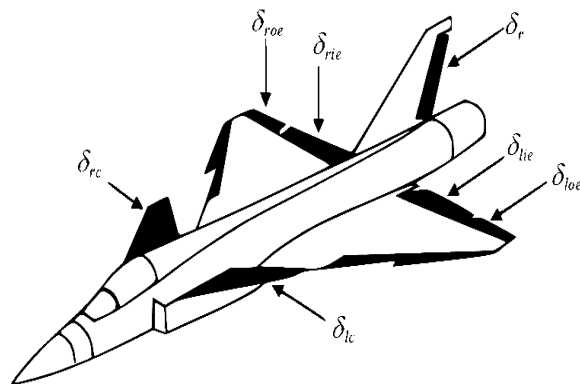


Fig. 5: ADMIRE aircraft configuration

coupling between rudder and elevons which effects yaw and roll control.

SIMULATION RESULTS

The simulation presented here shows a linear model trimmed at low speed flight condition of Mach 0.3 at an altitude of 3000 m. The state vector $x = [\alpha \beta p q r]^T$ consist of α angle of attack (rad), β angle of sideslip (rad), p roll rate (rad/sec), q pitch rate (rad/sec) and r yaw rate (rad/sec). The controlled output vector is $y = [\alpha \beta p]^T$. For controller design with control allocation strategy, we will use only seven control surfaces $[\delta_{lc}, \delta_{rc}, \delta_{roe}, \delta_{rie}, \delta_{lie}, \delta_{loe}, \delta_r]^T$.

Following are the respective linearized model matrices at trim point:

$$A = \begin{bmatrix} -0.6973 & 0.0090 & 0 & 0.9765 & 0 \\ 0 & -0.1680 & 0.1303 & 0 & -0.9816 \\ 0 & -14.595 & -1.3423 & 0 & 0.5268 \\ 3.7422 & -0.0053 & 0 & -0.7045 & 0 \\ 0 & 0.8291 & -0.0889 & 0 & -0.2992 \end{bmatrix}$$

$$B_u = \begin{bmatrix} 0.0003 & 0.0003 & -0.0508 \\ -0.0060 & 0.0060 & 0.0032 \dots \\ 0.7984 & -0.7984 & -4.5787 \\ 1.3841 & 1.3841 & -1.0906 \\ 0.3970 & 0.3970 & -0.2014 \\ -0.0813 & -0.0813 & -0.0508 & 0.0004 \\ 0.0135 & -0.0135 & -0.0032 & 0.0395 \\ -3.9413 & 3.9413 & 4.5787 & 2.6919 \\ -1.7433 & -1.7433 & -1.0906 & 0.0046 \\ -0.4256 & 0.4256 & 0.2014 & -1.6265 \end{bmatrix}$$

In this example, the actuators position limits are considered and the approximate model with allocator can be given, where:

$$B_u = B_v B_e$$

where,

$$B_v = [0_{2 \times 3} \quad I_{3 \times 3}]^T$$

$$B_e = \begin{bmatrix} 0.7984 & -0.7984 & -4.5787 \\ 1.3841 & 1.3841 & -1.0906 \dots \\ -0.3970 & 0.3970 & -0.2014 \\ -3.9413 & 3.9413 & 4.5787 & 2.6919 \\ -1.7433 & -1.7433 & -1.0906 & 0.0046 \\ -0.4256 & 0.4256 & 0.2014 & -1.6265 \end{bmatrix}$$

The control limits used in the simulation were as follows:

$$\bar{u} = \{25 \ 25 \ 30 \ 30 \ 30 \ 30 \ 30\}(\text{deg})$$

$$\underline{u} = \{-55 \ -55 \ -30 \ -30 \ -30 \ -30 \ -30\}(\text{deg})$$

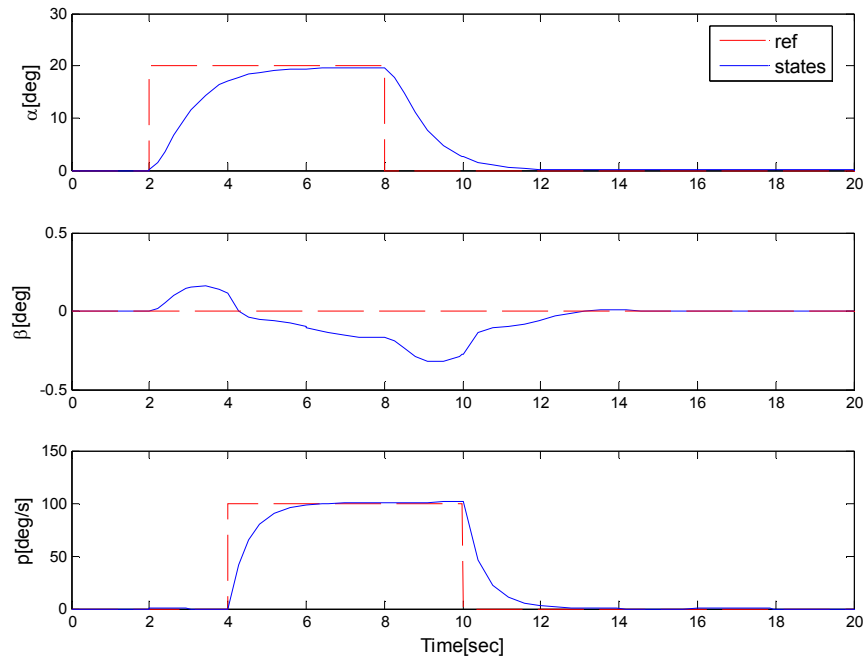


Fig. 6: Aircraft reference trajectories and actual trajectories

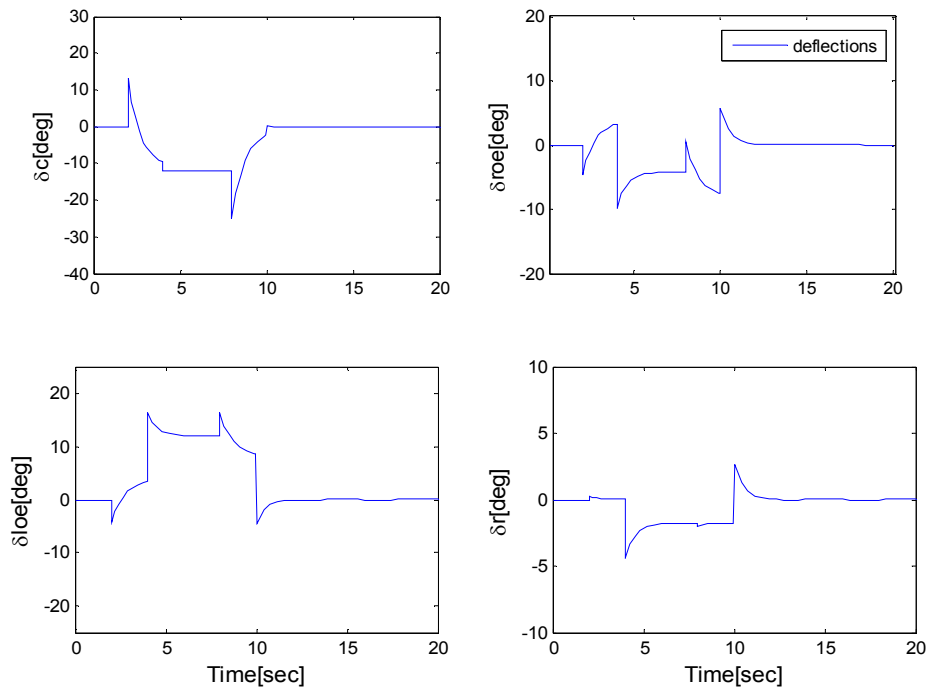


Fig. 7: Aircraft actuators deflection δ

The resulting virtual control for control allocator input $v = B_{\mathcal{M}}$ consist of pure moments in roll, pitch and yaw produced by the control effectors (Harkegard, 2003). The normal flight trajectories with actuator movements in ‘ α -roll’ manoeuvre are shown in Fig. 6 and 7, where a step demand of magnitude 20 deg is

applied to α during 2-8 sec and a step of 100 deg for p is applied during 4-10 sec.

The presence of control allocation module in the suggested modular approach, actuator faults can easily be accommodated with modified control effectiveness matrix instead of modifying the base-line controller as

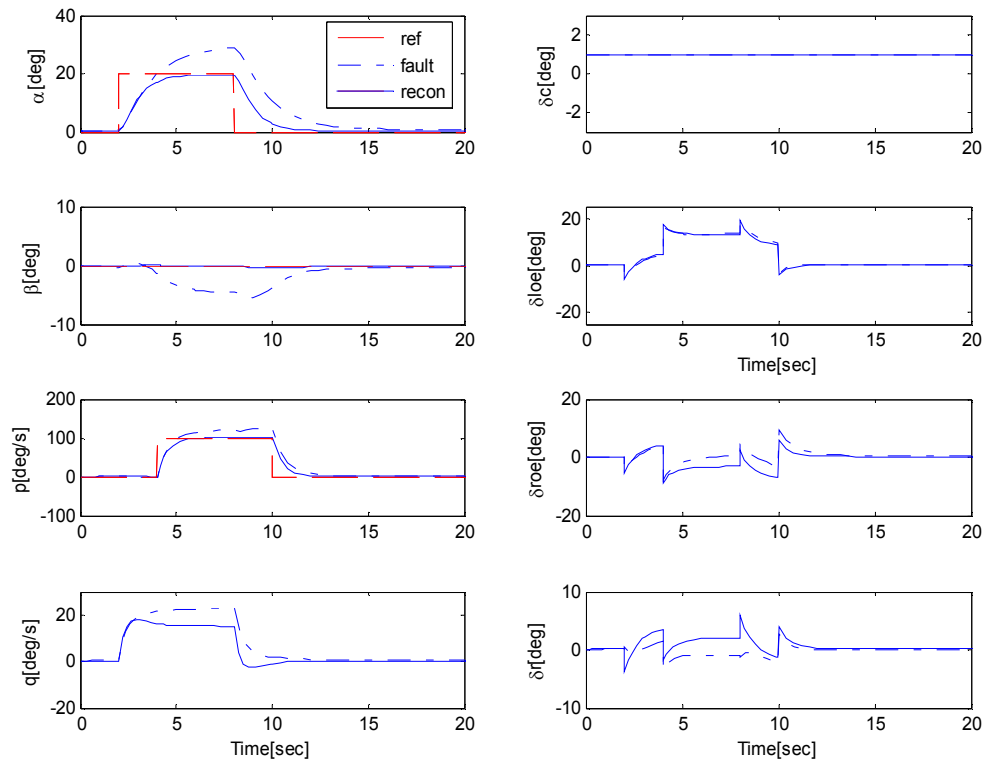


Fig. 8: Jammed canard (at 1 deg) fault: States and actuator deflection

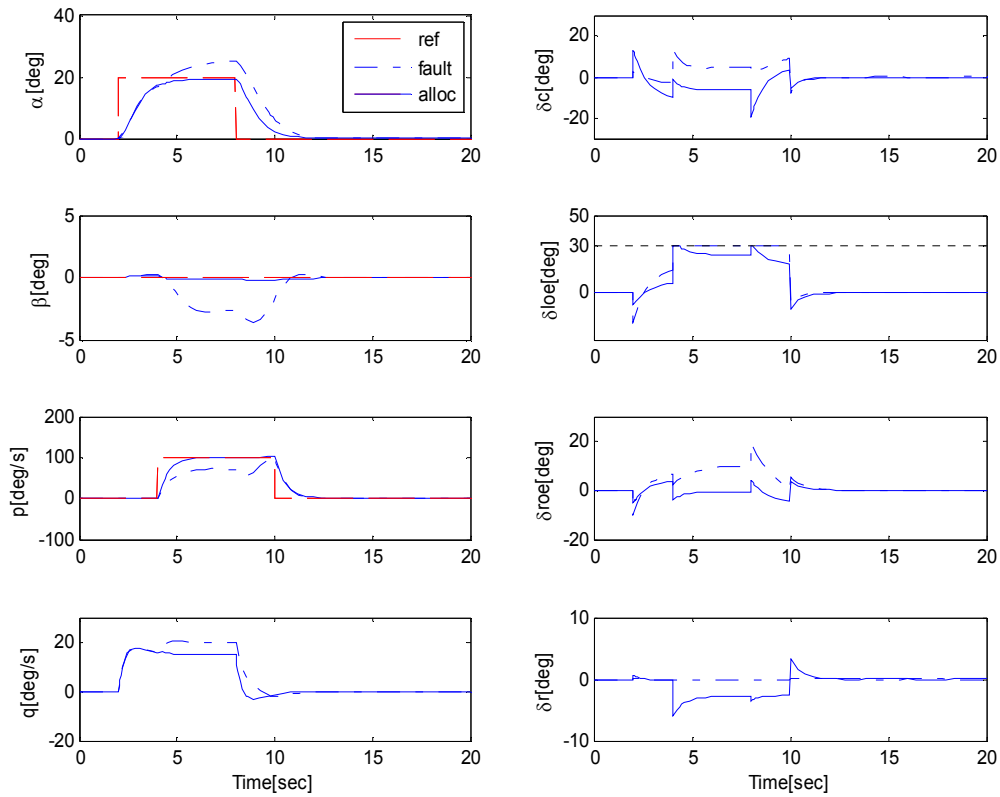


Fig. 9: Partial loss of control (left elevon) fault: States and actuator deflection

shown in Fig. 8 and 9. Because of faults in either canards or elevons cause an overshoot in pitch variables, Angle of Attack (AoA) and pitch rate. But due to the availability of redundancy in ADMIRE aircraft, pitch moment can be control by either the canard or elevons (left and right). We can see that in the event of faults or failures, healthy elevons can replace the damaged canard by redistribution of control effort to elevons as much as possible to achieve the desired pitch moment (Fig. 8) and left elevon saturation can be compensated by redistributing the lost control effect to the right elevons and canards (Fig. 9).

CONCLUSION

This study has presented a GA based optimized modular control design for reconfigurable flight control. The effectiveness level of control surfaces is used in control allocation to redistribute the control commands to the remaining active actuators in fault and failure situations. This strategy guarantees the optimal performance of controller with efficient distribution of the desired efforts between redundant control effectors. GA based cascaded weighted pseudo-inverse method is presented with ADMIRE benchmark model for actuator faults or failures handling in reconfigurable control.

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