Research Article Development of a Novel Fractional Order Sliding Mode Controller for a Gun Control System

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Abstract: To solve the nonlinearity phenomenon of a Gun Control System (GCS), a novel Fractional order Sliding Mode Control (FoSMC) strategy is proposed in this study. By inducing the fractional order calculus, a Fractional Order PID (FOPID) type sliding surface is especially designed and consequently an equivalent control discipline with fractional order dynamics is induced. The saturation function is employed as the switch function. By numerical simulation, the dynamic characteristics of the FoSMC based control system are analyzed and compared with Conventional Sliding Mode Control (CSMC) system. The results demonstrate that the FoSMC system could reach up to the equilibrium state more smoothly, which shall significantly suppress the inherent chatter effects. Besides, the FoSMC based gun control system is of high response rate, better positioning accuracy and high robustness, which is suitable for fast, smooth and accurate adjustments of the gun.

Keywords: Chattering effects, fractional order calculus, gun control system, sliding mode control

INTRODUCTION

Gun Control Equipment (GCE) is one of the key components of Fire Control Systems (FCSs), the motion stability and the motion accuracy of the gun are widely regarded as two main challenges associated with the developments of GCE with excellent performances (Kumar et al., 2009; Hao et al., 2011; Shen et al., 2011). Controlling the motion capacities of the gun is an ongoing topic in the accelerator community due to certain extremely complicated segments with strong nonlinearities and uncertainties (Feng et al., 2007; Shen et al., 2011; Wang et al., 2011), such as the timevarying parameters induced by the varying working conditions, the external applied loads with random features and the complex friction forces between the cannon and the trunnion, etc. To solve these nonlinear problems, a dominant method is the application of the well-known Proportional-Integral-Derivative (PID) control strategy (Da-Fu, 1997; Liang et al., 2010; Hao al., 2011). However, due to the inherent et nonlinearities existing in gun control systems as mentioned above, it is hard for the linear PID control strategy to achieve excellent control behaviors, resulting in deteriorating dynamic performances of the GCE.

Devote to compensating for the nonlinearities and achieving high robustness, Sliding Mode Controller (SMC) based strategies have recently been introduced in GCEs (Feng et al., 2007, 2010; Ma et al., 2011; Wei et al., 2011). The SMC can guarantee the stability and robustness of nonlinear control systems, which can be systematically achieved but just at the expense of chattering effects. Theoretically, chattering can be made negligible if the width of the boundary layer is chosen large enough, but it would significantly deteriorate the positioning accuracy of control systems (Delavari et al., 2010). Another method for the suppression of chattering effects is to embed certain adaptive tuning methods into the SMC, such as the Neuron Network (NN) based methods (Tsai et al., 2004; Nguyen et al., 2011), the fuzzy reasoning based methods (Feng et al., 2007, 2010; Wai, 2007) and so on. These adaptive schemes would well suppress the chattering and enhance the robustness of control systems, but they are just at expense of response speeds and positioning accuracies. Considering all these factors, the improvement of SMC based control systems could not just depend on embedding adaptive strategies. Recently, Fractional Order Calculus (FOC) theory has been introduced in SMC based control systems and some sort of Fractional order SMC (FoSMC) strategies have been proposed by designing fractional order sliding surfaces or fractional order switching law. For instance, the sliding surface was determined by Fractional Order PID (FOPID) and PI (FOPI) and this sort of FoSMC was

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Fig. 1: The diagram of the AC servo system for GCEs

originally proposed for the control of DC/DC buck converter in Calderón et al. (2006), Efe and Kasnakoğlu (2008) and Efe (2010) respectively designed a fractional sliding surface and an adaptive switching law for the control of a class of MIMO dynamic systems to enhance system performances (Efe and Kasnakoğlu, 2008; Efe, 2010). Furthermore, this sort of FoSMC is applied to chaotic systems with strong nonlinearities and uncertainties (Faieghi et al., 2012). Inspired by conventional adaptive SMC, fuzzy reasoning based adaptive scheme is also incorporated in FoSMC (Delavari *et al.*, 2010; Zhang *et al.*, 2012). The previously obtained results demonstrate that the FoSMC systems would well maintain high robustness and high accuracy and the inherent chattering effects of SMC could also be well attenuated by fractional order operations.

Motivated by this, the FOPID based FoSMC is employed for a gun control system in this study. The performances of the FoSMC system are carefully investigated and compared with Conventional SMC (CSMC) system, further exploring the unique characteristics of FoSMC and verifying the corresponding excellent performances.

METHODOLOGY

Modeling the AC servo system for GCES: The schematic of the AC servo system utilized in a certain sorts of typical GCEs is presented in Fig. 1. Where y_d and y represent the desired angle position and the real angle position of the cannon, respectively. U is the control voltage; K_a is the amplify gain; K_d is the motor torque factor. Td, Td and T_f are the motor torque, load torque disturbance and friction torque disturbance, respectively. R and L represent the resistance and inductance of the motor armature circuit, respectively. Ee is the Counter-Electromotive Force (CEMF) of the motor armature and Ce denotes the CEMF coefficient. J is the total moment of inertia to the rotor; B is the viscous friction coefficient; ω_d is the angular velocity of the motor, i is the moderating ratio and s denotes the Laplace operator.

The total moment of inertia J, the viscous friction coefficient B, the load torque disturbance T_L and friction torque disturbance T_f all contain obvious uncertainties and present strong time-varying nonlinearities with the working condition changes during the working process.

Generally, the current time constant is much smaller than the mechanical time constant, thus, the delay of the current response can be neglected and it yields:

$$\frac{1}{Ls+R} = \frac{1}{R} \frac{1}{Ls/R+1} \approx \frac{1}{R}$$
(1)

The motor torque T_d is given as follows:

$$T_d = -\frac{K_d C_e}{R} \omega_d + \frac{K_d K_a}{R} U$$
⁽²⁾

According to the equilibrium equation of the torques, we can obtain:

$$T_d - T_L - T_f = Ji\ddot{\beta} + Bi\dot{\beta} \tag{3}$$

Substituting Eq. (2) into (3) yields:

$$li\ddot{\beta} + Bi\dot{\beta} = -\frac{K_d C_e}{R}\omega_d + \frac{K_d K_a}{R}U - T_L - T_f$$
(4)

Multiply the two sides of Eq. (4) by the moderating ratio 1/i, the govern principle of the AC servo system can be obtained:

$$\ddot{\beta} = -\left(\frac{B}{J} + \frac{K_d C_e}{JR}\right)\dot{\beta} + \frac{K_d K_a}{iJR}U - \frac{T_L + T_f}{iJ}$$
(5)

We define the status variables as $X = [x_1 \ x_2]^T$ and set $x_1 = y, x_2 = \dot{y}$. The status equation can be written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(X, t) + gu(t) + d(t) \end{cases}$$
(6)

where,

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$$f(X,t) = -(\frac{B}{J} + \frac{K_d C_e}{JR})x_2, \ g = \frac{K_d K_a}{iJR}, \ d(t) = -\frac{T_L + T_f}{iJ}$$

f (X, t) and g denote the nonlinear dynamic equations and the control gain, respectively. d (t) is the external bounded disturbance, that is $|d(t)| \le C$ and C is a constant.

In working conditions, the desired state of the system could be defined as:

$$X_{d} = [x_{d1} \ x_{d2}]^{T} \in \mathbb{R}^{2}$$
(7)

The tracking error of the gun control system E (t) $\in R^2$ could be defined as:

$$E(t) = [e_1 \ e_2]^T = X_d - X$$
(8)

A preliminary to FOC: Fractional order calculus is a generalization of the conventional integration and differentiation to non-integer orders with the fundamental operator $_{t_0}D_t^{\alpha}f(t)$, which is defined as Monje *et al.* (2010), Zhiwei (2011) and Zhu *et al.* (2012):

$$_{t_0} D_t^{\alpha} f(t) = \begin{cases} d^{\alpha} / dt^{\alpha}, \operatorname{Re}(\alpha) > 0\\ 1, \qquad \operatorname{Re}(\alpha) = 0\\ \int_{t_0}^t (d\tau)^{\alpha}, \operatorname{Re}(\alpha) < 0 \end{cases}$$
(9)

where,

 $t_0 \& t$: The limits of the operation, respectively

 α : The order and $\alpha \in R$, but α could also be a complex number

Generally, there exist several well-known definitions of FOC operations including the Grunwald-Letnikov (G-L) definition, the Riemann-Liouville (R-L) definition and so on Monje *et al.* (2010), Zhiwei (2011) and Zhu *et al.* (2012). Attributing to the discrete features of G-L definition, it is employed to directly carry out the numerical computation of fractional order operations, which can be given as:

$${}_{t_0} D_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[(t-t_0/h)\right]} (-1)^j \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-j+1)\Gamma(j+1)} f(t-jh)$$
(10)

where,

 $\Gamma(\cdot)$ = The Gamma function h = The calculation step

To conduct the operation more conveniently in control process, an iteration formation of the G-L definition could also be expressed as Zhou *et al.* (2010):

$$\begin{cases} \int_{t_0} D_t^{\alpha} f(t) = h^{-\alpha} \sum_{j=0}^{[(t-t_0/h)]} b_j f(t-jh) \\ b_0 = 1, b_j = [1-(1+\alpha)/j] b_{j-1} \end{cases}$$
(11)

For zero initial conditions, the Laplace transform of G-L definition can be written as:

$$L[_{0}D_{t}^{\alpha}f(t)] = s^{\alpha}L[f(t)] = s^{\alpha}F(s)$$
(12)

Fractional order sliding mode controller: Design of the fractional order sliding surface: A typical PID sliding surface can be expressed as:

$$S(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{de(t)}{dt}$$
(13)

By introducing the FOC, the fractional order PID sliding surface can be expressed as:

$$\begin{cases} S(t) = k_{p}e(t) + k_{i}D^{-\lambda}e(t) + k_{d}D^{\mu}e(t) \\ 0 < \lambda < 1, \qquad 0 < \mu < 1 \end{cases}$$
(14)

where,

- k_p, k_i, & k_d: The proportional gain, integral gain and derivative gain, respectively
- $\lambda \& \mu$: The order of an integrator and a differentiator, respectively

e (t) : The tracking error at time t

The control objective can now be achieved by choosing the control input such that the sliding surface satisfies the following sufficient condition:

$$SS < -\eta \left| S \right| \tag{15}$$

where, η is a positive constant. To guarantee the stability, the energy of *S*(*t*) should decay towards zero. All trajectories are seen improved and approach to the sliding surface in a finite time and will stay on the surface for all future times. *S*(*t*) is called the sliding surface. Once the behavior of the system is settled on the surface, is called the sliding mode \dot{S} (t) = 0 is happened.

Taking the time derivative from both sides of Eq. (14), yielding:

$$\dot{S}(t) = k_p D e(t) + k_i D D^{-\lambda} e(t) + k_d D D^{\mu} e(t)$$
(16)

Combining the definition of tracking errors in Eq. (8), the Eq. (16) could be written as:

$$\dot{S}(t) = k_{p} D e_{1}(t) + k_{i} D^{1-\lambda} e_{1}(t) + k_{d} D^{\mu-1} \dot{e}_{2}(t)$$
(17)

Using $\dot{S}(t) = 0$ one can obtain:

$$D^{\mu-1}u_{eq}(t) = k_d^{-1}g^{-1}\{k_p De_1 + k_i D^{1-\lambda}e_1 + g^{-1}D^{\mu-1}[f(x_{d2}) - f(x_2) - d(t)]\}$$
(18)

Then, the equivalent input control signal could be obtained as:

$$u_{eq}(t) = k_d^{-1} g^{-1} [k_p D^{2-\mu} e_1 + k_i D^{2-\lambda-\mu} e_1] + g^{-1} [f(\dot{x}_{d1}) - f(\dot{x}_1) - d(t)]$$
(19)

Take advantage of Eq. (11), the digital control signal could be obtained:

$$u_{eq}(k) = k_d^{-1} g^{-1} [k_p h^{\mu-2} \sum_{j=0}^{(t/h)} p_j e_1(k-1) + k_i h^{\lambda+\mu-3} \sum_{j=0}^{(t/h)} q_j e_1(k-1)]$$
(20)
+ $g^{-1} [f(\dot{x}_{d1}(k)) - f(\dot{x}_1(k)) - d(k)]$

First, due to systems with memory are typically more stable and less sensitive than their memoryless counterpart, fractional order differential equations are, at least, as stable as their integer order counterpart (Zhiwei, 2011; Zhang *et al.*, 2012). Besides, as for conventional SMC, the system shall be convergent to the equilibrium state by the law of exp (t), while the FoSMC system shall be convergent to the equilibrium state by the law of t^{T} , where r depends on the equivalent order of the control system (Zhang and Pi, 2012; Zhang *et al.*, 2012). By selecting suitable value of the system order, the FoSMC system can possess smaller chatter layer and transfer energy by a more smooth way, which will attenuate the chattering effects. Take advantage of this sort of convergence, the FoSMC can well suppress the inherent chatter effects of sliding motions.

The improved switching strategy: To guarantee that the state trajectory can converge to the sliding surface, the corrective control should be employed and it is generally defined as:

$$u_{vs}(t) = -\Theta \operatorname{sgn}(S) \tag{21}$$

where, k is a positive constant and the sign function is a discontinuous function as follows:

$$\operatorname{sgn}(S)\begin{cases} 1, & S(k) > 0\\ 0, & S(k) = 0\\ -1, & S(k) < 0 \end{cases}$$
(22)

The discontinuous feature may cause high frequency oscillations, which is defined as chattering. The chattering effects are undesired due to the reason that it may excite the high frequency response of the system. Basically, the common methods to eliminate the chattering are usually adopting the following:

- Using a saturation function (Delavari *et al.*, 2010)
- Inserting a boundary layer so that an equivalent control can replace the corrective one when the system is inside the boundary layer. This method can give a chattering-free system, but a finite steady-state error would exist (Tsai *et al.*, 2004). Hence, to eliminate the chattering, a saturation function is employed in this study, which could be defined as:

$$\operatorname{sat}(S/\Phi) = \begin{cases} \operatorname{sgn}(S/\Phi), & |S/\Phi| \ge 1\\ S/\Phi, & |S/\Phi| < 1 \end{cases}$$
(23)

where, Φ is the width of boundary layer. The saturation function could significantly reduce the chattering effects, but a compromise should be made between small chattering and fine tracking precision in presence of parameter uncertainties. During the determination of the controller parameters, the goal is to seek a control signal that will minimize the tracking error and the time that the system state trajectory needs to reach the sliding surface.

SIMULATION RESULTS AND DISCUSSION

Simulation conditions: To investigate the performances and the unique characteristics of the proposed FoSMC based control strategy, a series of numerical simulations are conducted in this study. As for the simulation process, the parameters of the gun control system are chosen as follows:

$$\begin{split} J &= 0.0352 Kg \cdot m^2, \ K_d = 0.195 N \cdot m / A, \\ C_e &= 0.195 V / (rad \cdot s^{-1}), \ i = 315, \ R = 0.07 \Omega \\ B &= 0.000143 N \cdot m / (rad \cdot s^{-1}) \end{split}$$

As for the fractional order sliding surface, the orders are set as: $\lambda = 1/3$, $\mu = 1/2$; the gains are set as $k_p = 6$, $k_d = 5$ and $k_i = 3.2$. It should be noticed that the parameters are just chosen to simply investigate the characteristics of the FoSMC system. To achieve better performances, more elaborate methods or certain adaptive tuning methods should be employed to optimally determine these parameters, which shall be detailed in our future study. If we set the orders as $\lambda = \mu = 1$, then the Conventional PID based SMC (CSMC) could be obtained.

Performance of the FoSMC system:

Step responses of the FoSMC system: To investigate the converging characteristics and the chattering effects of FoSMC system, step response of the FoSMC based gun control system is performed in this study. In order to verify the superiority of FoSMC, the corresponding results are compared with the results obtained by CSMC. The responses are illustrated in Fig. 2a and the corresponding control voltages are further presented in Fig. 2b. The converging trajectories of the sliding surfaces S are illustrated in Fig. 3a. For sake of clarity, the zoom fit results of the trajectories are further presented in Fig. 3b.

As is evident from the results shown in Fig. 2a, the response time of CSMC system is about 0.8s, while it of the FoSMC is about 0.35s which is less than half of that of CSMC. The results show that the FoSMC could response more rapidly to the commands than CSMC. From the control voltages given in Fig. 2b, the Peak-to-Valley (PV) value of the chattering voltage of FoSMC system is about 0.25 V, which is about 40% of that of CSMC system. The results demonstrate that the FoSMC can significantly suppress the inherent chattering phenomenon of CSMC and a more smooth control



(a) Step responses of the servo system



(b) Control voltage for the step response process

Fig. 2: Step response process of the servo system



(a) Schematic of converging trajectories of sliding surfaces



(b) Zoom fit of the converging trajectories



process could be achieved. From the convergent trajectories of the sliding surfaces as shown in Fig. 3,



Fig. 4: Tracking errors of the servo system



Fig. 5: Converging trajectories of the corresponding sliding surfaces

the chattering phenomenon is just occurred when reaching the equilibrium state of the control system and the amplitude is far less than CSMC. The results suggest that the FoSMC system could reach up to the equilibrium state more smoothly and possess more excellent dynamics features than CSMC.

Positioning accuracy of the FoSMC system: To investigate the trajectory tracking performances of the gun control system, a harmonic wave with 40 mil amplitude and 0.5 Hz frequency is employed as the command signal. The tracking errors of the FoSMC and the CSMC based control systems are illustrated in Fig. 4. The converging trajectories of the corresponding sliding surfaces are presented in Fig. 5. From the tracking errors shown in Fig. 3, the steady error of the FoSMC system is about $\pm 0.63\%$ of the full span, while the error of the CSMC system is about $\pm 1.25\%$ of the full span, which is about twice of that of the FoSMC system. The results demonstrate that more accurate positioning performances could be achieved by employing FoSMC.

As is evident from the converging trajectories of the sliding surfaces, the FoSMC possesses smooth features when reaching the equilibrium state, whereas, obvious chatter phenomenon is observed during the converging process of the CSMC based control system and the chatter becomes stronger when getting closer to the equilibrium state. The results clearly verify that more stable and smooth positioning process could be obtained by employing FoSMC.



Fig. 6: Step responses with external disturbances

Robustness performance rejecting external load disturbance: In order to validate the robustness of the proposed FoSMC based gun control system, step responses (80 mil) with external load disturbance are investigated. Harmonic external load disturbance is selected as follows: d (t) = 2 sin (π t) mil. The positioning errors of the FoSMC and the CSMC system are illustrated in Fig. 6. From Fig. 6, it can be seen that the FoSMC system can well suppress external disturbances and present more robust characteristics comparing with CSMC.

CONCLUSION

A Fractional order Sliding Mode Control (FoSMC) scheme has been proposed and studied for a gun control system in this study. The principle of the AC servo system for a gun control system is introduced first. Then, a fractional order PID type sliding surface is designed for the sliding mode controller and consequently an equivalent control discipline with fractional order dynamics is introduced. The saturation function is employed as the switch function. The unique characteristics of the FoSMC system is analyzed and interpreted. By numerical simulation, the performances of FoSMC system including chatter suppression, positioning accuracy and system robustness are investigated and compared with Conventional Sliding Mode Control (CSMC) system. The results demonstrate that the FoSMC can well attenuate the inherent chatter effects of CSMC system, more accurate positioning performances and better robustness features can simultaneously be obtained by employing FoSMC.

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