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**Research Article**

# An Economic Reliability Efficient Group Acceptance Sampling Plans for Family Pareto Distributions

[1]Abdur Razzaque Mughal and [2]Muhammad Ismail
[1]Department of Statistics, Lahore Business School, University of Lahore, Pakistan
[2]Department of Statistics, COMSATS Institute of Information Technology, Lahore, Pakistan

**Abstract:** The present research study deals with an economic reliability efficient group acceptance sampling plan for time truncated tests which are based on the total number of failures assuming that the life time of a product follows the family for Pareto distribution. This research is proposed when a multiple number of products as a group can be observed simultaneously in a tester. The minimum termination time required for a given group size and acceptance number is determined such that the producer and consumer risks are satisfied for specific standard of quality level, while the number of groups and the number of testers are pre-assumed. Comparison studies are made between the proposed plan and the existing plan on the basis of minimum termination time. Two real examples are also discussed.

**Keywords:** Consumer risk, group sampling plan, pareto distribution, producer risk, truncated life test

## INTRODUCTION

Different techniques of life testing exist in Statistical Quality Control to ensure or improve the quality standard of a product. In the new era, to meet the growing advancement, the manufacturers wish to use the high standard quality manufacturing techniques. One of the most important techniques of statistical quality control to ensure the quality of the product is acceptance sampling plan. Acceptance sampling plan is an important scheme to examine the desire quality standard of the product observed in a sample taken from the whole batch or lot and on the basis of this judgment make an inference to accept or reject a submitted lot of the product by a vendor. When the inspection of the product is very costly one may refer to use Economic Reliability Accepting Sampling Plan (ERASP). Economic Reliability Accepting Sampling Plan (ERASP) is minimizing the total inspection also reduces cost, time, energy and labor. It is simply considered in the ordinary sampling plans that only a single product is put in a tester for inspection. Ordinary sampling plans of the various lifetime distributions are discussed by many researchers (Epstein, 1954; Grimshaw, 1993; Jun *et al*., 2006; Kantam *et al*., 2006; Baklizi, 2003; Tsai and Wu, 2006; Balakrishnan *et al*., 2007; Rosaiah *et al*., 2008; Schilling and Neubauer, 2008; Srinivasa Rao, 2011; Mughal *et al*., 2011b). If the manufacturer desires to test more than one product at a one time because experimental cost, time, energy and labor can be saved by testing these products simultaneously, the sampling plan follows this type of testing will be called an economic reliability group acceptance sampling plan basis on the truncated life test. According to Mughal *et al*. (2010a) a sampling plan with this kind of tester consists of the sample size is equivalent to the number of testers.

Aslam *et al*. (2010a, 2010b) proposed the group acceptance sampling plan on the truncated life test when the lifetime of a product follows the Pareto distribution of the second kind and generalized pare to distribution, respectively. The consumer and producer play a vital role in group acceptance sampling plan to lead the desired quality standards of the product. The chance of accepting the bad lot is called the consumer risk and the chance of rejecting the good lot is called the producer risk. Mughal *et al*. (2010b) develop the economic reliability group acceptance sampling plan for truncated life test having Weibull distribution by satisfying both the risks. The main objective of this study is to propose the efficient economic reliability group acceptance sampling plan for the truncated life test when the lifetime of a product follows the family of Pareto distribution.

## THE ECONOMIC RELIABILITY EFFICIENT GROUP ACCEPTANCE SAMPLING PLAN

Pareto (1897) suggested the Pareto distribution as a model for income and Baklizi (2003) developed an ordinary acceptance sampling plan for Pareto distribution of the 2$^{nd}$ kind. Choulakian and Stephens

**Corresponding Author:** Abdur Razzaque Mughal, Department of Statistics, Lahore Business School, University of Lahore, Pakistan

(2001), Zhang (2007) and Abd Elfattah *et al*. (2007) have worked on generalized Pareto distribution. Recently, Mughal and Aslam (2011a) developed an efficient Group Acceptance Sampling Plan (GASP) for family Pareto distribution. The Cumulative Distribution Function (CDF) and the Probability Density Function (PDF) of the Pareto distribution of the 2nd kind are as follow:

$$F(t;\sigma,\lambda)=1-\left(1+\frac{t}{\sigma}\right)^{-\lambda} \quad t>0,\sigma>0,\lambda>0. \tag{1}$$

$$f(t;\sigma,\lambda)=\frac{\lambda}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\lambda+1)} \quad t>0,\sigma>0,\lambda>0 \tag{2}$$

'λ' and 'σ' represents the shape and scale parameters, respectively. The mean of this distribution is:

$$\mu=\frac{\sigma}{\lambda-1}, \quad \lambda>1 \tag{3}$$

It is note that mean function is valid when the values of the shape parameter is higher than 1. The Cumulative Distribution Function (CDF), the Probability Density Function (PDF), survival S (t) and hazard function H (t) of the generalized Pareto distribution are:

$$F\left(t;\alpha,\beta,\lambda,\delta\right)=1-\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha} \tag{4}$$

$$f(t;\alpha,\beta,\lambda,\delta)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-(\alpha+1)} \tag{5}$$

$$S(t)=\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha} \tag{6}$$

$$H(t)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-1} \tag{7}$$

where, $\lambda<t<\infty$, $\beta>0$, $\alpha>0$, $\delta>0$, λ is the location parameter, β is the scale parameter and (α, δ) are shape parameters respectively. The mean and variance of generalized Pareto distribution are respectively:

$$\mu=\beta\frac{\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\Gamma(\alpha)}+\lambda \tag{8}$$

$$\sigma^2=\beta^2\left[\frac{\Gamma\left(1+\frac{2}{\delta}\right)\Gamma\left(\alpha-\frac{2}{\delta}\right)}{\Gamma(\alpha)}-\left(\frac{\Gamma\left(1+\frac{1}{\delta}\right)\Gamma\left(\alpha-\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right)^2\right] \tag{9}$$

The above defined generalized Pareto distribution (5) can be converted to different distributions such as:

- If δ = 1, (5) reduces to a three-parameter Pareto distribution (10) with PDF:

$$f(t,\alpha,\beta,\lambda)=\frac{\alpha}{\beta}\left[1+\left(\frac{t-\lambda}{\beta}\right)\right]^{-(\alpha+1)} \tag{10}$$

where $\lambda<t<\infty$, $\beta>0$, $\alpha>0$, where λ is the location parameter, β is the scale parameter and α is the shape parameter.

- If $\beta^{\delta}=\theta$, (5) reduces the 4-parameter compound Weibull-gamma distribution (11) with PDF:

$$f(t,\alpha,\theta,\lambda,\delta)=\delta\alpha\frac{(t-\lambda)^{\delta-1}}{\theta}\left[1+\frac{(t-\lambda)^{\delta}}{\theta}\right]^{-(\alpha+1)}$$
$$\lambda<t<\infty,\delta,\theta,\alpha>0 \tag{11}$$

- If $\beta^{\delta}=\theta$ and δ = 2, (5) reduces to 3-parameter compound Rayleigh-gamma distribution (12) with PDF:

$$f(t;\alpha,\theta,\lambda)=2\alpha\frac{(t-\lambda)}{\theta}\left[1+\left(\frac{t-\lambda}{\theta}\right)^{2}\right]^{-(\alpha+1)}$$
$$\lambda<t<\infty,\alpha,\theta>0 \tag{12}$$

- If $\beta^{\delta}=\theta$ and α = 1, (5) reduces to 3-parameter compound Weibull-exponential distribution (13) with PDF:

$$f(t;\alpha,\theta,\lambda,\delta)=\delta\left[1+\frac{(t-\lambda)^{\delta}}{\theta}\right]^{-2}$$
$$\lambda<t<\infty\ \delta,\theta>0 \tag{13}$$

- If $\beta^{\delta}=\theta$ and δ = α = 1, (5) reduces to 2-parameter compound exponential-exponential distribution (14) with PDF:

$$f(t;\theta,\lambda)=\frac{1}{\theta}\left[1+\frac{t-\lambda}{\theta}\right]^{-2},\lambda<t<\infty\ \theta>0 \tag{14}$$

- If β = 1 and λ = 0, (5) reduces the 2-parameter Burr XII distribution (15) with PDF:

$$f(t;\theta,\delta)=\delta\alpha\, t^{\delta-1}\left(1+t^{\delta}\right)^{-(\alpha+1)}$$
$$t>0,\alpha>0,\delta>0. \tag{15}$$

where, δ and α are the shape parameters.

- If δ = 1 and λ = 0, (5) reduces to the 2-parameter Lomax distribution (16) with PDF:

$$f(t;\alpha,\beta)=\frac{\alpha}{\beta}\left[1+\left(\frac{t}{\beta}\right)\right]^{-(\alpha+1)},t>0,\beta,\alpha>0 \quad (16)$$

- If β = δ = 1 and λ = 0, (5) reduces to beta type II distribution (17) with PDF:

$$f(t;\alpha,)=\alpha(1+t)^{-(\alpha+1)},\ t>0,\alpha>0 \quad (17)$$

For more details one may refer to Abd Elfattah *et al*. (2007). In the literature, Mughal *et al*. (2011b) proposed the efficient group acceptance sampling plan and made an inference on the information that a product under testing will be rejected if the number of defective items in each group is greater than or equal to the pre-specified acceptance number, otherwise the lot is accepted. By using this relationship, we developed the following economic reliability efficient group sampling plan based on the family for Pareto distributions. Consider μ denote the true mean life of a product and $\mu_0$ represent the specified mean life. A submitted product by the vendor is considered as good or reliable and accepted for consumer use if satisfies the hypothesis $H_0$: $\mu \geq \mu_0$ otherwise the same is rejected. In acceptance sampling technique, this hypothesis is inspected based on the number of failure occurs from a sample in an experimental truncation time which is denoting by $t_0$. The submitted lot is accepted if the number of failures less than the acceptance number c. The hypothesis $H_0$: $\mu \geq \mu_0$ is leading to a reliable inference at comprehensive quality level of both risks to accept the submitted lot. The proposed economic reliability efficient group acceptance sampling plan can be stated as:

**Step 1:** Find the minimum truncation or termination time and allocate r products (or testers) to each pre-specified g group so that the sample size is n = r×g.
**Step 2:** Specify the acceptance number *c* for each group.
**Step 3:** Terminate the experiment and reject the submitted lot if more than c failures are found during the experimental truncation time.

It is interesting to note that the proposed plan is the generalization of the ordinary acceptance sampling plan. If *r* = 1, this plan reduces the ordinary single sampling plan. If the total number of failures occurred from every group is less than or equal to the pre-assumed acceptance number c, then the lot acceptance probability of the proposed plan is written as follow (18):

$$L(p)=\sum_{i=0}^{c}\binom{rg}{i}p^{i}(1-p)^{rg-i} \quad (18)$$

where, p is the probability or chance that a product in any group fails during the truncation (termination) time $t_0$. It would be convenient to write the termination time $t_0$ as a multiple of the pre-assumed life $\mu_0$ and test termination ratio a. That is, we will suppose $t_0 = a\mu_0$ for a constant a. So, the lot acceptance probability of Pareto distribution of the second kind (19) and generalized Pareto distribution (20) can be written as:

$$p=F(t;\sigma,\lambda)=1-\left[1+\frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \quad (19)$$

$$p=F(t)=1-\left[1+\left(\frac{a\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\left(\frac{\mu}{\mu_0}\right)\Gamma(\alpha)}\right)^{\delta}\right]^{-\alpha} \quad (20)$$

The probability of rejecting a good product is called the producer risk denoting by α* and the probability of accepting a bad product is called the consumer risk denoting by β respectively. Economic reliability efficient group acceptance sampling plan under the true mean life to specified life $\mu/\mu_0$ is developed to find the minimum truncation or termination time when the following inequality (21) is satisfied:

$$L(p)=\left[\sum_{i=0}^{c}\binom{rg}{i}p^{i}(1-p)^{rg-i}\right]\geq 1-\alpha^{*} \quad (21)$$

## DESCRIPTION OF TABLES AND EXAMPLES

The constructed Table 1 to 3 represents the minimum termination time*,* the acceptance number c, number of groups g and the number of testers r required for the proposed economic reliability efficient group acceptance sampling plan according to various values of the producers risk (α* = 0.25, 0.10, 0.05, 0.01) when the true mean life equals to the specified mean life. We consider different values of the λ, α and δ to find the minimum termination time that can be obtained, if needed. These tables are constructed by using different values of the shape parameters of Pareto distribution of the second kind and generalized Pareto distribution for example (λ = 2, 3), (α, δ = 2) respectively to find the minimum termination time by using the proposed plan.

Consider that the lifetime of a product under consideration is known to follow a Pareto distribution of the second kind with the shape parameter (λ = 2). Suppose that it is desired to develop a proposed plan to assure that the mean life is greater than 5000 h through the experiment to be completed by 5000 h using testers equipped with five products each in three groups. It is assumed that the producer risk is 5% then the designed parameters of the proposed plan are (r, g, c, α*) =

Table 1: Test termination time for the Pareto distribution of the 2nd kind for λ = 2

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.0746 | 0.0492 | 0.0367 | 0.0292 | 0.0243 | 0.0208 | 0.0182 |
| 1 | 2 | 0.1494 | 0.0919 | 0.0664 | 0.0520 | 0.0428 | 0.0363 | 0.0316 |
| 2 | 3 | 0.1926 | 0.1149 | 0.0821 | 0.0638 | 0.0522 | 0.0442 | 0.0383 |
| 3 | 4 | 0.2209 | 0.1295 | 0.0918 | 0.0712 | 0.0581 | 0.0491 | 0.0425 |
| 4 | 5 | 0.2409 | 0.1398 | 0.0986 | 0.0763 | 0.0622 | 0.0525 | 0.0454 |
| 5 | 6 | 0.2562 | 0.1474 | 0.1037 | 0.0800 | 0.0652 | 0.0550 | 0.0476 |
| 6 | 7 | 0.2682 | 0.1534 | 0.1076 | 0.0830 | 0.0675 | 0.0569 | 0.0492 |
| 7 | 8 | 0.2780 | 0.1582 | 0.1108 | 0.0853 | 0.0694 | 0.0585 | 0.0506 |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.0267 | 0.0178 | 0.0133 | 0.0106 | 0.0089 | 0.0076 | 0.0067 |
| 1 | 2 | 0.0800 | 0.0498 | 0.0362 | 0.0285 | 0.0235 | 0.0200 | 0.0174 |
| 2 | 3 | 0.1187 | 0.0718 | 0.0516 | 0.0403 | 0.0330 | 0.0280 | 0.0243 |
| 3 | 4 | 0.1468 | 0.0874 | 0.0623 | 0.0484 | 0.0396 | 0.0336 | 0.0291 |
| 4 | 5 | 0.1683 | 0.0990 | 0.0709 | 0.0545 | 0.0445 | 0.0376 | 0.0236 |
| 5 | 6 | 0.1853 | 0.1081 | 0.0765 | 0.0592 | 0.0483 | 0.0408 | 0.0353 |
| 6 | 7 | 0.1992 | 0.1154 | 0.0815 | 0.0630 | 0.0513 | 0.0433 | 0.0375 |
| 7 | 8 | 0.2109 | 0.1215 | 0.0856 | 0.0661 | 0.0539 | 0.0455 | 0.0393 |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0129 | 0.0086 | 0.0065 | 0.0052 | 0.0043 | 0.0037 | 0.0033 |
| 1 | 2 | 0.0527 | 0.0330 | 0.0241 | 0.0189 | 0.0156 | 0.0133 | 0.0116 |
| 2 | 3 | 0.0867 | 0.0528 | 0.0380 | 0.0297 | 0.0244 | 0.0207 | 0.0180 |
| 3 | 4 | 0.1131 | 0.0678 | 0.0485 | 0.0377 | 0.0309 | 0.0262 | 0.0227 |
| 4 | 5 | 0.1341 | 0.0794 | 0.0565 | 0.0439 | 0.0359 | 0.0304 | 0.0263 |
| 5 | 6 | 0.1511 | 0.0887 | 0.0630 | 0.0488 | 0.0399 | 0.0337 | 0.0292 |
| 6 | 7 | 0.1654 | 0.0965 | 0.0683 | 0.0529 | 0.0431 | 0.0364 | 0.0316 |
| 7 | 8 | 0.1774 | 0.1030 | 0.0727 | 0.0562 | 0.0459 | 0.0387 | 0.0335 |
| α* = 0.01 | | | | | | | | |
| 0 | 1 | 0.0026 | 0.0017 | 0.0013 | 0.0011 | 0.0009 | 0.0008 | 0.0007 |
| 1 | 2 | 0.0217 | 0.0137 | 0.0100 | 0.0079 | 0.0065 | 0.0056 | 0.0048 |
| 2 | 3 | 0.0452 | 0.0278 | 0.0201 | 0.0157 | 0.0129 | 0.0110 | 0.0096 |
| 3 | 4 | 0.0665 | 0.0402 | 0.0289 | 0.0226 | 0.0185 | 0.0157 | 0.0136 |
| 4 | 5 | 0.0850 | 0.0508 | 0.0363 | 0.0283 | 0.0443 | 0.0196 | 0.0170 |
| 5 | 6 | 0.1006 | 0.0597 | 0.0426 | 0.0331 | 0.0271 | 0.0229 | 0.0199 |
| 6 | 7 | 0.1143 | 0.0674 | 0.0479 | 0.0372 | 0.0304 | 0.0257 | 0.0223 |
| 7 | 8 | 0.1262 | 0.0741 | 0.0525 | 0.0407 | 0.0333 | 0.0281 | 0.0244 |

Table 2: Test termination time for the Pareto distribution of the 2nd kind for λ = 3

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.0983 | 0.0650 | 0.0486 | 0.0388 | 0.0323 | 0.0276 | 0.0242 |
| 1 | 2 | 0.1945 | 0.1207 | 0.0876 | 0.0688 | 0.0566 | 0.0481 | 0.0418 |
| 2 | 3 | 0.2492 | 0.1504 | 0.1079 | 0.0842 | 0.0691 | 0.0585 | 0.0508 |
| 3 | 4 | 0.2846 | 0.1692 | 0.1206 | 0.0938 | 0.0767 | 0.0649 | 0.0563 |
| 4 | 5 | 0.3097 | 0.1822 | 0.1294 | 0.1004 | 0.0820 | 0.0694 | 0.0601 |
| 5 | 6 | 0.3285 | 0.1920 | 0.1360 | 0.1053 | 0.0860 | 0.0726 | 0.0629 |
| 6 | 7 | 0.3433 | 0.1996 | 0.1410 | 0.1091 | 0.0890 | 0.0752 | 0.0651 |
| 7 | 8 | 0.3553 | 0.2057 | 0.1451 | 0.1122 | 0.0915 | 0.0773 | 0.0668 |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.0355 | 0.0236 | 0.0177 | 0.0141 | 0.0118 | 0.0101 | 0.0088 |
| 1 | 2 | 0.1052 | 0.0659 | 0.0480 | 0.0378 | 0.0311 | 0.0265 | 0.0231 |
| 2 | 3 | 0.1553 | 0.0947 | 0.0682 | 0.0533 | 0.0438 | 0.0372 | 0.0323 |
| 3 | 4 | 0.1913 | 0.1148 | 0.0822 | 0.0641 | 0.0525 | 0.0445 | 0.0386 |
| 4 | 5 | 0.2185 | 0.1299 | 0.0926 | 0.0720 | 0.0589 | 0.0499 | 0.0432 |
| 5 | 6 | 0.2400 | 0.1416 | 0.1007 | 0.0781 | 0.0639 | 0.0540 | 0.0468 |
| 6 | 7 | 0.2574 | 0.1511 | 0.1072 | 0.0831 | 0.0679 | 0.0574 | 0.0497 |
| 7 | 8 | 0.2720 | 0.1589 | 0.1125 | 0.0872 | 0.0712 | 0.0601 | 0.0521 |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0172 | 0.0115 | 0.0086 | 0.0069 | 0.0058 | 0.0049 | 0.0043 |
| 1 | 2 | 0.0697 | 0.0438 | 0.0320 | 0.0252 | 0.0208 | 0.0177 | 0.0154 |
| 2 | 3 | 0.1140 | 0.0698 | 0.0504 | 0.0394 | 0.0324 | 0.0275 | 0.0239 |
| 3 | 4 | 0.1481 | 0.0893 | 0.0641 | 0.0500 | 0.0410 | 0.0347 | 0.0302 |
| 4 | 5 | 0.1750 | 0.1045 | 0.0747 | 0.0581 | 0.0476 | 0.0403 | 0.0349 |
| 5 | 6 | 0.1967 | 0.1166 | 0.0831 | 0.0646 | 0.0528 | 0.0447 | 0.0387 |
| 6 | 7 | 0.2147 | 0.1266 | 0.0900 | 0.0699 | 0.0571 | 0.0483 | 0.0418 |
| 7 | 8 | 0.2300 | 0.1350 | 0.0958 | 0.0743 | 0.0607 | 0.0513 | 0.0444 |

Table 2: (Continue)

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.01 | | | | | | | | |
| 0 | 1 | 0.0034 | 0.0023 | 0.0017 | 0.0014 | 0.0012 | 0.0010 | 0.0009 |
| 1 | 2 | 0.0288 | 0.0182 | 0.0133 | 0.0105 | 0.0087 | 0.0074 | 0.0064 |
| 2 | 3 | 0.0600 | 0.0369 | 0.0267 | 0.0209 | 0.0172 | 0.0146 | 0.0127 |
| 3 | 4 | 0.0877 | 0.0533 | 0.0383 | 0.0300 | 0.0246 | 0.0209 | 0.0181 |
| 4 | 5 | 0.1116 | 0.0671 | 0.0481 | 0.0375 | 0.0307 | 0.0261 | 0.0226 |
| 5 | 6 | 0.1320 | 0.0789 | 0.0563 | 0.0439 | 0.0359 | 0.0304 | 0.0264 |
| 6 | 7 | 0.1496 | 0.0889 | 0.0634 | 0.0493 | 0.0403 | 0.0341 | 0.0296 |
| 7 | 8 | 0.1649 | 0.0976 | 0.0694 | 0.0539 | 0.0441 | 0.0373 | 0.0323 |

Table 3: Test termination time for the generalized Pareto distribution for α = 2, δ = 2

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.2307 | 0.1873 | 0.1617 | 0.1444 | 0.1316 | 0.1218 | 0.1138 |
| 1 | 2 | 0. 3265 | 0.2561 | 0.2177 | 0.1927 | 0.1747 | 0.1609 | 0.1500 |
| 2 | 3 | 0.3708 | 0.2864 | 0.2420 | 0.2134 | 0.1931 | 0.1776 | 0.1654 |
| 3 | 4 | 0.3971 | 0.3041 | 0.2560 | 0.2253 | 0.2036 | 0.1872 | 0.1742 |
| 4 | 5 | 0.4148 | 0.3158 | 0.2653 | 0.2333 | 0.2106 | 0.1935 | 0.1800 |
| 5 | 6 | 0.4277 | 0.3243 | 0.2720 | 0.2390 | 0.2157 | 0.1981 | 0.1842 |
| 6 | 7 | 0.4375 | 0.3308 | 0.2772 | 0.2433 | 0.2195 | 0.2015 | 0.1874 |
| 7 | 8 | 0.4454 | 0.3360 | 0.2812 | 0.2468 | 0.2226 | 0.2043 | 0.1899 |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.1380 | 0.1125 | 0.0973 | 0.0870 | 0.0794 | 0.0735 | 0.0687 |
| 1 | 2 | 0.2389 | 0.1885 | 0.1607 | 0.1425 | 0.1293 | 0.1192 | 0.1112 |
| 2 | 3 | 0.2910 | 0.2264 | 0.1918 | 0.1695 | 0.1535 | 0.1413 | 0.1316 |
| 3 | 4 | 0.3237 | 0.2497 | 0.2108 | 0.1859 | 0.1682 | 0.1547 | 0.1440 |
| 4 | 5 | 0.3466 | 0.2658 | 0.2239 | 0.1972 | 0.1782 | 0.1638 | 0.1525 |
| 5 | 6 | 0.3636 | 0.2777 | 0.2336 | 0.2055 | 0.1856 | 0.1706 | 0.1587 |
| 6 | 7 | 0.3770 | 0.2870 | 0.2411 | 0.2120 | 0.1914 | 0.1758 | 0.1636 |
| 7 | 8 | 0.3879 | 0.2945 | 0.2471 | 0.2172 | 0.1960 | 0.1801 | 0.1675 |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0960 | 0.0783 | 0.0678 | 0.0606 | 0.0553 | 0.0512 | 0.0479 |
| 1 | 2 | 0.1939 | 0.1534 | 0.1310 | 0.1162 | 0.1055 | 0.0973 | 0.0908 |
| 2 | 3 | 0.2487 | 0.1941 | 0.1647 | 0.1456 | 0.1319 | 0.1215 | 0.1132 |
| 3 | 4 | 0.2841 | 0.2199 | 0.1859 | 0.1641 | 0.1485 | 0.1366 | 0.1272 |
| 4 | 5 | 0.3093 | 0.2380 | 0.2008 | 0.1770 | 0.1600 | 0.1472 | 0.1370 |
| 5 | 6 | 0.3284 | 0.2517 | 0.2120 | 0.1866 | 0.1687 | 0.1551 | 0.1443 |
| 6 | 7 | 0.3435 | 0.2624 | 0.2207 | 0.1942 | 0.1754 | 0.1612 | 0.1500 |
| 7 | 8 | 0.3558 | 0.2711 | 0.2278 | 0.2003 | 0.1809 | 0.1662 | 0.1546 |
| α* = 0.01 | | | | | | | | |
| 0 | 1 | 0.0423 | 0.0346 | 0.0299 | 0.0268 | 0.0244 | 0.0226 | 0.0212 |
| 1 | 2 | 0.1243 | 0.0986 | 0.0843 | 0.0749 | 0.0680 | 0.0627 | 0.0585 |
| 2 | 3 | 0.1796 | 0.1408 | 0.1196 | 0.1059 | 0.0960 | 0.0885 | 0.0824 |
| 3 | 4 | 0.2179 | 0.1694 | 0.1436 | 0.1268 | 0.1148 | 0.1057 | 0.0985 |
| 4 | 5 | 0.2461 | 0.1903 | 0.1609 | 0.1420 | 0.1285 | 0.1182 | 0.1101 |
| 5 | 6 | 0.2680 | 0.2064 | 0.1742 | 0.1536 | 0.1389 | 0.1278 | 0.1189 |
| 6 | 7 | 0.2856 | 0.2193 | 0.1849 | 0.1629 | 0.1472 | 0.1354 | 0.1260 |
| 7 | 8 | 0.3002 | 0.2299 | 0.1936 | 0.1704 | 0.1540 | 0.1416 | 0.1318 |

(5, 2, 0.05). This means that a total of 15 products are needed and that five products will be allocated to each of the three groups. We will accept the lots if no more than two failures occur before 149 (5000×0.0297) h in all groups, otherwise the lot is rejected.

## COMPARATIVE STUDY

In Table 4 to 6 the upper values of cells showing the proposed plan and lowest values presenting the existing plan. In Table 4 to 6, the minimum termination time finding by a proposed plan is minimal when compared to an existing plan. Consider that we want to develop an economic reliability efficient group acceptance sampling plan based on the generalized

Table 4: Comparison of test termination time for the Pareto distribution of the 2nd kind for λ = 2

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.0746 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0919 | | | | | |
| | | | 0.7000 | | | | | |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.0267 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0498 | | | | | |
| | | | 1.2000 | | | | | |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0129 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0330 | | | | | |
| | | | 1.5000 | | | | | |

Table 4: (Continue)

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.01 | | | | | | | | |
| 1 | 2 | | 0.0137 | | | | | |
| | | | 2.0000 | | | | | |
| 2 | 3 | | | 0.0201 | | | | |
| | | | | 0.8000 | | | | |

Table 5: Comparison of test termination time for the Pareto distribution of the 2nd kind for λ = 3

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.0983 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.1207 | | | | | |
| | | | 0.8000 | | | | | |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.0355 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0659 | | | | | |
| | | | 1.2000 | | | | | |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0172 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0438 | | | | | |
| | | | 1.5000 | | | | | |
| 2 | 3 | | | 0.0504 | | | | |
| | | | | 0.7000 | | | | |
| α* = 0.01 | | | | | | | | |
| 1 | 2 | | 0.0182 | | | | | |
| | | | 2.0000 | | | | | |
| 2 | 3 | | | 0.0267 | | | | |
| | | | | 1.0000 | | | | |

Table 6: Comparison of test termination time for the generalized Pareto distribution for α = 2, δ = 2

| c | r/g | 2 g | 3 g | 4 g | 5 g | 6 g | 7 g | 8 g |
|---|---|---|---|---|---|---|---|---|
| α* = 0.25 | | | | | | | | |
| 0 | 1 | 0.2307 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.2561 | | | | | |
| | | | 0.7000 | | | | | |
| α* = 0.10 | | | | | | | | |
| 0 | 1 | 0.1380 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.1885 | | | | | |
| | | | 0.8000 | | | | | |
| α* = 0.05 | | | | | | | | |
| 0 | 1 | 0.0960 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.1534 | | | | | |
| | | | 1.0000 | | | | | |
| α* = 0.01 | | | | | | | | |
| 0 | 1 | 0.0423 | | | | | | |
| | | 2.0000 | | | | | | |
| 1 | 2 | | 0.0986 | | | | | |
| | | | 1.2000 | | | | | |
| 2 | 3 | | | 0.1196 | | | | |
| | | | | 0.7000 | | | | |

Pareto distribution with pre-specified mean ratio is $\mu/\mu_0$ = 10000 h and the value of the shapes parameters are α = 2, δ = 2 when the producers risk is 5%. If c = 0, r = 2 and g = 1, the termination time from the propose approach is 2307 h and from the existing approach it is 20000 h.

## CONCLUSION

In this research study, minimum test truncation or termination time is determined for the specified values of producer's risk considering that the lifetime of the product follows the family Pareto distribution. The test truncation or termination time obtained by proposed plan is less than the exiting plan, so we can conclude that the present plan is more economical in the sense of saving cost, time, energy and labor.

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