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Research Article

Heat Transfer in the Formation

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Abstract: Heat loss from the wellbore fluid depends on the temperature distribution in the formation; since it is necessary to know the formation temperature distribution as a function of radial distance and time for computing the fluid flow temperature in the wellbore. This study solves energy balance equation in the formation by analytical and numerical solutions. Comparison of analytical and numerical solutions results figured out good agreement between those solutions, except early times. This study further showed the temperature changes in small area, only near the wellbore in the formation.

Keywords: Analytical solution, bottomhole temperature, early times, numerical solution, radial temperature distribution, wellbore

INTRODUCTION

Injection and production wells have been used in several industries, especially petroleum and geothermal industries for many decades (Hasan and Kabir, 2002). An appropriate wellbore completion design requires a knowledge temperature profile along the depth of the well, especially during hot fluid flow in the wellbore (e.g., steam injection) (Paterson *et al*., 2008).

The formation is the part that surrounds the wellbore system and acts as a heat source or sinks for the wellbore. The temperature difference between the wellbore fluid and the formation causes a transfer of heat between the fluid and the surrounding earth and the amount of heat transfer from/to the wellbore fluid depends on the temperature distribution in the formation around the well, since it is necessary to know the formation temperature distribution as a function of radial distance and time for computing the fluid flow temperature in the wellbore (Ramey, 1962; Hasan and Kabir, 1991, 2002). This study is going to investigate heat transfer in the formation as a function of radial distance and time by using analytical and numerical solutions to find application criteria for using each solution.

Heat transfer in the formation: An energy balance equation on shown element in Fig. 1 during a small time interval, Δt, can expressed as (Kreith and Bown, 2000):

Rate of heat transfer at (r) -Rate of heat transfer at (r+Δr)+Rate of heat generation inside the element = Rate of change of the energy content with in the element

Or

$$Q_r - Q_{r+\Delta r} + Q_{gen} = \frac{\Delta E_{element}}{\Delta t} \quad (1)$$

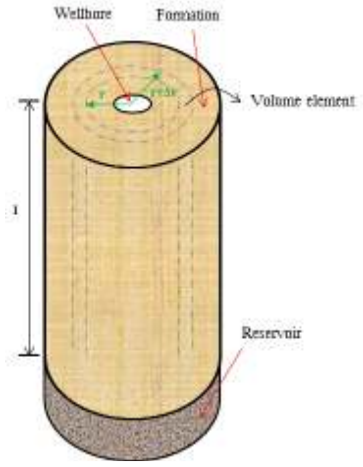

Fig. 1: Schematic representation of the system under study

Rate of heat generation inside the element is zero for shown system in Fig. 1(Farouq Ali, 1981). Rate of change of the energy content within the element can expressed as (Kreith and Bown, 2000):

$$E_{t+\Delta t} - E_t = mC_e\left(T_{t+\Delta t} - T_t\right) = \rho_e A C_e \Delta r\left(T_{t+\Delta t} - T_t\right) \quad (2)$$

where,

$$A = 2\pi l(r + \frac{\Delta r}{2}) \quad (3)$$

The conduction mechanism plays the main role in heat transfer in the formation around the wellbore

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compared to heat transfer by conduction mechanism (Paterson *et al*., 2008; Hasan and Kabir, 1991; Ramey, 1962; Hagoort, 2004). For a symmetrical cylinder, heat diffusion in a three-dimensional may be mathematically treated as a two-dimensional, as in the case of the petroleum wellbore shown in Fig. 1. In addition, if a small increment in the vertical direction of the well is considered, the problem simplifies to one-dimensional heat diffusion because the vertical heat-transfer in the formation can be neglected due to small vertical temperature gradient (Hasan and Kabir, 1991; Paterson *et al*., 2008). In consideration of Fourier's law (Willhite, 1967):

$$Q_r = 2\pi r l k_e (\frac{dT}{dr})|_r \tag{4}$$

and,

$$Q_{r+\Delta r} = 2\pi (r+\Delta r) l k_e (\frac{dT}{dr})|_{r+\Delta r} \tag{5}$$

By substituting (2)-(5) in (1) and rearranging it:

$$2\pi r l k_e \left(\frac{dT}{dr}\right)|_r - 2\pi (r+\Delta r) l k_e (\frac{dT}{dr})|_{r+\Delta r} = \frac{2\pi \rho_e (r+\frac{\Delta r}{2}) l C_e \Delta r \left(T_{t+\Delta t} - T_t\right)}{\Delta t} \tag{6}$$

By rearranging (6):

$$\frac{r\left(\frac{dT}{dr}\right)|_r - (r+\Delta r) l (\frac{dT}{dr})|_{r+\Delta r}}{\Delta r} = \frac{\rho_e (r+\frac{\Delta r}{2}) C_e \left(T_{t+\Delta t} - T_t\right)}{k_e \Delta t} \tag{7}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields:

$$\frac{1}{r}\left(\frac{d}{dr}\left(r\frac{dT_e}{dr}\right)\right) = \frac{1}{\alpha}\frac{dT_e}{dt} \tag{8}$$

where,

$$\alpha = \frac{k_e}{\rho_e c_e} \tag{9}$$

Three conditions are needed for the solution (8). Initial formation temperature is known; in this study it is assumed that at t = 0, the formation temperature profile is linear based on the local geothermal gradient.

$$t = 0 : T = T_e \tag{10}$$

It is also assumed that at the outer boundary of the formation, temperature does not change (Hasan and Kabir, 2002; Hagoort, 2004; Hasan and Kabir, 1994).

$$\lim_{r\to\infty} T = T_e \tag{11}$$

In addition, Fourier's law of heat conduction is used at the wellbore/formation interface, since heat transfer rate is (Hasan and Kabir, 2002; Hagoort, 2004; Hasan and Kabir,1994):

$$Q = 2\pi l k_e \frac{r dT_e}{dr}|_{r=r_w} \tag{12}$$

Through literature, several investigators have presented analytical and numerical solutions for solving (8) to find the heat transfer rate at the wellbore/formation interface.

ANALYTICAL SOLUTION

The solution of (8) is analogous to that used for pressure diffusion defined by Everdingen Van and Hurst (1949). Using the Laplace transformation, we can present an equation for the temperature distribution as a function of distance and time. The general solution is a combination of Bessel functions (Ramey, 1962). Some researchers have tried to find the correlation that best fits the rigorous solution due to complicated application of Bessel functions. The most commonly used correlations in literature were developed by Ramey Jr (1962) and Hasan and Kabir (1991).

Ramey Jr (1962) used a line source solution to transient heat conduction in 1D cylinder proposed by Carslaw and Jaeger (1950); which the wellbore/formation interface temperature becomes log-linear with time at large times (Ramey, 1962; Hasan and Kabir, 1991):

$$f(t) = -\ln\left(\frac{r_w}{2\sqrt{\alpha t}}\right) - 0.290 \tag{13}$$

And the heat transfer rate at the wellbore/formation interface:

$$Q = \frac{2\pi k_e (T_{cemo} - T_e)}{f(t)} \Delta l \tag{14}$$

Hasan and Kabir (1991), stated that Ramey Jr (1962) model outcomes in errors at early times. They considered the wellbore diameter in their solution and solved the resulting (8) with the Laplace transform, following the approach suggested by Everdingen Van and Hurst (1949) for a similar set of equations used for pressure transients. They found the following algebraic expressions for dimensionless temperature, T_D, in terms of dimensionless time, t_D, to represent the solutions quite accurately:

$$If\ t_D \le 1.5 : T_D = 1.1281\sqrt{t_D}\ (1-0.3\sqrt{t_D}) \quad (15)$$

$$If\ t_D > 1.5 : T_D = \left(0.4063 + 0.5ln(t_D)\right)\left(1+\frac{0.6}{t_D}\right) \quad (16)$$

where,

$$t_D = \frac{k_e t}{\rho_e C_e r_w^2} \quad (17)$$

And the heat transfer rate at the wellbore/formation interface:

$$Q = \frac{2\pi k_e (T_{cemo} - T_e)}{T_D}\Delta l \quad (18)$$

NUMERICAL SOLUTION

In this part of the study (8) is solved by numerical method due to analytical solution cannot model heterogeneous layers, variable heat transfer and shut in through injection or production processes (e.g., cyclic injection processes). Some investigators stated the numerical solution as an accurate solution if there is selected suitable timestep and grid size (Farouq Ali, 1981; Cronshaw and John, 1982).

To solve problems that involve (8), the finite difference method can be used. The system is discretized to *N* cells as shown in Fig. 2. By implicit definite difference, approximations for dT/dt, dT/dt, and 1/r (d/dr(rdT/dr)) are:

$$\frac{dT}{dt} \cong \frac{T_i^{n+1} - T_i^n}{\Delta t} \quad (19)$$

$$\frac{dT}{dr} \cong \frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta r} \quad (20)$$

And

$$\frac{1}{r}\left(\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right) \cong \frac{r_{i+\frac{1}{2}}\frac{dT}{dr}\Big|_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}\frac{dT}{dr}\Big|_{i-\frac{1}{2}}}{r_i\Delta r} = \frac{r_{i+\frac{1}{2}}\left(\frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta r}\right) - r_{i-\frac{1}{2}}\left(\frac{T_i^{n+1} - T_{i-1}^{n+1}}{\Delta r}\right)}{r_i\Delta r} \quad (21)$$

With those notations, the implicit definite difference approximations form of (8) is:

$$\frac{T_i^{n+1} - T_i^n}{\alpha\Delta t} = \frac{r_{i+\frac{1}{2}}\left(\frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta r}\right) - r_{i-\frac{1}{2}}\left(\frac{T_i^{n+1} - T_{i-1}^{n+1}}{\Delta r}\right)}{r_i\Delta r} \quad (22)$$

Equation (22) is the basic finite difference equation for the one dimensional diffusivity equation. It is solved for all the new temperature, T^{n+1}, simultaneously. Once these new temperatures are solved, they become old temperatures for the next timestep. In this manner, solutions to (22) are solved in a timestep sequence for as many timesteps as required.

By applying boundary conditions in (22), it can be modified for every cell in Fig. 2 as expressed in the following:

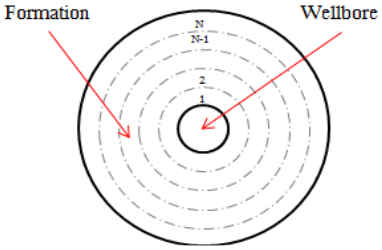

Fig. 2: A schematic of discretized formation (top view)

For cell = 1:

$$\frac{T_1^{n+1} - T_1^n}{\alpha\Delta t} = \frac{r\frac{dT}{dr}\Big|_{1+\frac{1}{2}} - r\frac{dT}{dr}\Big|_{\frac{1}{2}}}{r_1\Delta r} \quad (23)$$

The heat transfer at wellbore/formation interface is expressed by Fourier's law (Kabir and Hasan, 1991):

$$Q = -2\pi r_{\frac{1}{2}} lk \frac{dT}{dr}\Big|_{\frac{1}{2}} \quad (24)$$

By rearranging (24):

$$\frac{dT}{dr}\Big|_{\frac{1}{2}} = \frac{-Q}{2\pi r_{\frac{1}{2}} lk} \quad (25)$$

By substituting (25) in (23):

$$\frac{T_1^{n+1} - T_1^n}{\alpha\Delta t} = \frac{r_{1+\frac{1}{2}}\left(\frac{T_2^{n+1} - T_1^{n+1}}{\Delta r}\right) + \frac{Q}{2\pi lk}}{r_1\Delta r} \quad (26)$$

After rearranging (26):

$$\left(1 + r_{1+\frac{1}{2}}\frac{\alpha\Delta t}{r_1\Delta r^2}\right)T_1^{n+1} - \left(r_{1+\frac{1}{2}}\frac{\alpha\Delta t}{r_1\Delta r^2}\right)T_2^{n+1} = T_1^n + \frac{\alpha\Delta t Q}{2\pi r_1\Delta r lk} \quad (27)$$

For cells = 2 to N-1:

$$\frac{T_i^{n+1} - T_i^n}{\alpha\Delta t} = \frac{r_{i+\frac{1}{2}}\left(\frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta r}\right) - r_{i-\frac{1}{2}}\left(\frac{T_i^{n+1} - T_{i-1}^{n+1}}{\Delta r}\right)}{r_i\Delta r} \quad (28)$$

After rearranging (28):

$$-\left(r_{i-\frac{1}{2}}\frac{\alpha\Delta t}{r_i\Delta r^2}\right)T_{i-1}^{n+1} + \left(1 + (r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}})\frac{\alpha\Delta t}{r_i\Delta r^2}\right)T_i^{n+1} - \left(r_{i+\frac{1}{2}}\frac{\alpha\Delta t}{r_i\Delta r^2}\right)T_{i+1}^{n+1} = T_i^n \quad (29)$$

For cell = N:

$$\frac{T_N^{n+1} - T_N^n}{\alpha\Delta t} = \frac{r_{N+\frac{1}{2}}\left(\frac{T_{N+1}^{n+1} - T_N^{n+1}}{\Delta r}\right) - r_{N-\frac{1}{2}}\left(\frac{T_N^{n+1} - T_{N-1}^{n+1}}{\Delta r}\right)}{r_N\Delta r} \quad (30)$$

By considering the diameter of the outer boundary of the formation to be large, it can be assumed that, temperature does not change there:

$$-\left(r_{N-\frac{1}{2}}\frac{\alpha\Delta t}{r_N\Delta r^2}\right)T_{N-1}^{n+1}+\left(1+(2r_{N+\frac{1}{2}}+r_{N-\frac{1}{2}})\frac{\alpha\Delta t}{r_N\Delta r^2}\right)T_N^{n+1}=T_N^n+2\left(r_{N+\frac{1}{2}}\frac{\alpha\Delta t}{r_N\Delta r^2}\right)T_e \quad (31)$$

This set of Eq. (27), (29) and (31) can be represented by a matrix equation which can simply be written as:

$$A\vec{T}=\vec{d} \quad (32)$$

where,

A = Coefficient matrix

$\vec{T}$ and $\vec{d}$ = Column vectors

Graphically, the matrix may be presented as:

$$\begin{bmatrix} b_1 & c_1 & & & & & \\ a_2 & b_2 & c_2 & & & & \\ & a_3 & b_3 & c_3 & & & \\ & \ddots & \ddots & \ddots & & & \\ & & a_{N-2} & b_{N-2} & c_{N-2} & & \\ & & & a_{N-1} & b_{N-1} & c_{N-1} & \\ & & & & a_N & b_N & \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{N-2} \\ T_{N-1} \\ T_N \end{bmatrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N-2} \\ d_{N-1} \\ d_N \end{bmatrix} \quad (33)$$

The rows of the matrix represent equations and the columns represent unknowns. Equation (33) shows only the nonzero elements of the matrix. Notice that the non-zero elements follow a diagonal trend, lying in three adjacent diagonals. This is called a tridiagonal matrix. The values of a, b, c and d are:

$$b_1=1+r_{1+\frac{1}{2}}\frac{\alpha\Delta t}{r_1\Delta r^2} \quad (34)$$

$$c_1=-r_{1+\frac{1}{2}}\frac{\alpha\Delta t}{r_1\Delta r^2} \quad (35)$$

$$d_1=T_1^n+\frac{\alpha\Delta tQ}{2\pi r_1\Delta rlk} \quad (36)$$

For i = 2 to N-1:

$$a_i=-r_{i-\frac{1}{2}}\frac{\alpha\Delta t}{r_i\Delta r^2} \quad (37)$$

$$b_i=1+(r_{i+\frac{1}{2}}+r_{i-\frac{1}{2}})\frac{\alpha\Delta t}{r_i\Delta r^2} \quad (38)$$

$$c_i=-r_{i+\frac{1}{2}}\frac{\alpha\Delta t}{r_i\Delta r^2} \quad (39)$$

$$d_i=T_i^n \quad (40)$$

and,

$$a_N=-r_{N-\frac{1}{2}}\frac{\alpha\Delta t}{r_N\Delta r^2} \quad (41)$$

$$b_N=1+(2r_{N+\frac{1}{2}}+r_{N-\frac{1}{2}})\frac{\alpha\Delta t}{r_N\Delta r^2} \quad (42)$$

$$d_N=T_N^n+2\left(r_{N+\frac{1}{2}}\frac{\alpha\Delta t}{r_N\Delta r^2}\right)T_e \quad (43)$$

The tridiagonal matrix has a very efficient solution procedure called the Thomas algorithm (Kreith and Bown, 2000), since it is used to solve (33).

COMPARISON

To compare empirical and numerical solutions, temperature profiles versus injection time were computed using Ramey Jr (1962) and Hasan and Kabir (1991) and numerical solutions for a case of hot water injection at a surface temperature of 171.11 ºC into a 304.8 m vertical wellbore at a rate of 0.0088 m^3/s through 0.1629696 m casing diameter with overall heat transfer coefficient of 17.034 $W/m^2/K$. The temperatures at the wellbore/formation interface and bottom hole predicted by the models have been presented in Fig. 3 and 4 respectively. There is the good agreement between these solutions, except at early times. It was found that there are mismatchs in the time early because Ramey Jr (1962) and Hasan and Kabir (1991) solutions have been developed based on constant heat transfer rate between the wellbore and formation. However, heat flow between the wellbore and formation will decrease with time, especially at the initial stages of injection whereby the temperature difference between the fluid and the earth is quite large. Ramey Jr (1962) assumed the radius of the wellbore is negligible and the temperature becomes log-linear with time at large times at the wellbore/formation interface; due to it, errors for Ramey Jr (1962) model are larger than Hasan and Kabir (1991) model in early times as shown in Fig. 3 and 4.

Figure 5 presents the temperature distribution in the formation. It can be figured out that assuming unsteady state heat transfer in the formation and the constant temperature versus time at outer boundary of formation are correct.

Further it, Fig. 5 shows the formation temperature changes in small area, only near the wellbore, since it is reasonable to consider 20 (m) or a smaller value for the outer boundary radius of the formation to conduct faster numerical computations. Setting a suitable outer boundary radius of the formation is necessary in software packages programmed on basis of

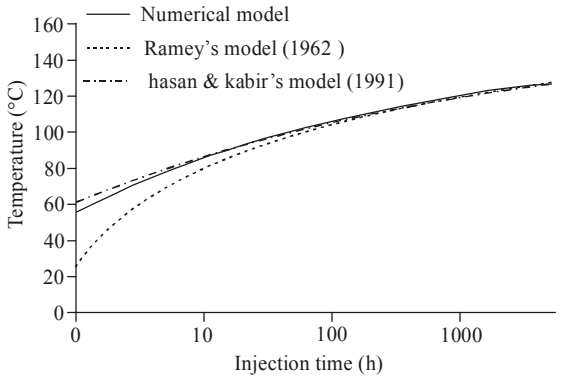

Fig. 3: The temperature at the wellbore/formation interface versus injection time

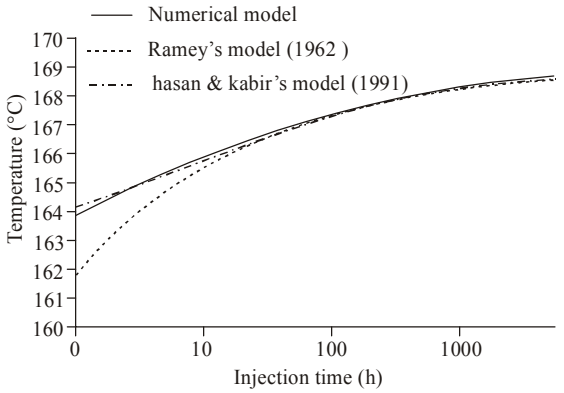

Fig. 4: The fluid flow bottomhole temperature versus injection time

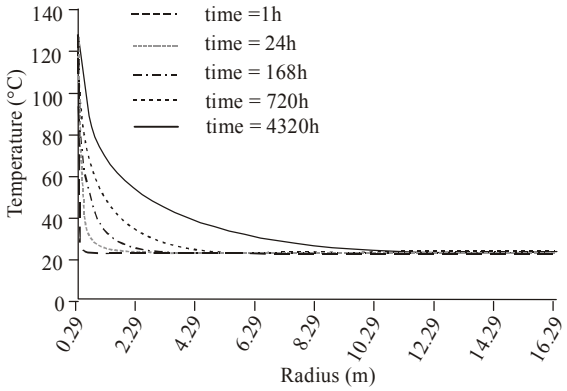

Fig. 5: The temperature distribution in the formation

Computational Fluid Dynamic (CFD) solutions to reduce CPU running time.

Figure 6 illustrates CPU running times for the models. It can be observed that the CPU running times for Ramey Jr (1962) and Hasan and Kabir (1991) models are much shorter than the numerical model, especially in long times because Ramey Jr (1962) and Hasan and Kabir (1994) assumed that the constant heat transfer rate for each time step and the calculations of each time step is independent of the last timestep calculations. However the heat transfer rate is considered variable in

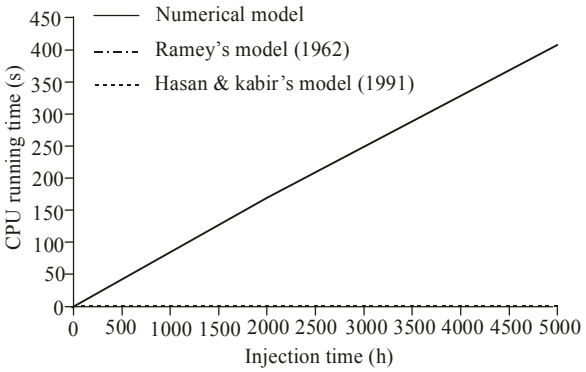

Fig. 6: CPU running time versus injection time

the numerical solution and the calculations of each time step is dependent on the last time step calculations.

Supposedly the numerical solution should be used in the early injection times or when there is a shut in through injection process (e.g., cyclic injection processes). It is more appropriate to use Hasan and Kabir (1991) model for predicting the temperature profile in the long times because it is enough accurate and its speed is faster than the numerical solution as shown in Fig. 3 and 6 respectively.

CONCLUSION

The main conclusions from the present study can be summarised as follows:

- Heat transfer occurs in unsteady state in the formation.
- For modelling heat transfer in the formation, it is recommended to use the numerical solution in the early injection times or when there is a shut in through injection process (e.g., cyclic injection processes) and it had better use Hasan and Kabir (1991) model in the long times because its accuracy is acceptable for petroleum engineering studies and its speed is much faster than the numerical solution.
- The change in the formation temperature is limited to small area, only near the wellbore; thereby it is reasonable to consider 25 (m) or smaller value for the outer boundary radius of the formation to conduct faster CFD and numerical computations.

NOMENCLATURE

A = Coefficient matrix
CFD = Computational fluid thermodynamic
C_e = Heat capacity of the earth (J/(kg/°C))
E_t = Energy content within the element at time t (W)
$E_{t+\Delta t}$ = Energy content within the element at time t+Δt (W)

$f(t)$ = Dimensionless temperature defined by Ramey Jr (1962)
i = Number of element in the radius direction
k_e = Thermal conductivity of earth (W/(m/°C))
l = Length (m)
m = Mass (kg)
n = Number of time step
Q_{gen} = Rate of heat generation inside the element (W)
Q_r = Rate of heat transfer at r (W)
$Q_{r+\Delta r}$ = Rate of heat transfer at r+Δr (W)
r_w = Radius of wellbore (m)
t = Time (s)
T_{cemo} = Temperature at interface of wellbore/formation (ºC)
T_D = Dimensionless temperature defined by Hasan and Kabir (1991)
t_D = Dimensionless time
T_t = Temperature of the earth at time t (°C)
$T_{t+\Delta t}$ = Temperature of the earth at time t+Δt (°C)
$\Delta E_{element}$ = Rate of change of the energy content within the element (W)
Δr = Increment of radius length (m)
Δt = Time interval (s)
α = Thermal diffusivity of the earth (m^2/s)
ρ_e = Density of the earth (kg/m^3)

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